School of Mechanical & Aerospace Engineering
MA3010 – Thermodynamics & Heat Transfer
AY2021 – Continual Assessment 1 Solution Guide

MCQs:

1. An air-conditioner is used to cool the air from $\boldsymbol{T_1}$°C to $\boldsymbol{T_2}$°C at a steady rate of $\dot{\boldsymbol{m}}$ kg/s. If the coefficient of performance of the air-conditioner is $\boldsymbol{COP_R}$, what is its power consumption ($\dot{\boldsymbol{W}}_{\boldsymbol{in}}$)? Utilize the properties of air at 300K.

$$\dot{Q}_L = \dot{m}c_p(T_2 - T_1)$$

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}}, \qquad \rightarrow \dot{W}_{in} = \frac{\dot{Q}_L}{COP_R}_{\#}$$

Makeup CA:

$$\dot{Q}_L = \frac{\dot{m}(\text{kg/min})}{60} \times c_p(T_2 - T_1)$$

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}}, \qquad \rightarrow \dot{W}_{in} = \frac{\dot{Q}_L}{COP_R}_{\#}$$

2. An inventor has developed a cyclical engine for use in winter climates that taps into a geothermal energy source whose temperature is $\boldsymbol{T_H}$°C while rejecting heat to the environment at $\boldsymbol{T_L}$°C. It is claimed that the engine produces $\dot{\boldsymbol{W}}_{\boldsymbol{net,out}}$ kW of power while the heat rejection rate is at $\dot{\boldsymbol{Q}}_{\boldsymbol{L}}$ kJ/h. Is this claim valid?

$$\eta_{TH} = \frac{\dot{W}_{net,out}}{\dot{Q}_H} = \frac{\dot{W}_{net,out}}{\dot{W}_{net,out} + \frac{\dot{Q}_L}{3600}}$$

$$\eta_{TH,Carnot} = 1 - \frac{T_L + 273}{T_H + 273}$$

check: $\eta_{TH} \leq \eta_{TH,Carnot}{}_{\#}$

3. Makeup CA: A Carnot refrigerator and a Carnot heat engine are operating between the same thermal energy reservoirs. If the COP of the refrigerator is $\boldsymbol{COP_R}$, what is the thermal efficiency of the heat engine?

$$COP_R(Carnot) = \frac{1}{\frac{T_H}{T_L} - 1}, \qquad \rightarrow \frac{T_H}{T_L} = 1 + \frac{1}{COP_R} = \frac{COP_R + 1}{COP_R}$$

$$\eta_{TH,Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{COP_R}{COP_R + 1}_{\#}$$

4. $\boldsymbol{m}$ kg of steam at $\boldsymbol{P_1}$ MPa and $\boldsymbol{T_1}$°C undergoes isenthalpic expansion/compression and its pressure drops/rises to $\boldsymbol{P_2}$ kPa. What is the total entropy change of steam?

Obtain values from property tables (steam):

Initial: h_1, s_1 at given P_1, T_1

Final: interpolate s_2 using $P_2, h_2 = h_1 (\because$ isenthalpic)

$$\Delta S = m(s_2 - s_1)_{\#}$$

5. Makeup CA: What is the power consumed for an ideal two-stage polytropic (n = $\boldsymbol{n}$) compression of argon gas at $\boldsymbol{T_1}$°C and $\boldsymbol{P_1}$ kPa to $\boldsymbol{P_2}$ MPa at a rate of $\dot{\boldsymbol{m}}$ kg/s?

$$P_x = \sqrt{P_1 \times \frac{P_2(\text{MPa})}{1000}}$$

$$W_{in} = 2\dot{m}\left\{\frac{nRT_1}{1-n}\left[\left(\frac{P_x}{P_1}\right)^{\frac{n-1}{n}} - 1\right]\right\}_{\#}$$

6. Pressurized air is expanded steadily at a rate of $\dot{\boldsymbol{m}}$ kg/s by an adiabatic turbine from $\boldsymbol{P_1}$ MPa and $\boldsymbol{T_1}$°C at the inlet to $\boldsymbol{P_2}$ kPa at the outlet. If the turbine produces $\dot{\boldsymbol{W}}_{\boldsymbol{out}}$ kW of power, calculate the isentropic efficiency of this turbine. Assume an average temperature of 177°C.

$$T_{2s} = (T_1 + 273)\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\eta_T = \frac{\dot{W}_{out}}{\dot{m}c_p(T_1 - T_{2s})}_{\#}$$

