MA2011: W3 DC motors

```
date
```

```
ans = 
'01-Nov-2020'
```

State-Space Representation

The order of a system depends on the highest derivative present, in mechanical systems we typically have second-order derivatives due to the acceleration in Netwon's law $f(t) = m\ddot{x} + b\dot{x} + kx$ which we can rewrite simply as

$$\ddot{x} = \frac{-b\dot{x} - kx + f(t)}{m}$$

However, a different way to present a system is to augment its state, e.g. define

$v := \dot{x}$ which also means $\dot{v} = \ddot{x}$

and rewrite Newton's law as a pair of equations

- $\dot{x} = v$
- $\dot{v} = \dfrac{-bv - kx - f(t)}{m}$

or in matrix format, by defining an augmented state vector $Y = [x \ \ v]^T$

$$\frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix}\begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} f(t)$$

$$\frac{d}{dt}X = A.X + B.f(t)$$

```
syms x(t) v(t) f(t)
syms m b k real

% augmented state
X = [x; v]
dXdt = diff(X)
A = [0 1; -k/m -b/m]
B = [0; 1/m]
myNewton = dXdt == A* X + B*f
```

Y(t) =

$$\begin{pmatrix} x(t) \\ v(t) \end{pmatrix}$$

dYdt(t) =

$$\begin{pmatrix} \frac{\partial}{\partial t}\, x(t) \\ \frac{\partial}{\partial t}\, v(t) \end{pmatrix}$$

```
A =
```

$$\begin{pmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{pmatrix}$$

```
B =
```

$$\begin{pmatrix} 0 \\ \frac{1}{m} \end{pmatrix}$$

```
myNewton(t) =
```

$$\begin{pmatrix} \frac{\partial}{\partial t}\, x(t) = v(t) \\ \frac{\partial}{\partial t}\, v(t) = \frac{f(t)}{m} - \frac{b\, v(t)}{m} - \frac{k\, x(t)}{m} \end{pmatrix}$$

numerical analysis - try: help ss

```
m=1, b=0.1, k=1
```

```
m = 1
b = 0.1000
k = 1
```

```
A = [0 1; -k/m -b/m]
```

```
A = 2×2
         0    1.0000
   -1.0000   -0.1000
```

```
B = [0; 1/m]
```

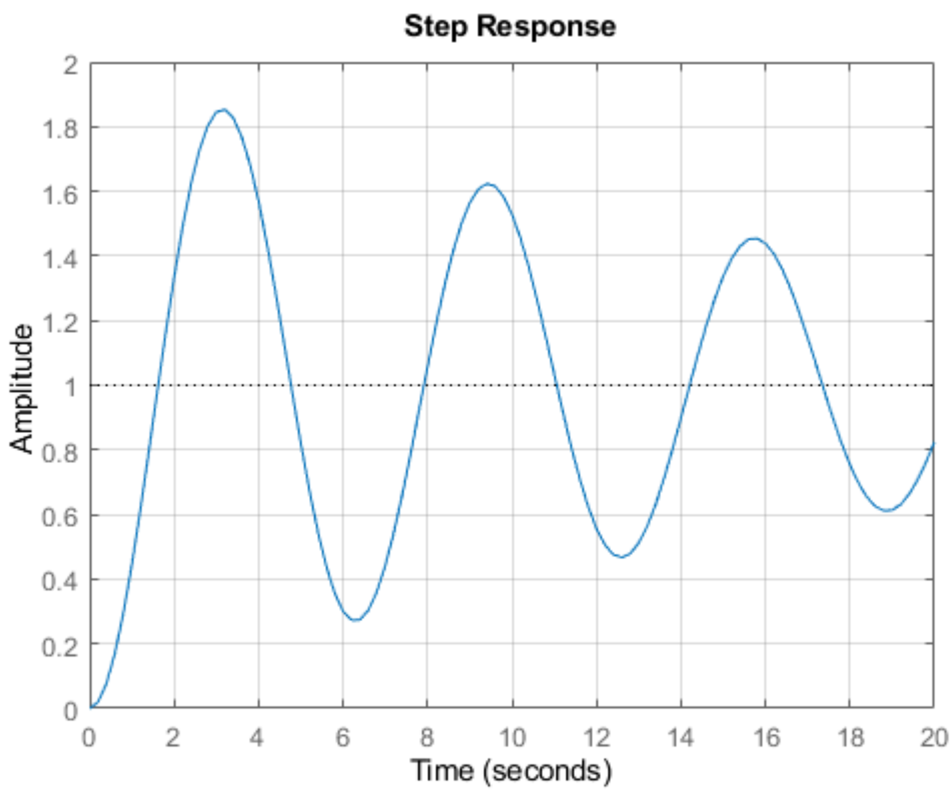
```
B = 2×1
     0
     1
```

```
mySystem = ss(A, B, [1 0], 0)
```

