NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

MA3005 Control Theory

Tutorial 2

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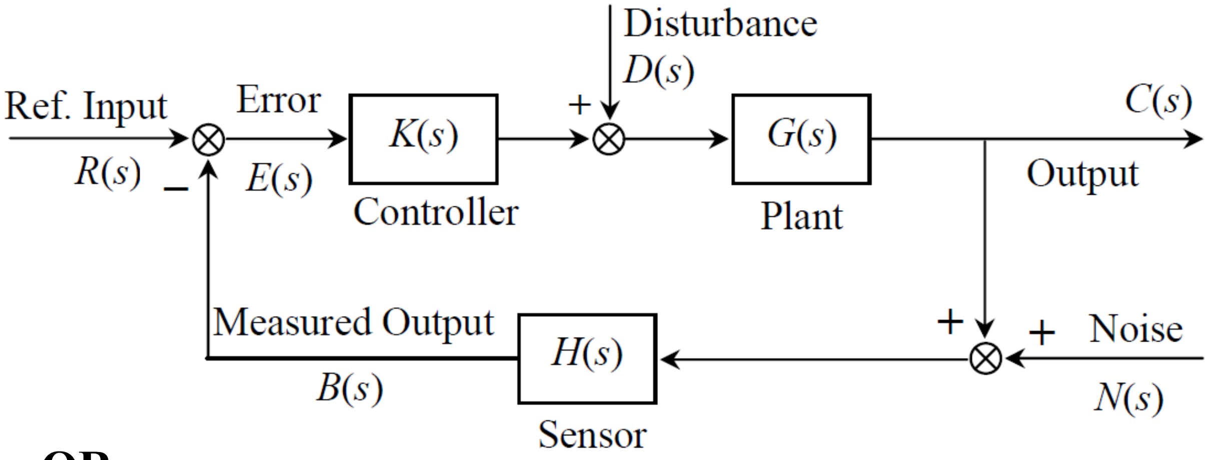
Question 1

Evaluate the transfer functions relating the output $C(s)$ to each of the inputs $R(s)$, $D(s)$ and $N(s)$.
Hint: *Since the system in linear, use the property of superposition*



We can compute the transfer functions from each input to the respective output individually. We can then obtain the overall input output function by adding the individual contribution.

For $R(s)$ to $C(s)$: Disregard other inputs, i.e. $D(s) = N(s) = 0$

$$\therefore C_R(s) = G(s)K(s)E(s) = G(s)K(s)[R(s) - B(s)]$$

$$= G(s)K(s)R(s) - G(s)K(s)H(s)C_R(s)$$

$$\Rightarrow \quad C_R(s) = \frac{G(s)K(s)}{1 + G(s)K(s)H(s)}R(s)$$

OR…

For **negative (positive)** feedback system,

$$CLTF = \frac{Forward\ Loop\ TF}{1 \pm Open\ Loop\ TF} \rightarrow \begin{cases} (+) \text{ for negative feedback} \\ (-) \text{ for positive feedback} \end{cases}$$

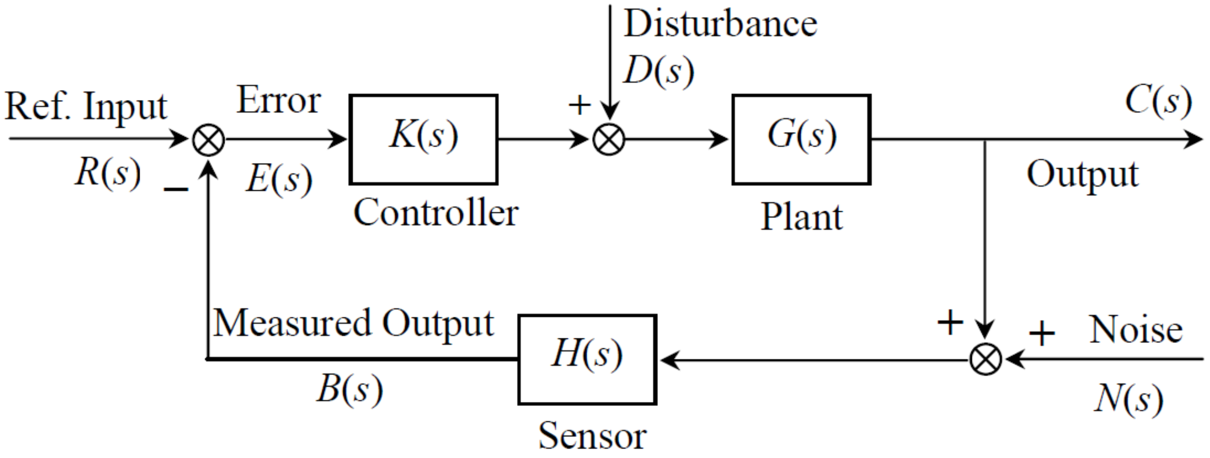
$$\Rightarrow \quad C_R(s) = \frac{G(s)K(s)}{1 + G(s)K(s)H(s)}R(s)$$

Similarly, for $D(s)$ to $C(s)$:

$$\Rightarrow \quad C_D(s) = \frac{G(s)}{1 + G(s)K(s)H(s)}D(s)$$

Question 1

Evaluate the transfer functions relating the output $C(s)$ to each of the inputs $R(s)$, $D(s)$ and $N(s)$.
Hint: *Since the system in linear, use the property of superposition*



Assume that $H(s) = 1$ and $K(s) = K$. Assume also that there is no noise or disturbance ($N(s) = 0$ and $D(s) = 0$). Derive a simple expression for the error $E(s) = R(s) – B(s)$, in terms of $R(s)$, $G(s)$ and K.

