NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

MA3010 – Entropy

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Contents

Reference: Thermodynamics Chapter 7

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Introduction: Entropy

- 1st Law of Thermodynamics – concept of energy
- 2nd Law of Thermodynamics – quality of energy & definition of entropy
- Entropy is a property
  - A measure of the degree of **microscopic disorder** in the system
  - Entropy can be **transferred by heat** but **not by work**
  - Entropy is **not conserved**
  - Entropy can be generated/created but **not destroyed**
  - Total entropy for isolated systems (system + surroundings) **increases** for **all real processes**

Clausius Inequality

Cyclic integral of $\delta Q/T$ is always less than or equal to zero for all cycles regardless of reversible, irreversible cycles

$$\boxed{\oint \frac{\delta Q}{T} \le 0}$$

- Unit for T must be in absolute temperature (K)
- For reversible cycles:

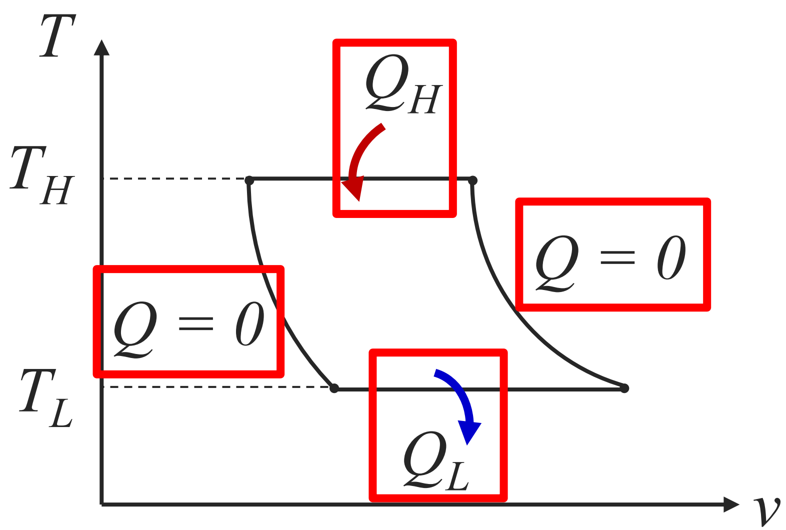
$$\oint \left(\frac{\delta Q}{T}\right)_{rev} = 0$$

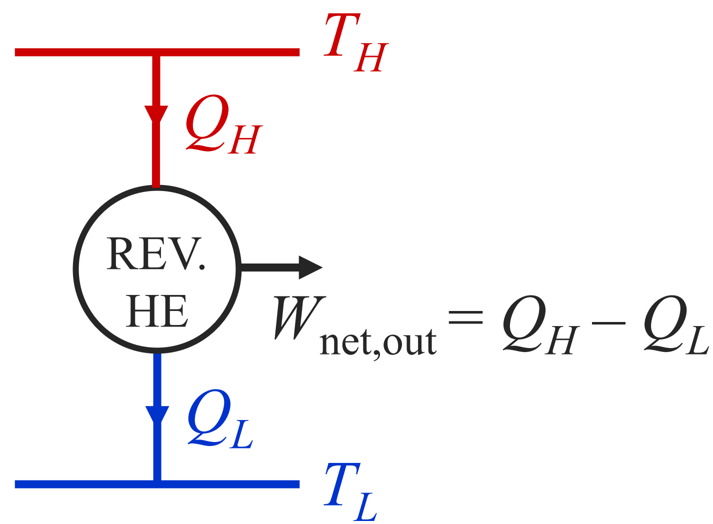
- For irreversible cycles:

$$\oint \left(\frac{\delta Q}{T}\right)_{irrev} < 0$$

Clausius Inequality – Proof

For a Carnot heat engine (rev. cycle):





$$\left(\frac{Q_H}{Q_L}\right)_{rev} = \frac{T_H}{T_L} \quad \rightarrow \frac{Q_H}{T_H} \overset{rev}{=} \frac{Q_L}{T_L}$$

$$\boxed{\oint\left(\frac{\delta Q}{T}\right)_{rev}} = \cancel{\frac{Q_H}{T_H}} + 0 - \cancel{\frac{Q_L}{T_L}} + 0 \boxed{= 0}$$

Clausius Inequality – Proof

$$\frac{Q_H}{T_H} \overset{rev}{=} \frac{Q_L}{T_L}$$

For any other heat engine (irrev. cycle):