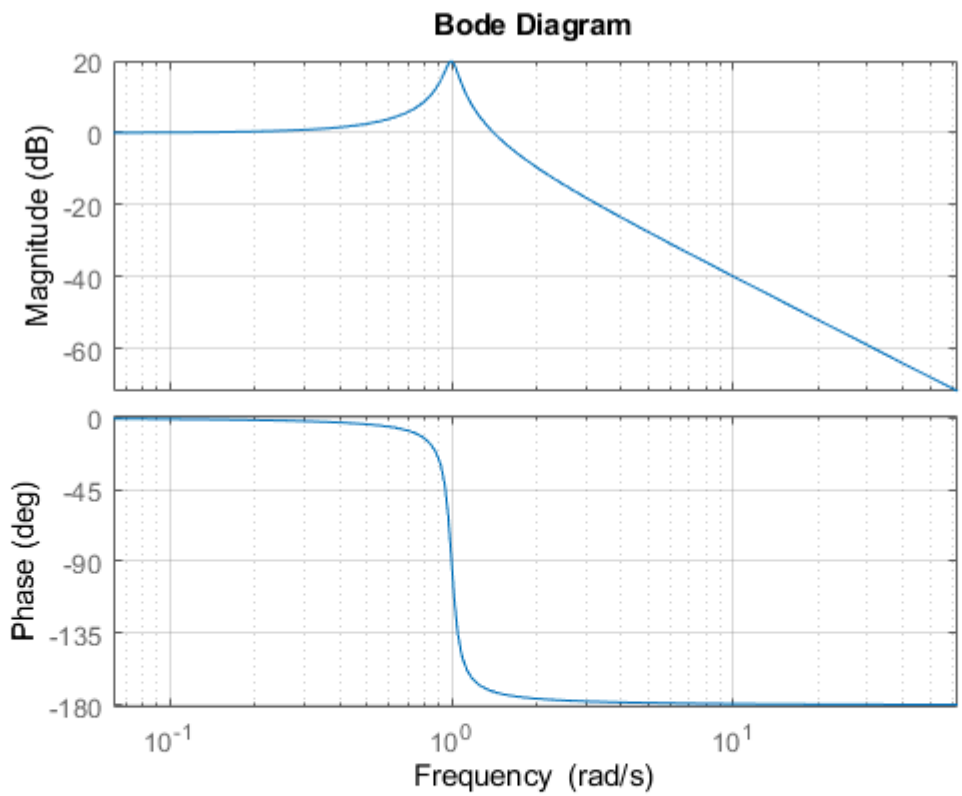
```
mySystem =

  A =
         x1    x2
   x1     0     1
   x2    -1  -0.1

  B =
        u1
   x1    0
   x2    1
```

```
C =
      x1  x2
  y1   1   0

D =
      u1
  y1   0

Continuous-time state-space model.
```

```
figure
step(mySystem, 20); grid on
```




```
figure
bode(mySystem, {2*pi*1e-2, 2*pi*10}); grid on
```



Block Diagram Representation

a general ODE written as

```
eqn_Newton
```

eqn_Newton =

$$f(t) = m\,\frac{\partial^2}{\partial t^2}\,x(t) + b\,\frac{\partial}{\partial t}\,x(t) + k\,x(t)$$

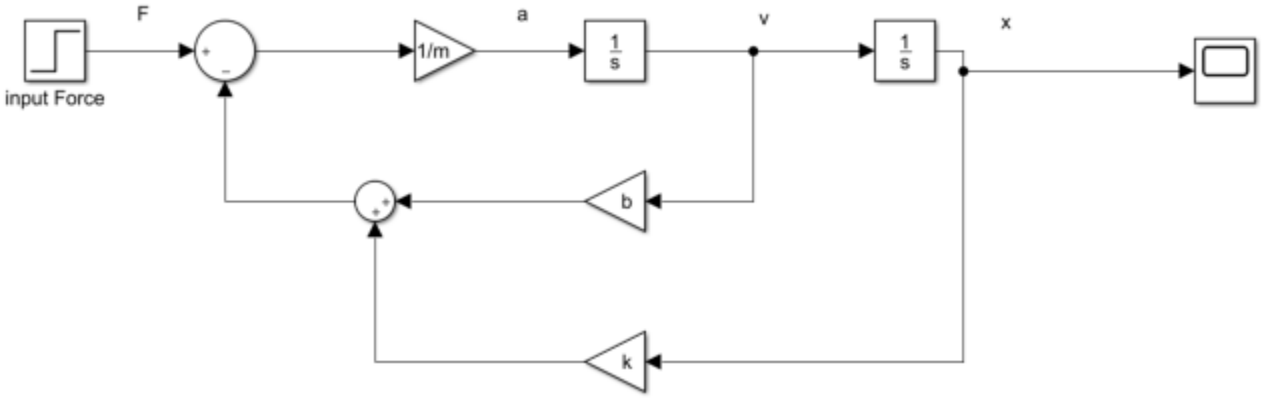
```
m =1, b = 0.1, k = 1
```

```
m = 1
b = 0.1000
k = 1
```

