NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

MA3005 Control Theory

Tutorial 9

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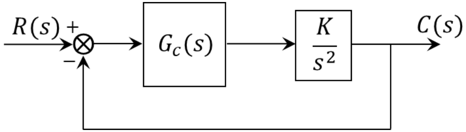
Question 1

Consider the following model of a spacecraft

I. Determine the unit-step time response of the system for $G_c(s) = 1$ and $K = 1$. Is it possible to tune the gain K to obtain a stable system?

II. Design the following controllers to give a pair of dominant closed-loop poles at $s = -1 \pm j$

- Proportional-Derivative controller
- Lead Compensator



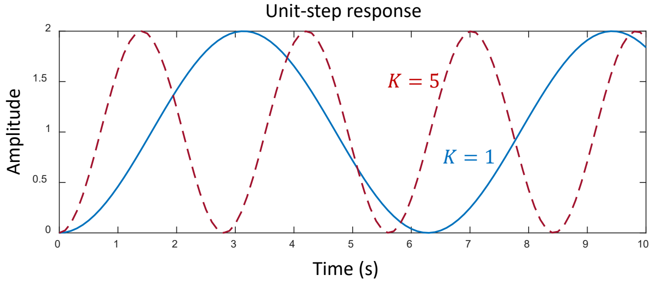
I. For $G_c(s) = 1$,

$$\text{CLTF:} \quad \frac{C(s)}{R(s)} = \frac{\frac{K}{s^2}}{1 + \frac{K}{s^2}} = \frac{\frac{K}{s^2}}{\frac{s^2 + K}{s^2}} = \frac{K}{s^2 + K}$$

$$\text{CE:} \quad s^2 + K = 0 \quad \Rightarrow s = \pm j\sqrt{K}$$

For $K = 1$, $s = \pm j$ $\quad \Rightarrow$ oscillatory response

$\Rightarrow$ Not possible to tune K to obtain a stable system as the poles only move along the imaginary axis



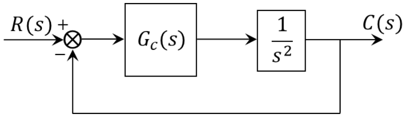
MATLAB script:

```
num1=1;den1=[1 0 1];
num2=5;den2=[1 0 5];

sys1=tf(num1,den1);
sys2=tf(num2,den2);
time=0:0.1:10;
y1=step(sys1,time);
y2=step(sys2,time);

figure
plot(time,y1,time,y2);
```

Question 1

II. Design the following controllers to give a pair of dominant closed-loop poles at $s = -1 \pm j$

- Proportional-Derivative Controller
- Lead Compensator



Note that the gain K is incorporated with $G_c(s)$

PD Controller: $G_c(s) = K(s + z_c)$

CLTF: $$\frac{C(s)}{R(s)} = \frac{K(s+z_c)(\frac{1}{s^2})}{1 + K(s+z_c)(\frac{1}{s^2})} = \frac{K(s+z_c)}{s^2 + Ks + Kz_c}$$

CE: $s^2 + Ks + Kz_c = 0$

Subst $s = -1 + j$: $(-1+j)^2 + K(-1+j) + Kz_c = 0$

$$-2j - K + Kj + Kz_c = 0$$

$$-K + Kz_c + j(K-2) = 0$$

$$\Rightarrow -K + Kz_c = 0 \qquad \Rightarrow K - 2 = 0$$

$$z_c = 1 \qquad K = 2$$

Changing poles to increase stability and speed up settling time with increased damping

Lead Compensator: $G_c(s) = K\dfrac{(s+z_c)}{(s+p_c)}$, $p_c > z_c$

Choose: $z_c \le 1$ (z_c of PD controller) $\Rightarrow$ $z_c = 0.5$

$$\frac{C(s)}{R(s)} = \frac{K\frac{(s+z_c)}{(s+p_c)}(\frac{1}{s^2})}{1 + K\frac{(s+z_c)}{(s+p_c)}(\frac{1}{s^2})} = \frac{K(s+z_c)}{s^2(s+p_c) + K(s+z_c)}$$

CE: $s^2(s+p_c) + K(s+z_c) = 0$

Subst $z_c = 0.5$ & $s = -1 + j$: $(-1+j)^2(-1+j+p_c) + K(-1+j+0.5) = 0$

$$-2j(-1+j+p_c) + K(0.5+j) = 0$$

$$2 - 0.5K + j(-2p_c + K + 2) = 0$$

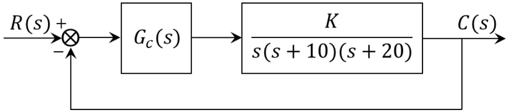
$$\Rightarrow 2 - 0.5K = 0 \qquad \Rightarrow -2p_c + K + 2 = 0$$

$$K = 4 \qquad p_c = 3$$

Question 2

Consider the following angular position control system

1. Compute the steady-state error for a ramp input of $2\pi \cdot \sec^{-1}$ and $K = 820$. *Assume* $G_c(s)$=*1*
2. Design a lag compensator to decrease the steady-state error to one-tenth of the value computed in the previous question. How did the compensator affect the root locus and the dominant closed-loop poles at $K = 820$?



$K = 820$

1. System type 1, ramp input (Unity feedback system) $\Rightarrow e_{ss} = \frac{1}{K_v} \times 2\pi$ ← scaling input

$$= \frac{1}{\lim_{s\to 0} sG(s)} \times 2\pi = \frac{1}{\frac{820}{(10)(20)}} \times 2\pi = 1.53 \text{ rad}$$

2. Lag Compensator:

$$G_c(s) = K\frac{(s+z_c)}{(s+p_c)}$$

$$p_c \ll z_c \ll 1$$

$$G(s) = K\frac{(s+z_c)}{(s+p_c)}\frac{1}{s(s+10)(s+20)}$$

Open loop system still type 1, ramp input

$$\Rightarrow e_{ss} = \frac{1}{K_v} \times 2\pi = \frac{1}{\lim_{s\to 0} sG(s)} \times 2\pi$$

$$= \frac{1}{820\frac{(z_c)}{(p_c)}\frac{1}{(10)(20)}} \times 2\pi = 1.53\frac{p_c}{z_c} \text{ rad}$$

Question 2

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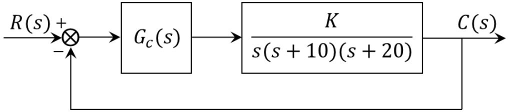
2. Design a lag compensator to decrease the steady-state error to one-tenth of the value computed in the previous question. How did the compensator affect the root locus and the dominant closed-loop poles at $K = 820$?



$e_{ss} = 1.53\dfrac{p_c}{z_c}$ rad $\Rightarrow$ For the new e_{ss} to be $\frac{1}{10}$ of the previous one,

$$\frac{p_c}{z_c} = \frac{1}{10} \quad \Rightarrow \quad \text{Choose e.g. } p_c = 0.05 \;\&\; z_c = 0.5$$

Open Loop TF for Root Locus (with K):

Previous: $G(s) = \dfrac{K}{s(s+10)(s+20)}$

New: $G(s) = K\dfrac{(s+0.5)}{(s+0.05)}\dfrac{1}{s(s+10)(s+20)}$

When $K = 820$, closed-loop poles:

Previous: $s = -3.6 \pm 4.8j$

New: $s = -3.36 \pm 4.54j$

Root locus does not change much!

Reduces e_{ss}, small change in transient response

MATLAB script:

```
Z = -0.5;
P = [0 -0.05 -10 -20];
K = 1;
G = zpk(Z,P,K);
figure
rlocus(G)
```