For $N(s)$ to $C(s)$: Let $R(s) = D(s) = 0$

$$-[C_N(s) + N(s)]H(s)\,K(s)G(s) = C_N(s)$$

$$\Rightarrow \quad C_N(s) = \frac{-G(s)K(s)H(s)}{1 + G(s)K(s)H(s)}N(s)$$

The overall input output function:

$$C(s) = C_R(s) + C_D(s) + C_N(s)$$

$$= \frac{1}{1 + G(s)K(s)H(s)}[G(s)K(s)R(s) + G(s)D(s) - G(s)K(s)H(s)N(s)]$$

$$E(s) = R(s) - B(s) = R(s) - C(s)$$

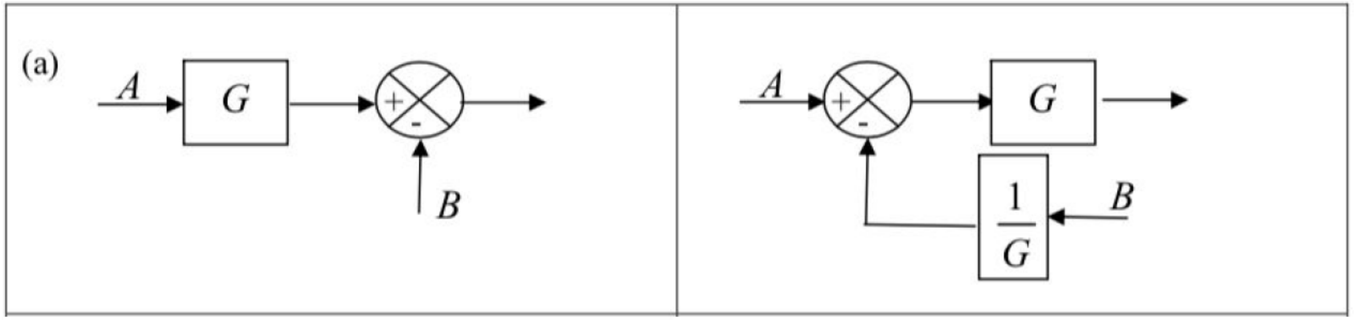
$$= R(s) - \frac{1}{1 + G(s)K(s)H(s)}[G(s)K(s)R(s)]$$

$$= R(s) - \frac{G(s)K}{1 + G(s)K}R(s) = \frac{1}{1 + G(s)K}R(s)$$

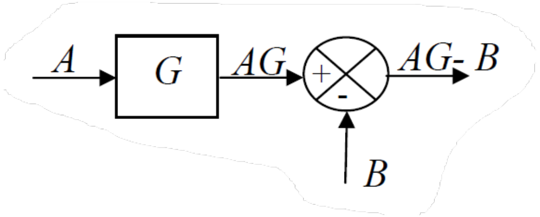
Question 2

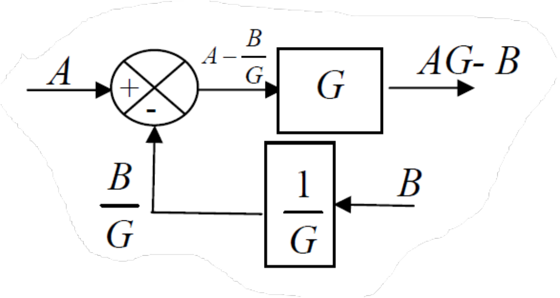
Show that the following block diagrams are equivalent.

To show block diagram equivalence, consider inputs and outputs in different cases and show that they remain the same.



"Moving a summing point ahead of a block"



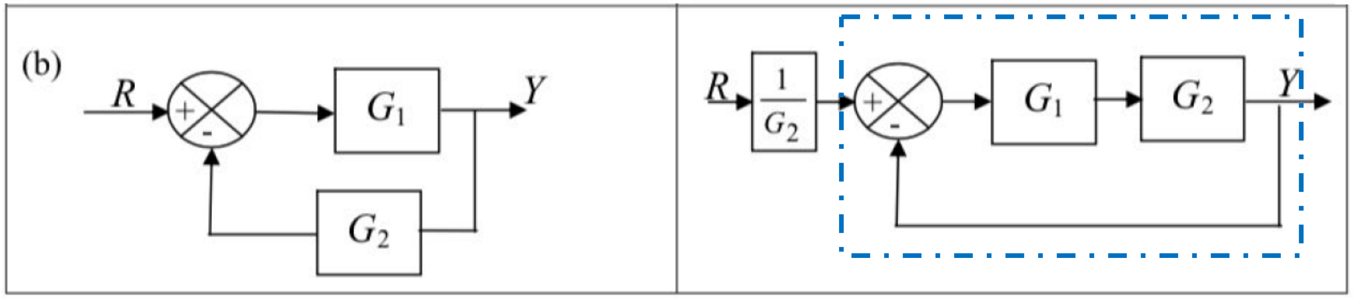


Input 1- A
Input 2- B
Output 1: AG-B

Input 1- A
Input 2- B
Output 1: AG-B

Hence the block diagrams are equivalent.