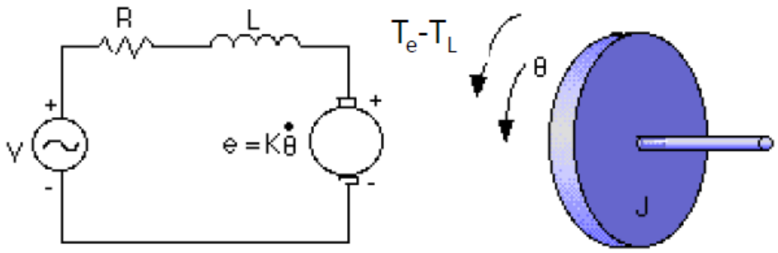
a = 1/m * (F(t) - b*v - k*x)

m=1, b=1, k=1

v = dx / dt
a = dv / dt



DC Motor - State-Space representation



$$u \rightarrow \boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}} \rightarrow y$$

$$\begin{cases} V = Ri + L\dfrac{di}{dt} + K_a\,\omega \\ J\,\dfrac{d\omega}{dt} + b\,\omega = T_m + K_a\,i \end{cases}$$

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \omega\\ i\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & -\frac{b}{J} & \frac{K_a}{J}\\ 0 & -\frac{K_a}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\theta\\ \omega\\ i\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{1}{L}\end{bmatrix}V-\begin{bmatrix}0\\ \frac{1}{J}\\ 0\end{bmatrix}T_m$$

Analog driving

```
% PHYSICAL PARAMETERS
J = 3.2284E-6 % rotor inertia
```

```
J = 3.2284e-06
```

```
    b = 3.5077E-6 % mech. damping
```

```
b = 3.5077e-06
```

```
Ke = 0.0274 % e.m.f constant
```

```
Ke = 0.0274
```

```
R = 4 % electric resistance
```

```
R = 4
```

```
L = 2.75E-6 % electric inductance
```

```
L = 2.7500e-06
```

```

%% PLANT STATE SPACE
A=[ 0 1 0;
    0 -b/J  Ke/J;
    0 -Ke/L -R/L;];
B=[ 0; 0; 1/L];

%% EIGENVALUES
eig(A)
```

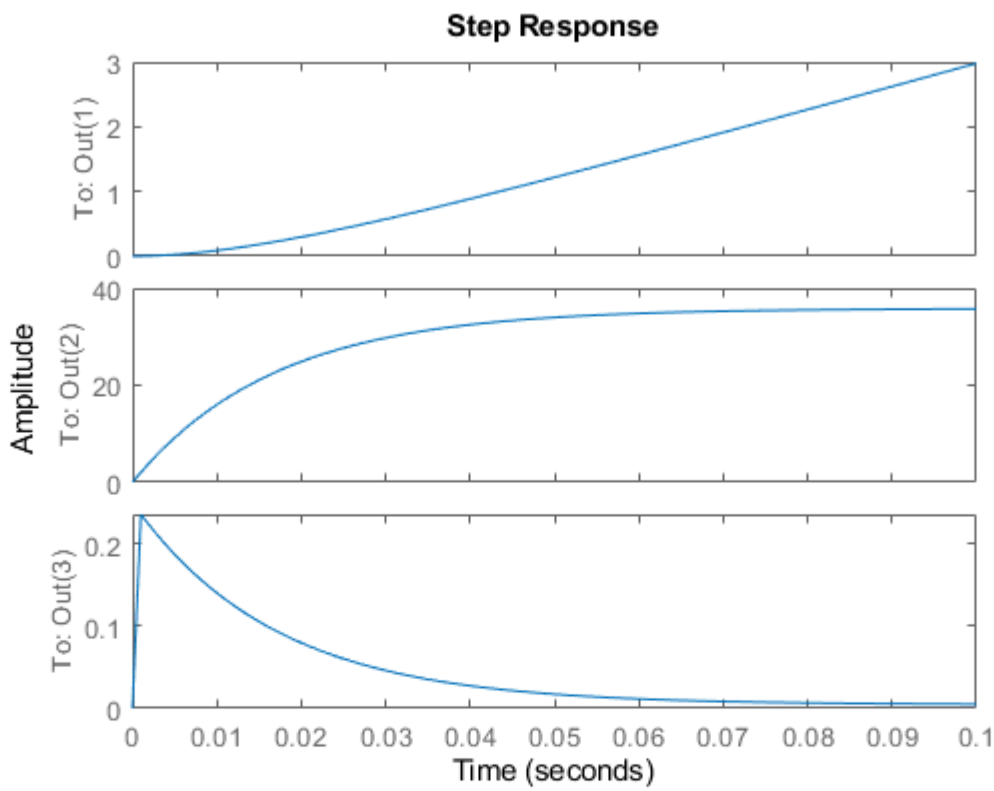
```
ans = 3×1
10^6 ×
        0
  -0.0001
  -1.4545
```

```

C=eye(3); % y=[theta; omega; i];
myDCM = ss(A,B,C,0)
```