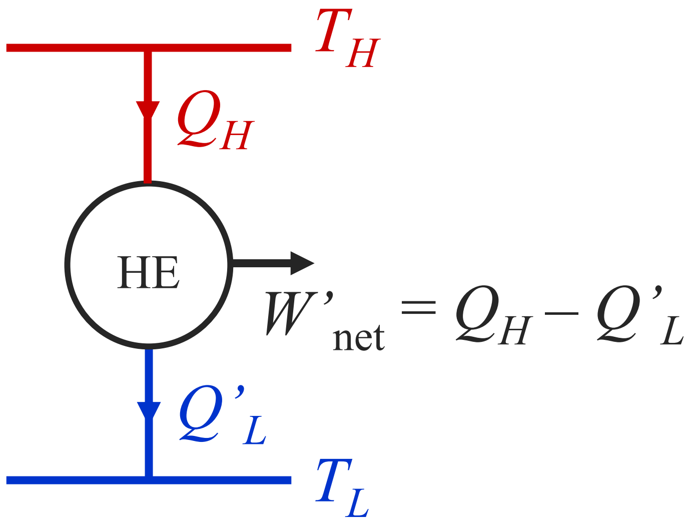
Thermal efficiency is *lower* :

$$W'_{net} < W_{net}$$

$$Q_H - Q'_L < Q_H - Q_L$$

$$Q'_L > (Q_L)_{rev}$$

$$\therefore \frac{Q_H}{T_H} \overset{irrev}{<} \frac{Q'_L}{T_L}$$



$$\boxed{\oint \left(\frac{\delta Q}{T}\right)_{irrev}} = \frac{Q_H}{T_H} + 0 - \frac{Q'_L}{T_L} + 0 \boxed{< 0}$$

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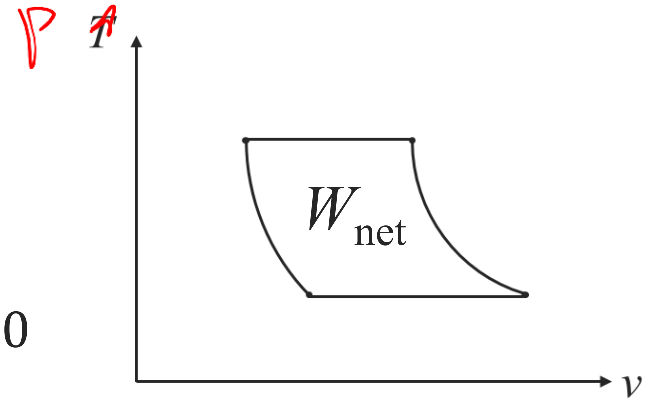
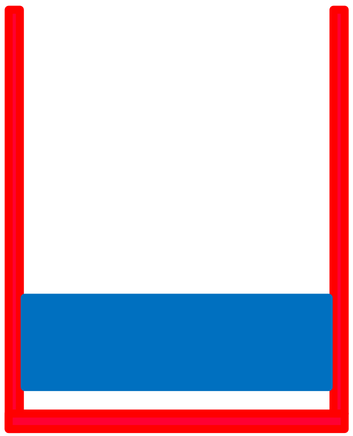
Entropy



- Examining again closely: $\oint \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = 0$
- Cyclic integral of Work and cyclic integral of Heat is not zero
- However, cyclic integral of volume (a property) is zero

$$\oint dV = \Delta V_{cyc} = 0$$



- More generally, cyclic integrals of any property is zero

$$\oint d(\text{property}) = 0$$

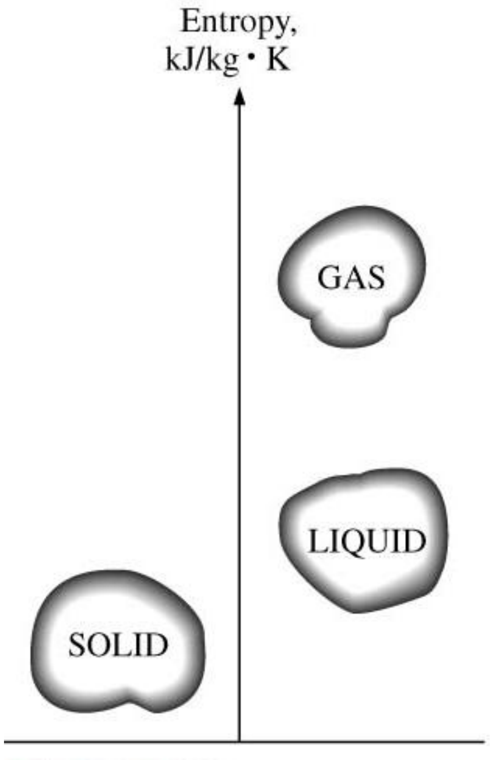
- Hence, $(\delta Q/T)_{\text{int rev}}$ represents a property:
  Entropy, in the differential form:

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \quad (\text{kJ/K})$$

Entropy

Entropy is a measure of molecular disorder

- As a system becomes more disordered, the position of the molecules become less predictable and thus entropy increases
- Quantity of energy is always conserved during an actual process (1st law) but the quality decreases (2nd law)
- There is no entropy transfer associated with energy transfer in the form of work
- Heat is a form of disorganised energy and some disorganisation (entropy) will flow with heat



Entropy of Pure Substances

- Entropy data for pure substances (including real gases) can be obtained from property tables
- Specific entropy (kJ/kg·K) values are assigned to reference states for each substance
  - E.g. water/steam: $s_f = 0$ @ 0.01°C,
    R134a refrigerant: $s_f = 0$ @ – 40°C
- Entropy change of a specified mass during a process:

$$\Delta S = S_2 - S_1 = m(s_2 - s_1)$$

TABLE A–6

Superheated water

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg·K
	P = 0.01 MPa (45.81°C)*				P = 0.05 MPa (81.32°C)				P = 0.10 MPa (99.61°C)			
Sat.†	14.670	2437.2	2583.9	8.1488	3.2403	2483.2	2645.2	7.5931	1.6941	2505.6	2675.0	7.3589
50	14.867	2443.3	2592.0	8.1741								
100	17.196	2515.5	2687.5	8.4489	3.4187	2511.5	2682.4	7.6953	1.6959	2506.2	2675.8	7.3611
150	19.513	2587.9	2783.0	8.6893	3.8897	2585.7	2780.2	7.9413	1.9367	2582.9	2776.6	7.6148
200	21.826	2661.4	2879.6	8.9049	4.3562	2660.0	2877.8	8.1592	2.1724	2658.2	2875.5	7.8356
250	24.136	2736.1	2977.5	9.1015	4.8206	2735.1	2976.2	8.3568	2.4062	2733.9	2974.5	8.0346
300	26.446	2812.3	3076.7	9.2827	5.2841	2811.6	3075.8	8.5387	2.6389	2810.7	3074.5	8.2172
400	31.063	2969.3	3280.0	9.6094	6.2094	2968.9	3279.3	8.8659	3.1027	2968.3	3278.6	8.5452
500	35.680	3132.9	3489.7	9.8998	7.1338	3132.6	3489.3	9.1566	3.5655	3132.2	3488.7	8.8362
600	40.296	3303.3	3706.3	10.1631	8.0577	3303.1	3706.0	9.4201	4.0279	3302.8	3705.6	9.0999
700	44.911	3480.8	3929.9	10.4056	8.9813	3480.6	3929.7	9.6626	4.4900	3480.4	3929.4	9.3424
800	49.527	3665.4	4160.6	10.6312	9.9047	3665.2	4160.4	9.8883	4.9519	3665.0	4160.2	9.5682
900	54.143	3856.9	4398.3	10.8429	10.8280	3856.8	4398.2	10.1000	5.4137	3856.7	4398.0	9.7800
1000	58.758	4055.3	4642.8	11.0429	11.7513	4055.2	4642.7	10.3000	5.8755	4055.0	4642.6	9.9800
1100	63.373	4260.0	4893.8	11.2326	12.6745	4259.9	4893.7	10.4897	6.3372	4259.8	4893.6	10.1698
1200	67.989	4470.9	5150.8	11.4132	13.5977	4470.8	5150.7	10.6704	6.7988	4470.7	5150.6	10.3504
1300	72.604	4687.4	5413.4	11.5857	14.5209	4687.3	5413.3	10.8429	7.2605	4687.2	5413.3	10.5229