Question 2



"Converting non-unity feedback to unity feedback system"

$$Y = \frac{\text{Forward Loop TF}}{1 + \text{Open Loop TF}} R$$

$$= \frac{G_1}{1 + G_1 G_2} R$$

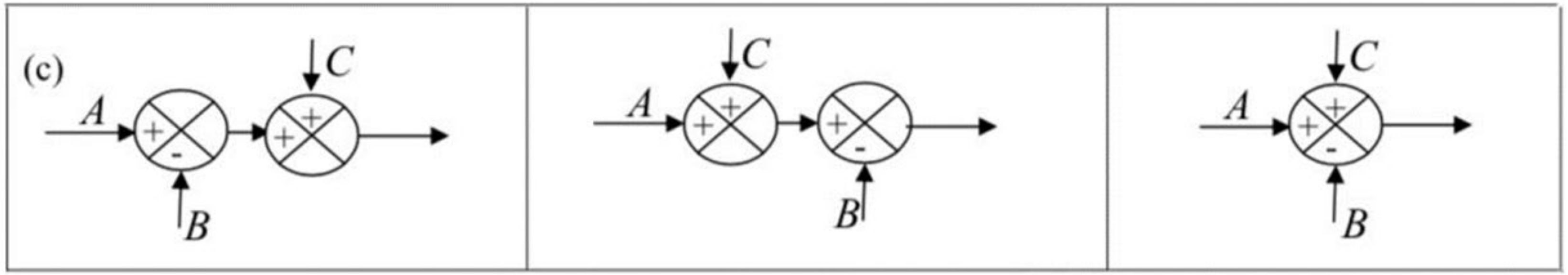
Input $\frac{R}{G_2}$ is being applied to the shaded feedback loop.

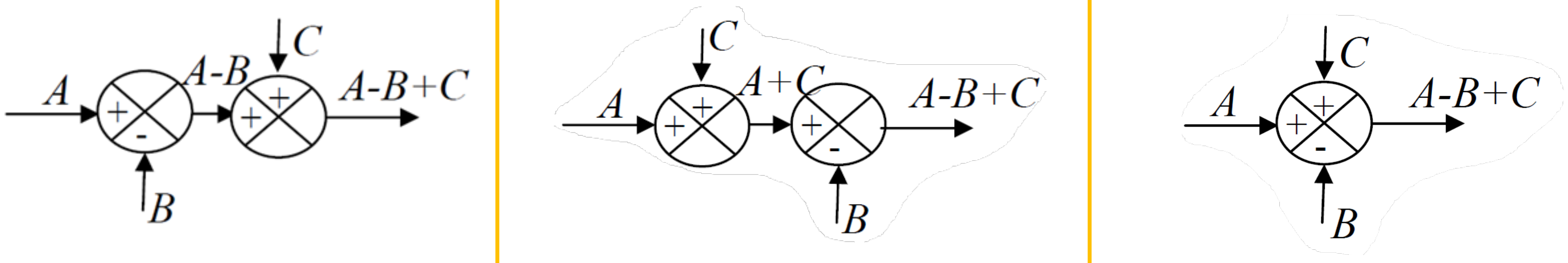
$$Y = \underbrace{\frac{\text{Forward Loop TF}}{1 + \text{Open Loop TF}}}_{\text{For shaded feedback loop}} \underbrace{\frac{R}{G_2}}_{\text{Input to shaded feedback loop}}$$

$$\Rightarrow Y = \frac{G_1 G_2}{1 + G_1 G_2} \frac{R}{G_2} = \frac{G_1}{1 + G_1 G_2} R$$

Same inputs and outputs. Given systems are equivalent

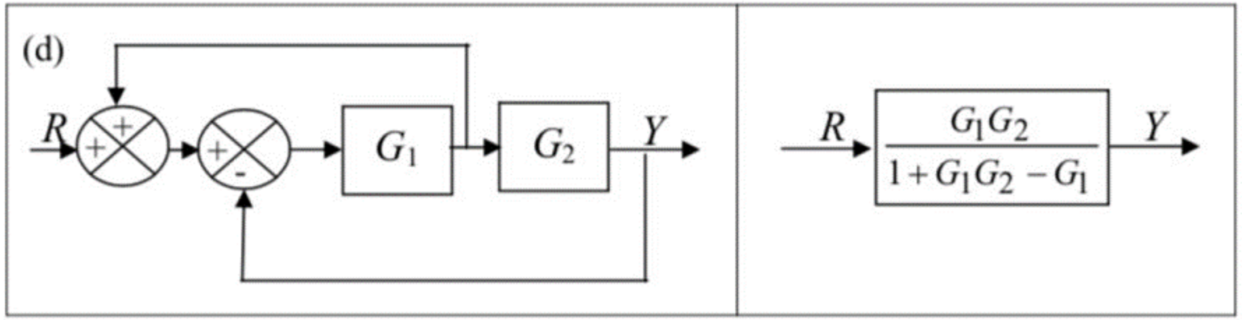
Question 2





Same inputs and outputs. Given systems are equivalent

Question 2

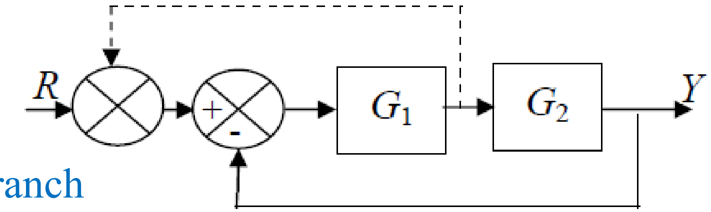


"To create another equivalent system through block diagram reduction"