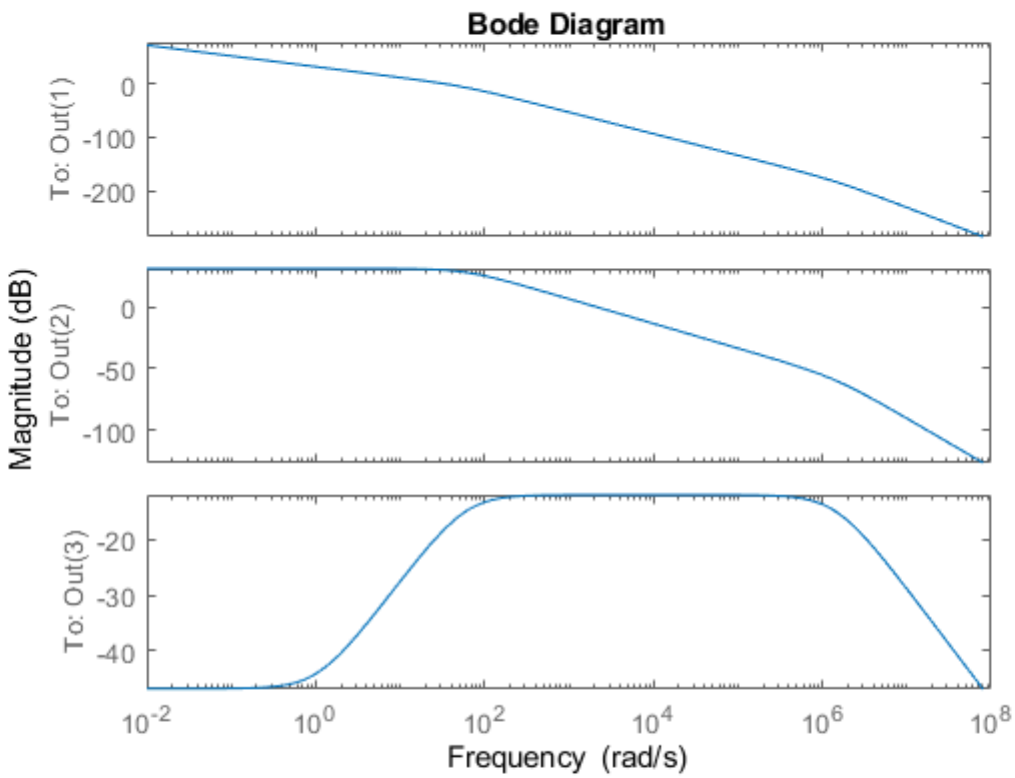
```
myDCM =

  A =
                x1          x2          x3
   x1            0           1           0
   x2            0      -1.087        8487
```

```
  x3            0       -9964   -1.455e+06

B =
            u1
  x1         0
  x2         0
  x3  3.636e+05

C =
      x1  x2  x3
  y1   1   0   0
  y2   0   1   0
  y3   0   0   1

D =
      u1
  y1   0
  y2   0
  y3   0

Continuous-time state-space model.
```

```
%% STEP
step(myDCM, 0.1)
```