Makeup CA (compressor):

$$\eta_C = \frac{\dot{m}c_p(T_{2s} - T_1)}{\dot{W}_{in}}_{\#}$$

7. A rigid 30-litre canister containing $\boldsymbol{m}$ kg of nitrogen/oxygen initially at $\boldsymbol{T_1}$°C is left in the surrounding ambient temperature at $\boldsymbol{T_{sur}}$°C. What is the entropy generation after the canister has reached thermal equilibrium? Utilize the properties of nitrogen/oxygen at 300K.

$$\Delta S_{sys} = m\left(c_v \ln\frac{T_{sur} + 273}{T_1 + 273}\right)$$

$$Q_{in} = mc_v(T_{sur} - T_1)$$

$$S_{gen} = \Delta S_{sys} + \frac{-Q_{in}}{T_{sur} + 273}_{\#}$$

Makeup CA (piston-cylinder, v_2/v_1 is given):

$$\Delta S_{sys} = m\left(c_v \ln\frac{T_{sur} + 273}{T_1 + 273} + R\ln\frac{v_2}{v_1}\right)$$

$$Q_{in} = mc_v(T_{sur} - T_1)$$

$$S_{gen} = \Delta S_{sys} + \frac{-Q_{in}}{T_{sur} + 273}_{\#}$$

Long Questions:

8. A heat exchanger facilitates the transfer of heat between refrigerant R134a and river water. R134a Qat $\boldsymbol{T_1}$°C and $\boldsymbol{P}$ MPa enters the heat exchanger and leaves as a saturated liquid. River water (c = 4.18 kJ/kg) enters the heat exchanger at $\boldsymbol{T_3}$°C at a rate of $\dot{\boldsymbol{m}}_{\boldsymbol{w}}$ kg/s and leaves at $\boldsymbol{T_4}$°C. The surrounding temperature is at $\boldsymbol{T_{sur}}$°C and the refrigerant loses $\boldsymbol{x}$ % of its heat to the surroundings. Assuming no pressure losses, calculate (a) the rate of entropy change of the river water, (b) the rate of entropy change of the refrigerant, and (c) the total rate of entropy generation. State any additional assumptions made.

   a) Entropy change of river water:

$$\Delta\dot{S}_w = \dot{m}_w c \ln \frac{T_4 + 273}{T_3 + 273}_{\#}$$

   b) Entropy change of refrigerant:

$$\dot{Q}_w = \dot{m}_w c(T_4 - T_3)$$

Obtain values from property tables (R134a):

Inlet: h_1, s_1 at given T_1, P

Outlet: $h_2 = h_f, s_2 = s_f$ at given P, sat. liq.

Heat transfer from refrigerant is split into two paths, one to river water, one to the surroundings.

$$\dot{Q}_R = \frac{\dot{Q}_w}{1-x} = \dot{m}_R(h_1 - h_2), \qquad \rightarrow \dot{m}_R = \frac{\dot{Q}_w}{(1-x)(h_1 - h_2)}$$

$$\Delta\dot{S}_R = \dot{m}_R(s_2 - s_1)_{\#}$$

   c) Entropy generation:

$$\dot{Q}_{loss} = x\dot{m}_R(h_1 - h_2)$$

$$\Delta\dot{S}_{sur} = \frac{\dot{Q}_{loss}}{T_{sur} + 273}$$

$$\dot{S}_{gen} = \Delta\dot{S}_w + \Delta\dot{S}_R + \Delta\dot{S}_{sur\#}$$

9. Makeup CA: Superheated steam at $\boldsymbol{P_1}$ MPa and $\boldsymbol{T_1}$°C from a boiler tank is steadily accelerated through an adiabatic nozzle at a rate of $\dot{\boldsymbol{m}}$ kg/min. The exit velocity of steam at the nozzle is measured to be $\boldsymbol{v_2}$ m/s and the pressure at the exit is measured to be $\boldsymbol{P_2}$ MPa. Assuming that the steam is stationary at the nozzle inlet, (a) calculate the temperature of steam at the exit, (b) the isentropic efficiency of the nozzle and (c) the rate of entropy generation in the nozzle.

a) Obtain values from property table (steam):

Inlet: h_1, s_1 at given P_1, T_1

Energy balance:

$$h_1 = h_{2a} + \frac{v_2^2}{2}, \qquad \rightarrow h_{2a}(\mathrm{J}) = h_1(\mathrm{kJ}) \times 1000 - \frac{v_2^2}{2}$$

Using the property tables, T_{2a} is then calculated by interpolation using given P_2 and h_{2a}.#

b) Obtain values from property table (steam):

Outlet: interpolate h_{2s} using $P_2, s_{2s} = s_1 (\because$ ideal nozzle)

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}}_{\#}$$

c) Obtain values from property table (steam):

Outlet: interpolate s_{2a} using P_2, h_{2a}

$$\dot{m}s_1 - \dot{m}s_{2_a} + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \dot{m}(s_{2a} - s_1)_{\#}$$