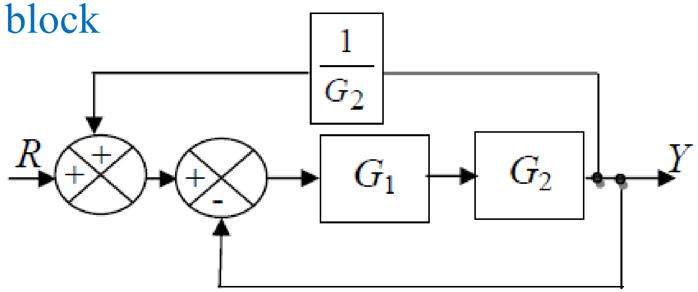
$$CLTF = \frac{Forward\ Loop\ TF}{1 \pm Open\ Loop\ TF} \rightarrow \begin{cases} \textbf{(+) for negative feedback} \\ \textbf{(–) for positive feedback} \end{cases}$$

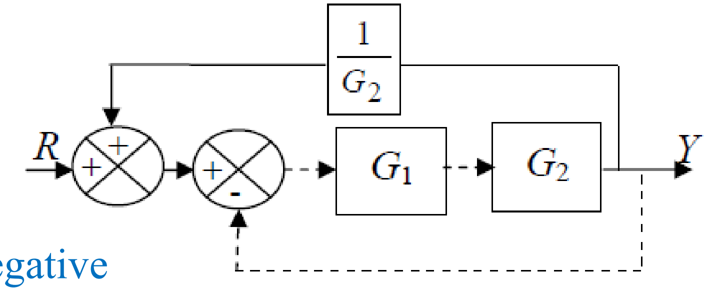
$$\Rightarrow \ Y(s) = \frac{\dfrac{G_1G_2}{1+G_1G_2}}{1 - \dfrac{G_1G_2}{1+G_1G_2}\dfrac{1}{G_2}} R(s)$$

$$= \frac{G_1G_2}{1+G_1G_2-G_1} R(s)$$

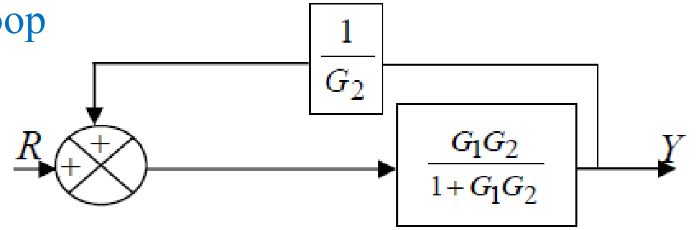


Moving a branch point behind a block

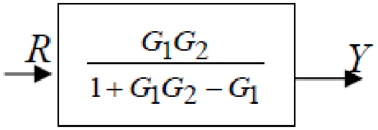




Eliminate negative feedback loop



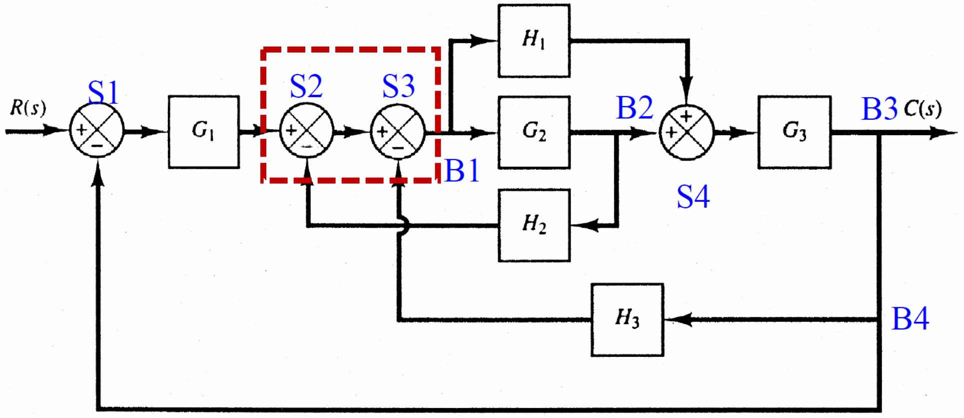
Eliminate positive feedback loop



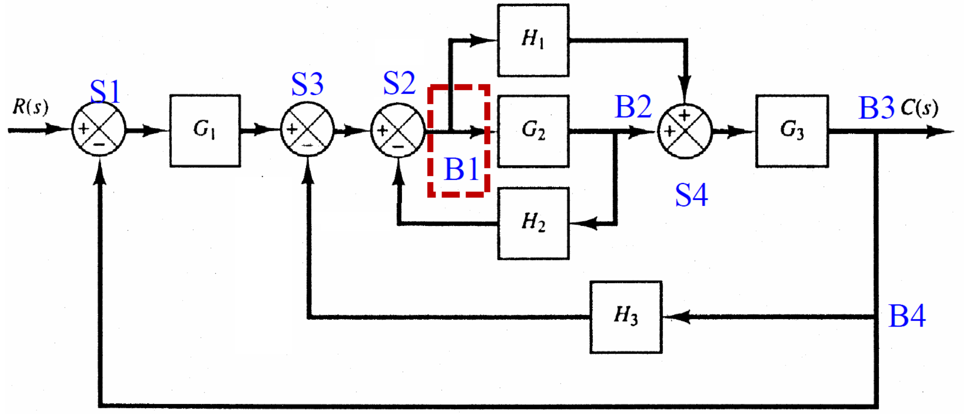
Question 3

Simply the block diagram and obtain the closed loop transfer function $\dfrac{C(s)}{R(s)}$