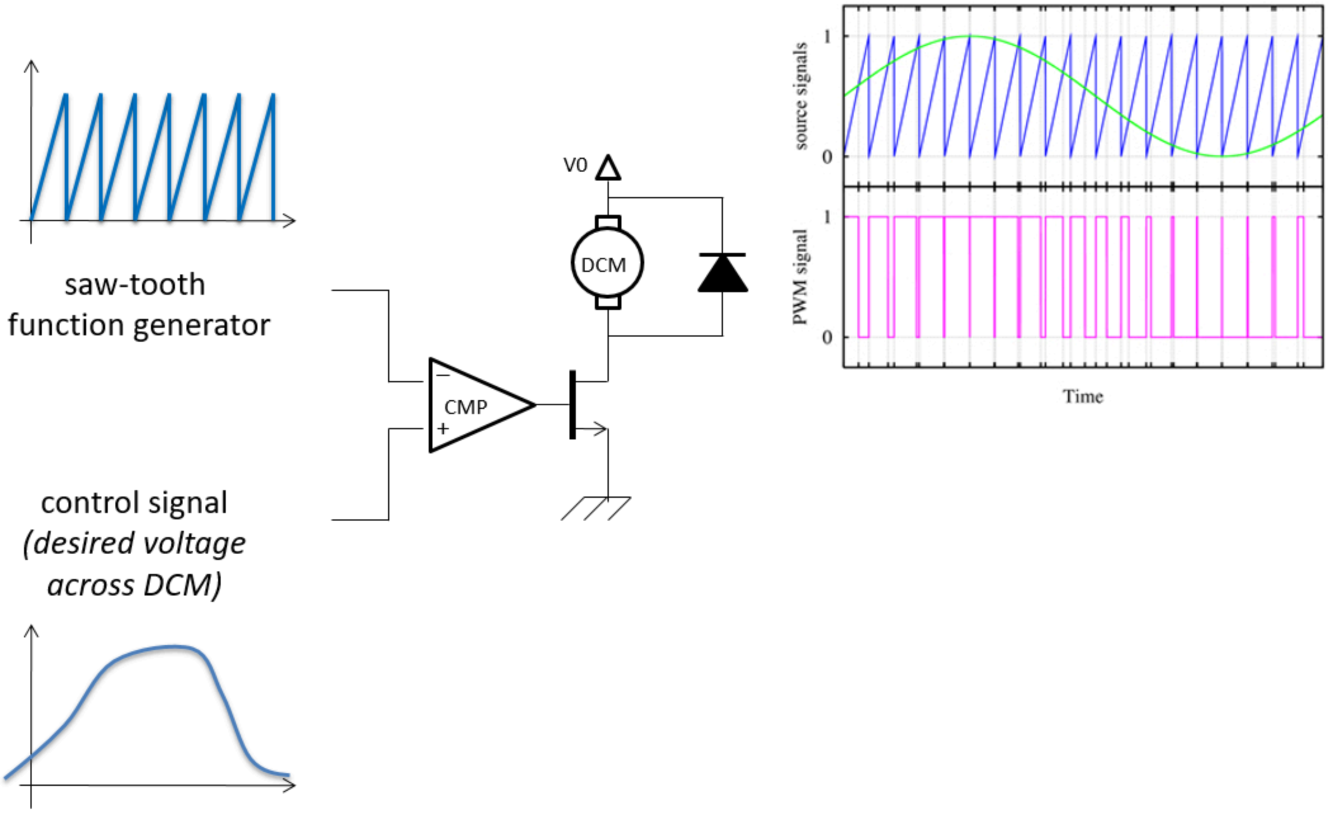



```
%% Bodel plot
bodemag(myDCM)
```



PWM: Pulse Width Modulation

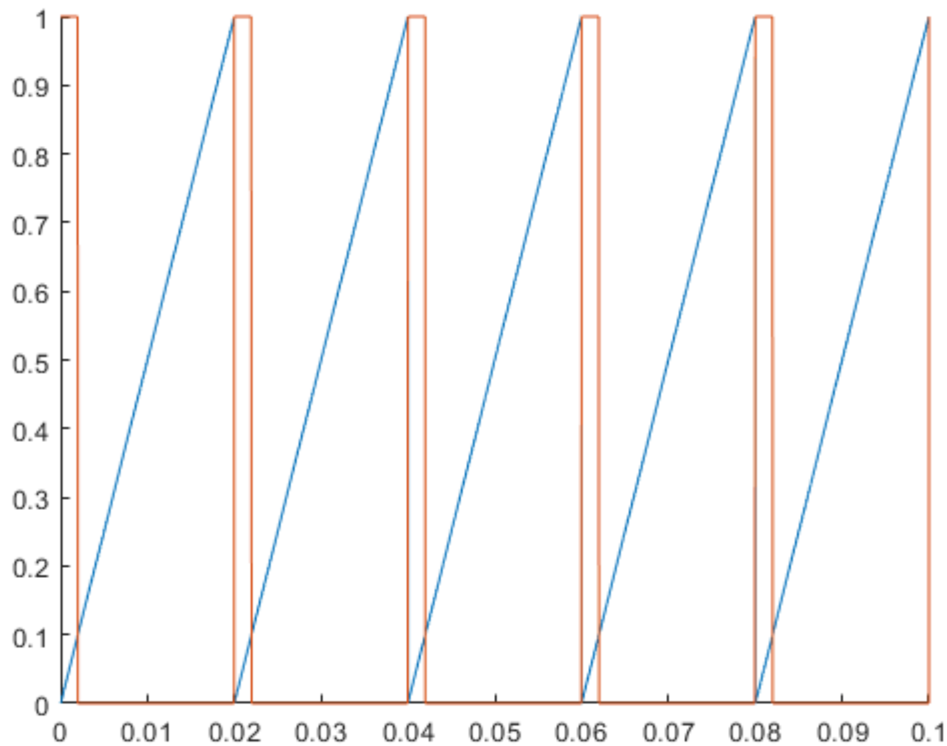
Driving loads such as a DC motor or a heater (which is basically just a high-power resistor) with analog devices, such as a transitor in active region, is usually very inefficient.