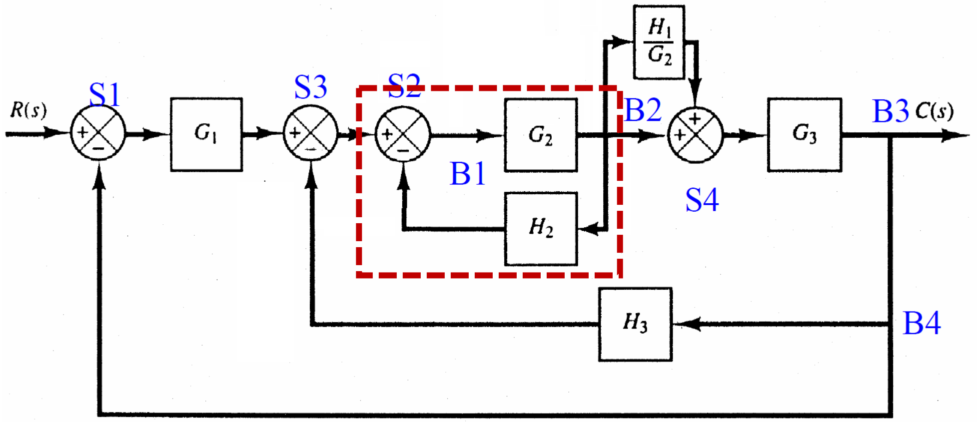
(Branch points and summation junctions labelled for ease of understanding)



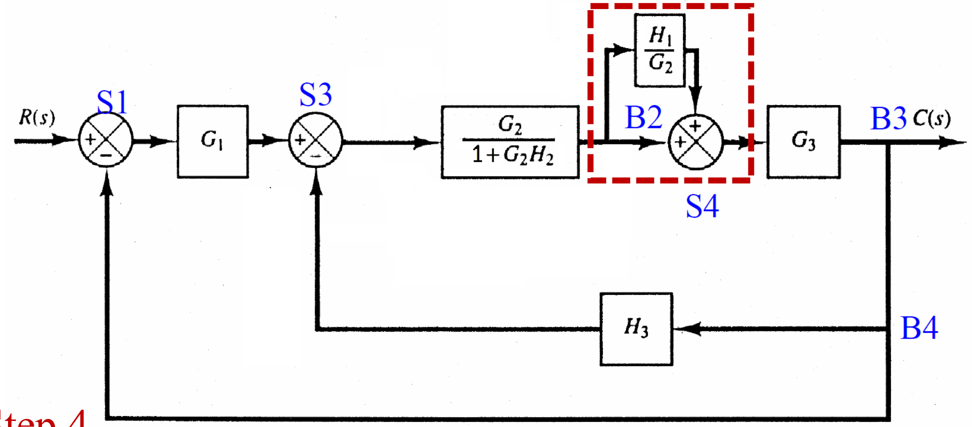
Step 1.
Consecutive summation blocks are swappable.
(Swap S2 and S3 to simplify H_2)



Step 2.
Moving a branch B1 behind a block (to form feedback for G_2 & H_2)

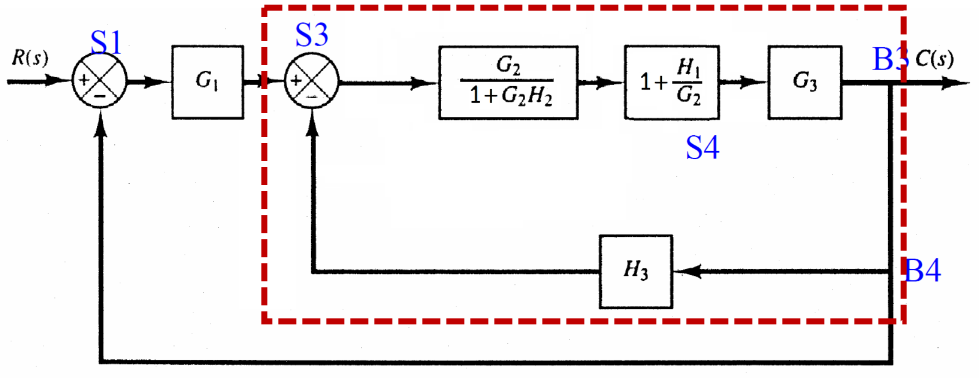


Step 3.
Eliminate feedback loop



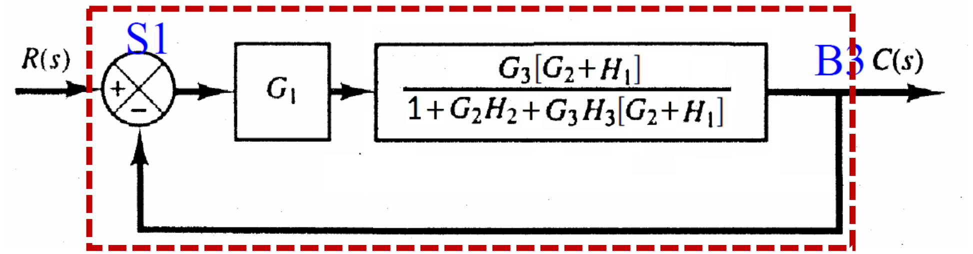
Step 4.
Combine feedforward term with unity transfer function