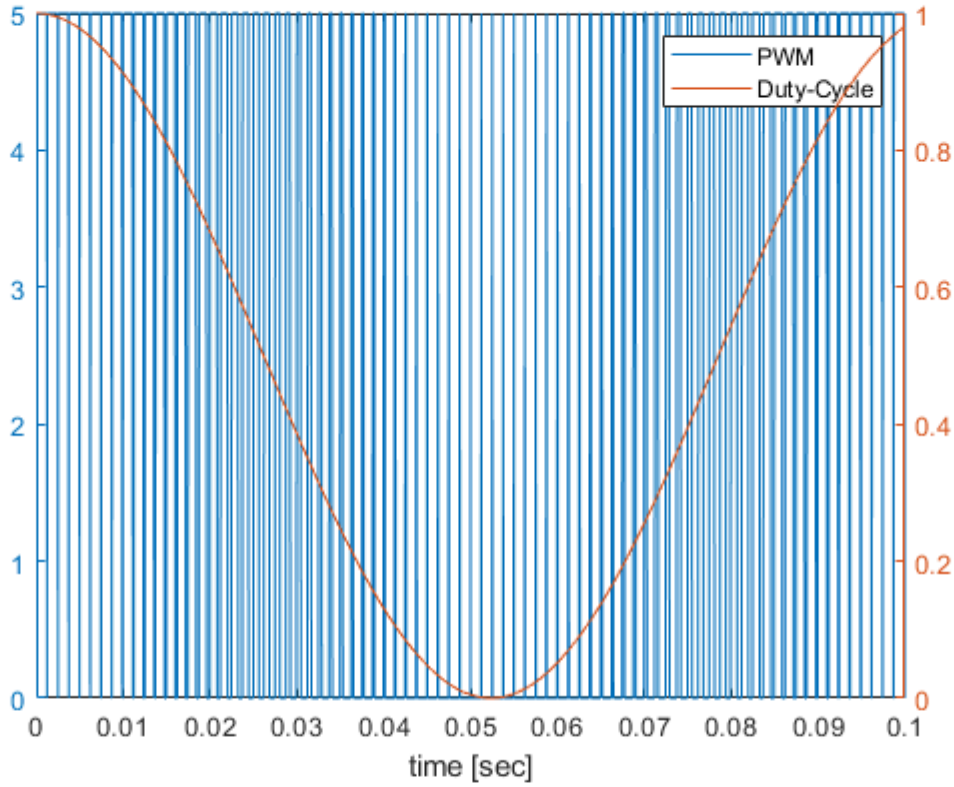

```
t = 0: 1e-6 : 0.1;

PWM = @(duty, f0, time) duty > mod(f0*time , 1)      %% this function generates a PWM
```

```
PWM = function_handle with value:
    @(duty,f0,time)duty>mod(f0*time,1)
```

```
figure
hold on
f0 = 50;
plot(t, mod(f0*t, 1))
plot (t, PWM(.1, f0, t))
```

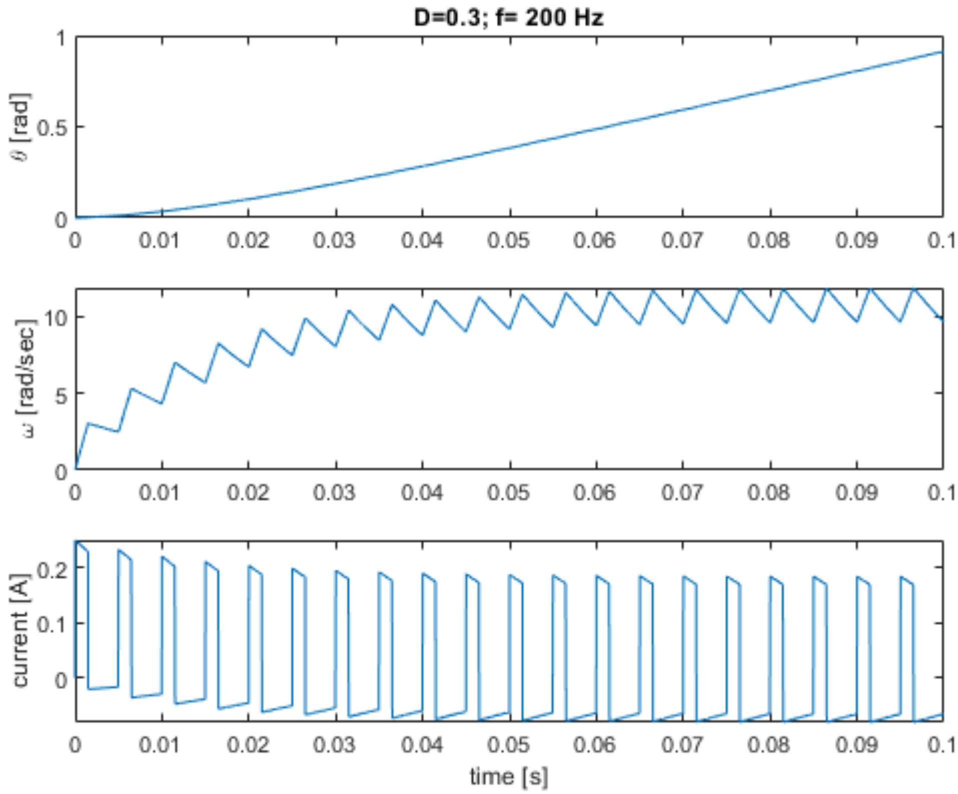



```
figure
modulating_signal = cos(30*t).^2;
plotyy(t, 5*PWM(modulating_signal , 800, t), t, modulating_signal )
legend('PWM', 'Duty-Cycle')
xlabel('time [sec]')
```