Question 3



Step 5.
Eliminate feedback loop

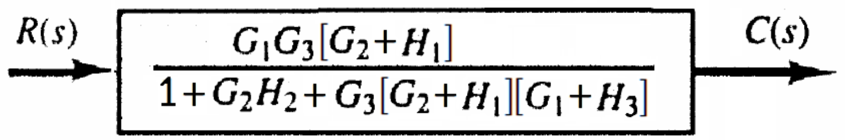
$$\frac{\frac{G_2}{1+G_2H_2}\left(1+\frac{H_1}{G_2}\right)G_3}{1+\frac{G_2}{1+G_2H_2}\left(1+\frac{H_1}{G_2}\right)G_3H_3}=\frac{G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)}$$



Step 6.
Eliminate feedback loop

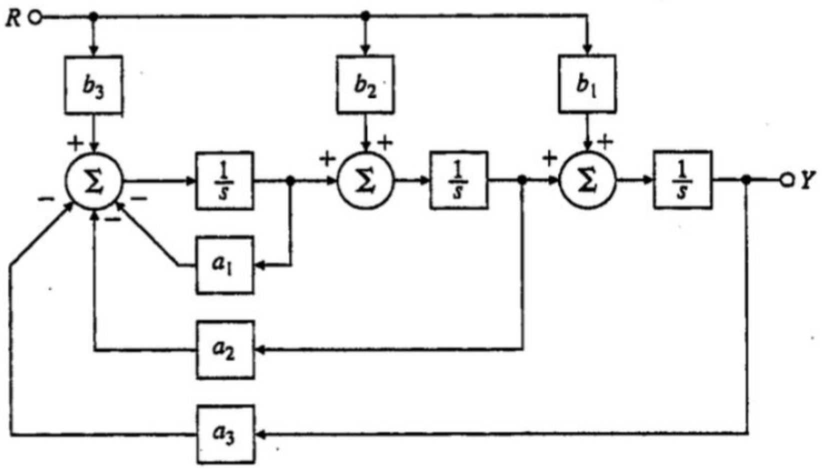
$$\frac{\frac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)}}{1+\frac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)}}=\frac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)+G_1G_3(G_2+H_1)}$$

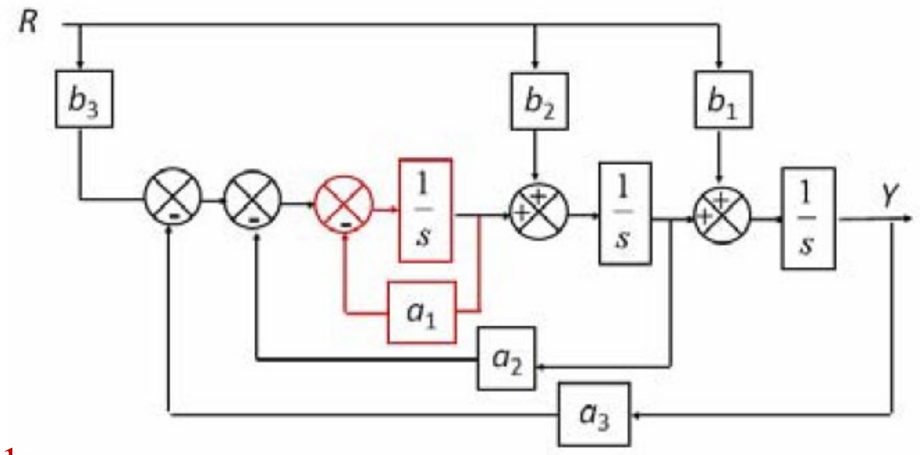
$$=\frac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3(G_2+H_1)(G_1+H_3)}$$



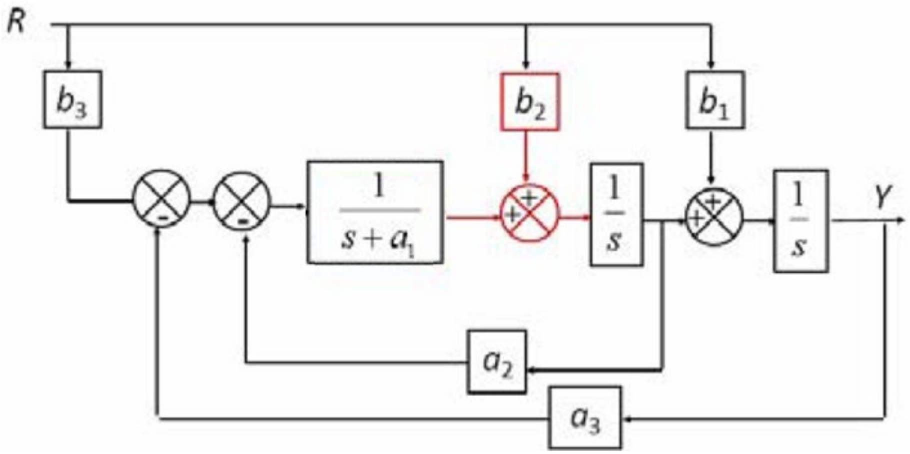
Question 4

Find the transfer function $\frac{Y(s)}{R(s)}$ for the following block diagram

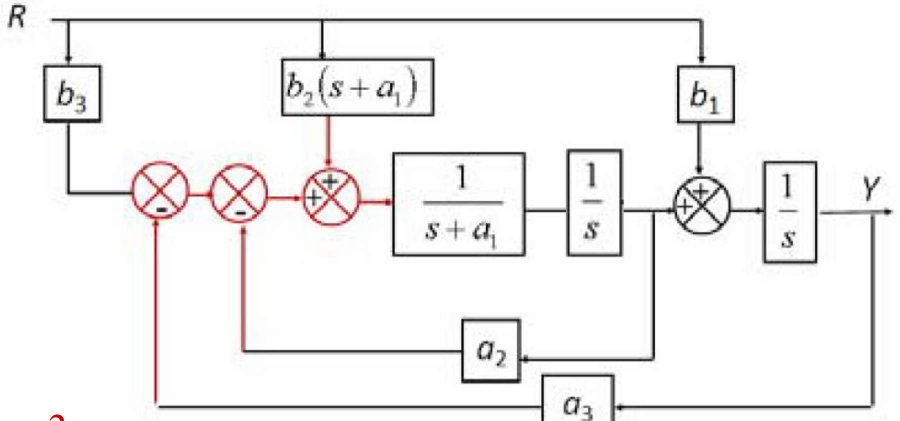




Step 1.
Eliminate feedback loop

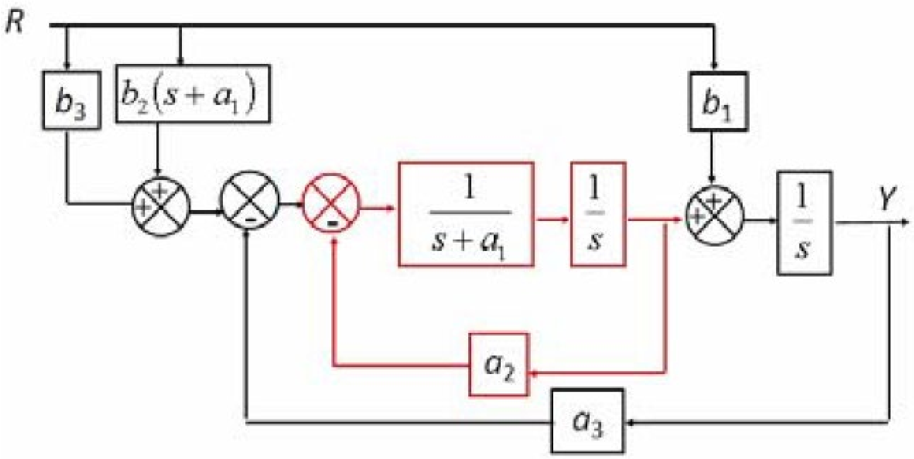


Step 2.
Moving summation point ahead of block

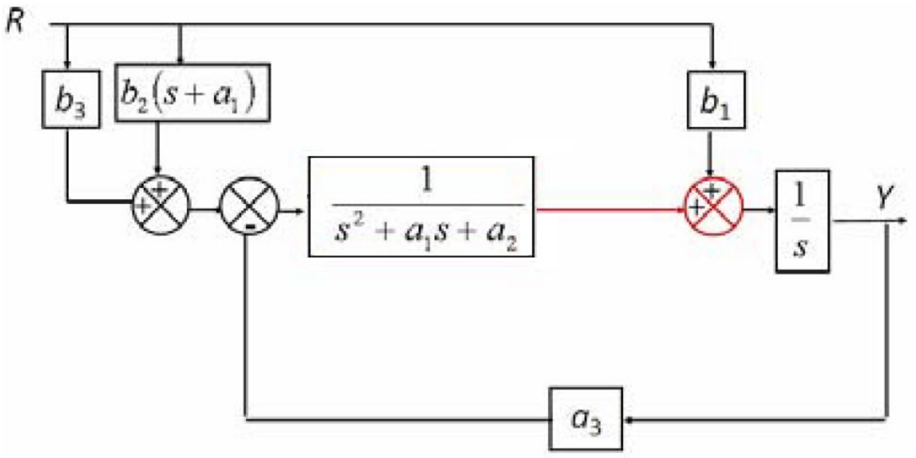


Step 3.
Consecutive summation blocks are swappable.
Move summation to the front

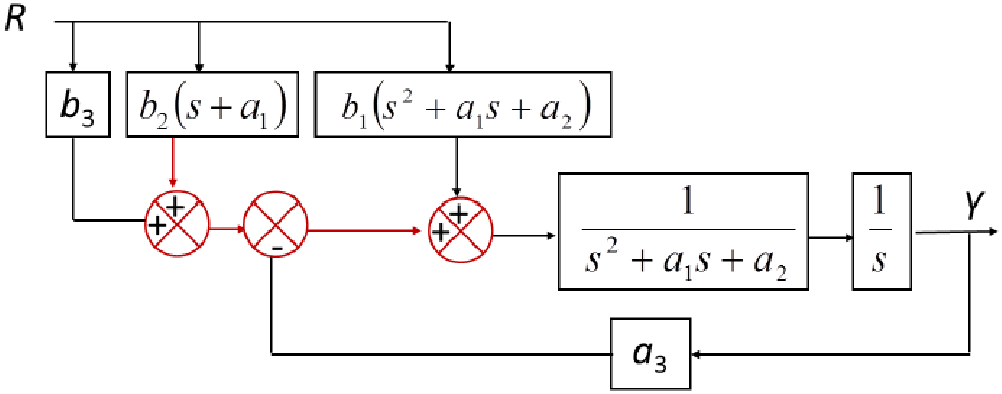
Question 4