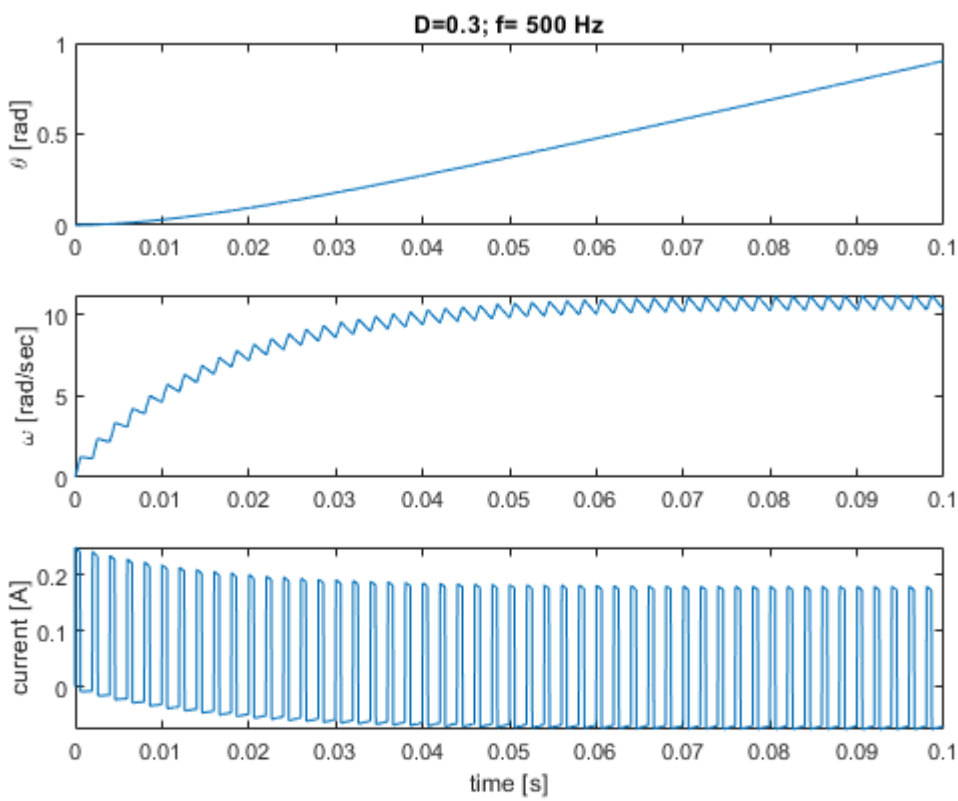



```
% now use the PWM to drive your motor
```

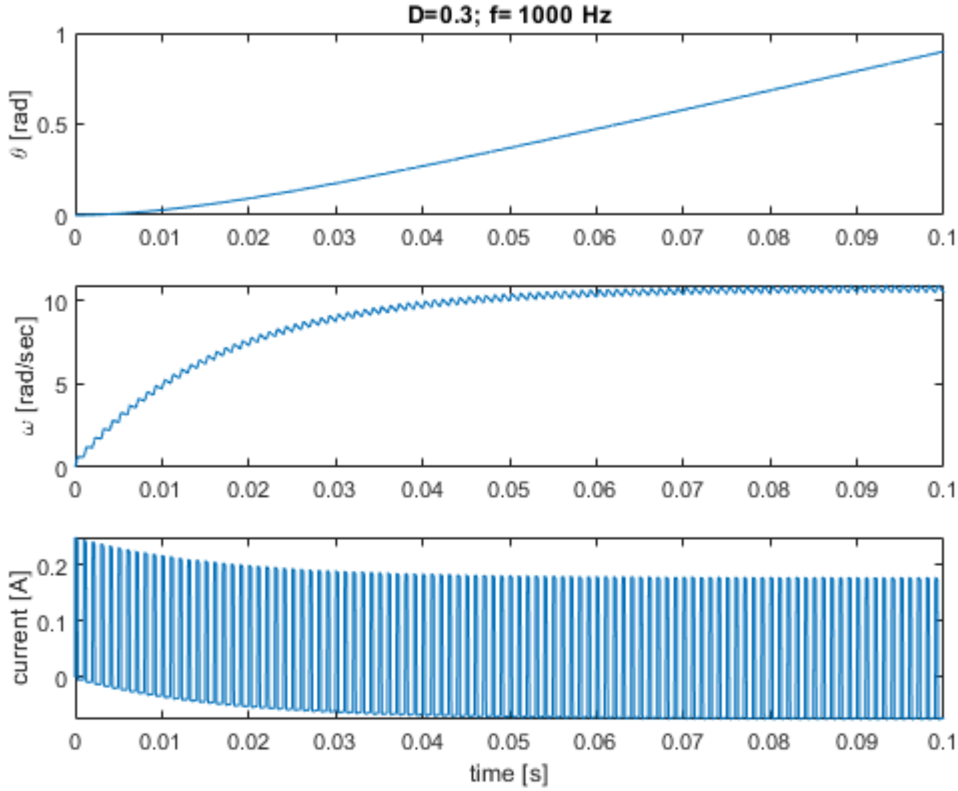
```
for freq= [200 500 1000]
    duty = 0.3;
    y= lsim(myDCM, 1*PWM(duty, freq, t) ,t);
    theta = y(:,1);
    omega = y(:,2);
    current = y(:,3);
figure;
    subplot(3,1,1)
    plot(t, theta); ylabel('\theta [rad]')
    title(['D=' num2str(duty) '; f= ' num2str(freq) ' Hz'])
    subplot(3,1,2)
    plot(t, omega); ylabel('\omega [rad/sec]')
    subplot(3,1,3)
    plot(t, current); ylabel('current [A]')
    xlabel('time [s]')
end
```