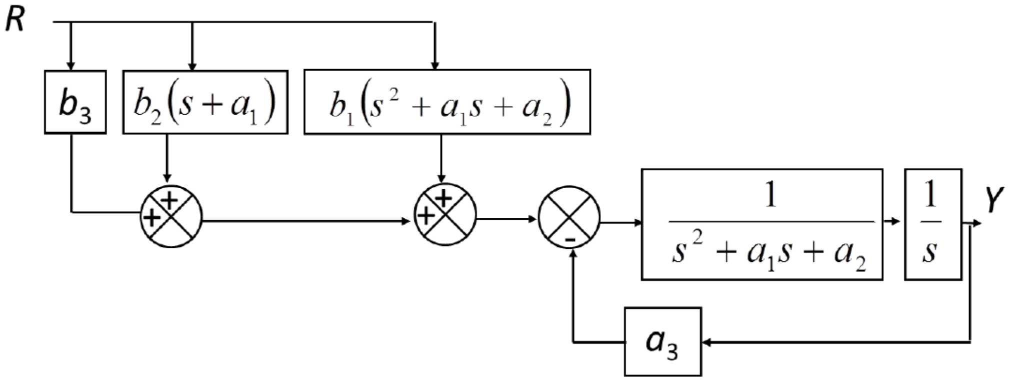
Step 4.
Eliminate feedback loop



Step 5.
Moving summation point ahead of block

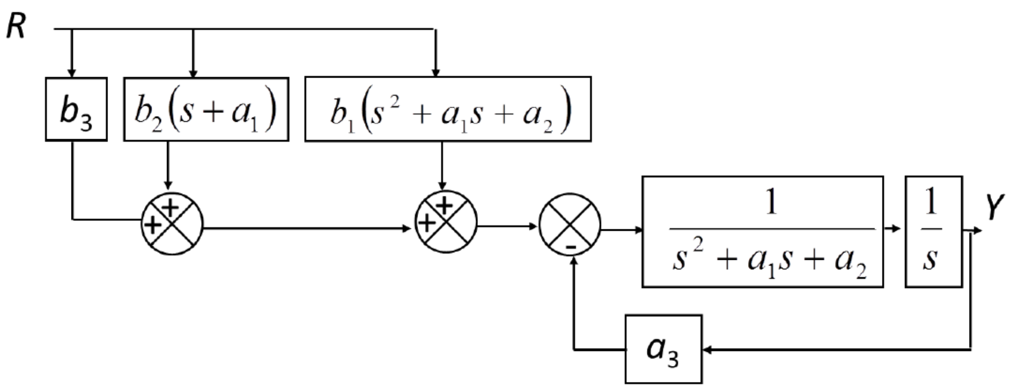


Step 6.
Consecutive summation blocks are swappable.
Move negative summation behind



Step 7.
Combine summations and eliminate feedback loop

Question 4



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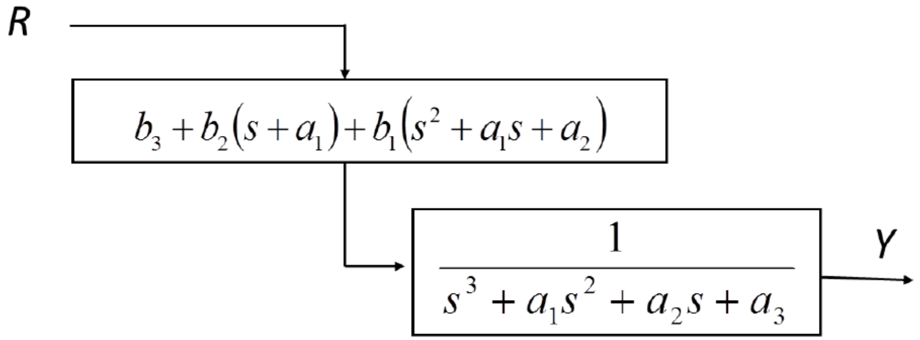

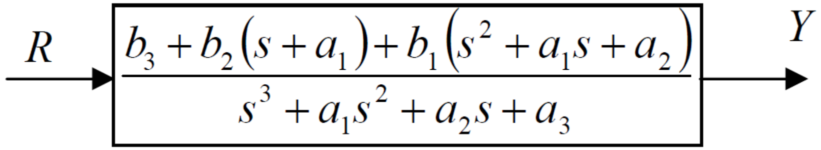

Step 8.
Combine transfer functions