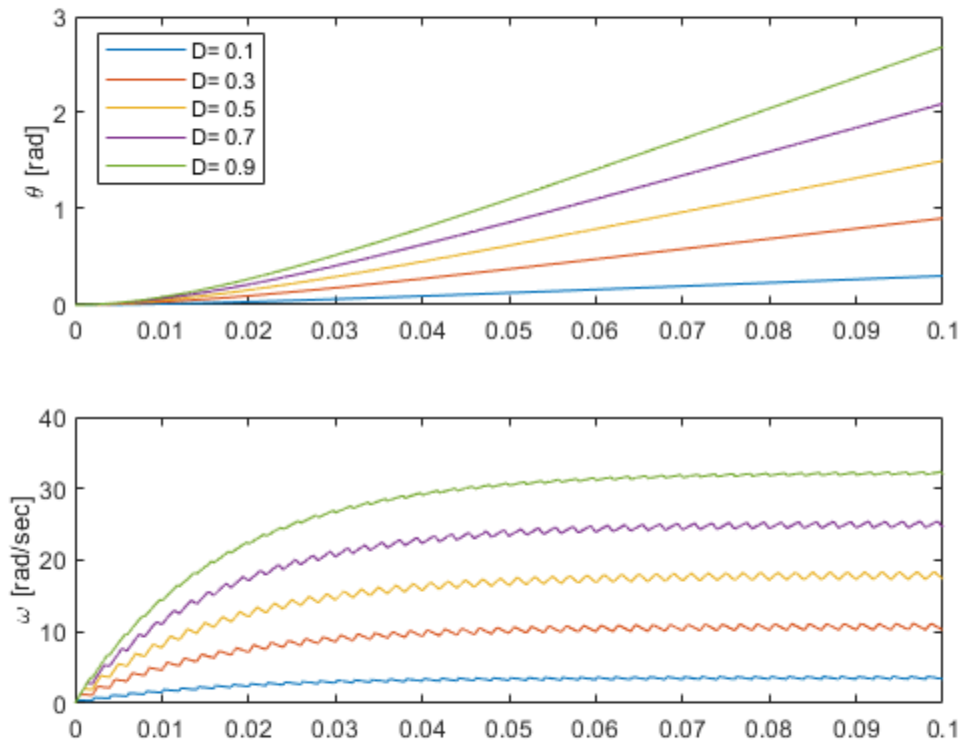
D=0.3; f= 200 Hz
θ [rad]
ω [rad/sec]
current [A]
time [s]



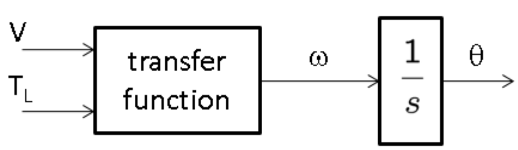
D=0.3; f= 500 Hz
θ [rad]
ω [rad/sec]
current [A]
time [s]



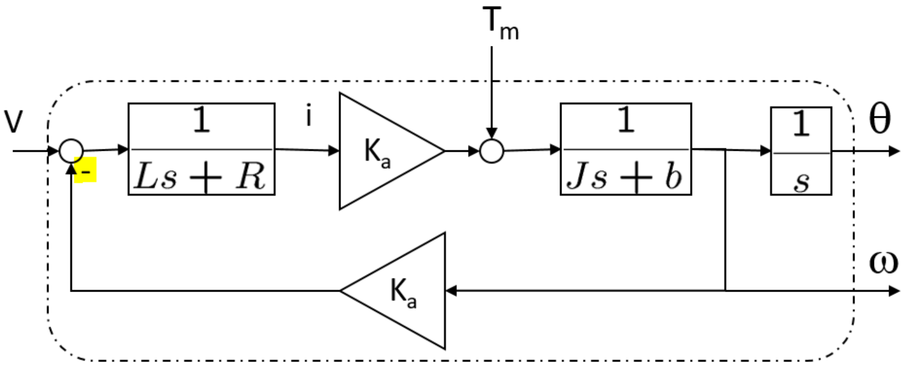

```
figure
myLegend = {};
for duty= [.1 .3 .5 .7 .9]
    myLegend = {myLegend{:}, ['D= ' num2str(duty)]};
    y= lsim(myDCM, 1*PWM(duty, 500, t) ,t);
    theta = y(:,1);
    omega = y(:,2);
    current = y(:,3);
    subplot(2,1,1)
    plot(t, theta); ylabel('\theta [rad]')
    hold on
    legend(myLegend, 'Location','northwest')
    subplot(2,1,2)
    plot(t, omega); ylabel('\omega [rad/sec]')
    hold on
end
```



DC Motor: block diagram



$$\begin{cases} V = Ri + L\dfrac{di}{dt} + K_a\,\omega \\ J\,\dfrac{d\omega}{dt} + b\,\omega = T_m + K_a\,i \end{cases} \xrightarrow{\text{Laplace}} \begin{cases} V - K_a\,\omega = (R + Ls)\,i \\ (Js + b)\,\omega = T_m + K_a\,i \end{cases}$$



T_m
V
$\frac{1}{Ls+R}$
i
K_a
$\frac{1}{Js+b}$
$\frac{1}{s}$
θ
ω
-
K_a