Entropy of Pure Substances

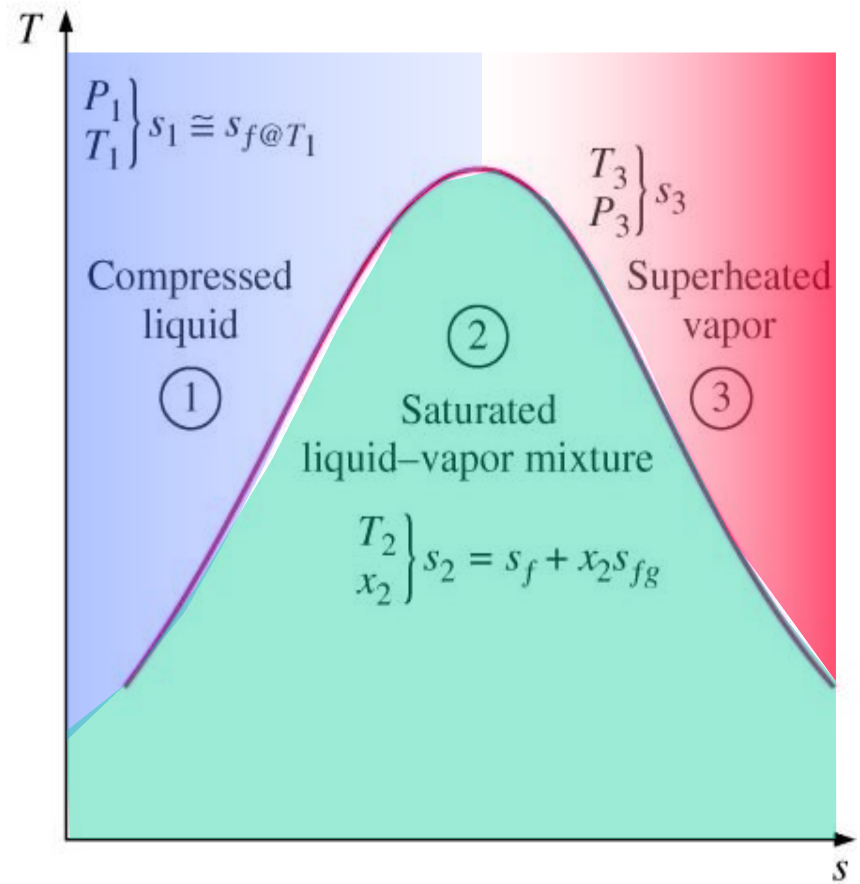
Entropy of a substance is determined from the tables just like any other property

- Superheated vapour region: tabulated versus P and T
- Saturated liquid-vapour region: s_f, s_g, s_{fg} given at P_{sat} and T_{sat}

$$s = s_f + x \cdot s_{fg}$$

- Compressed liquid region: tabulated versus P and T
  - Fluid is almost incompressible; entropy considered dependent only on temperature

$$s \cong s_f @ T$$



Property Diagrams Involving Entropy

- Useful visual aids for analysis of thermodynamics processes
  - 1st Law analysis: P-v & T-v diagrams
  - 2nd Law analysis: T-s & h-s diagrams
- Differential form:

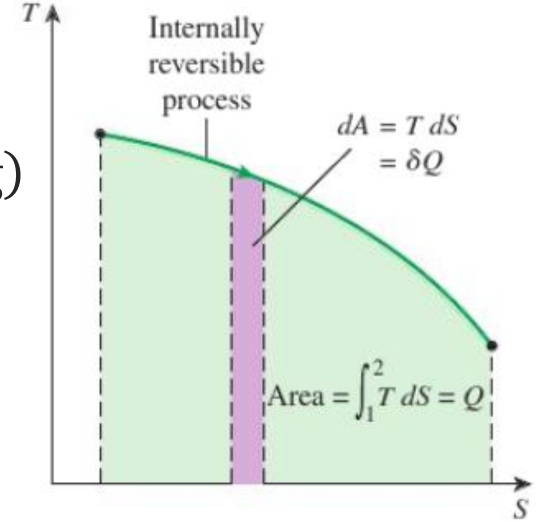
$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \quad (\text{kJ/K})$$

$$\rightarrow \delta Q_{\text{int rev}} = TdS \ (\text{kJ})$$

Or: $\delta q_{\text{int rev}} = Tds \ (\text{kJ/kg})$

- Integrating:

$$Q_{\text{int rev}} = \int_1^2 TdS \ \ (\text{kJ}) \qquad q_{\text{int rev}} = \int_1^2 Tds \ \ (\text{kJ/kg})$$



- Area under the process curve in the T-s diagram represents the heat transfer in an **internally reversible process**

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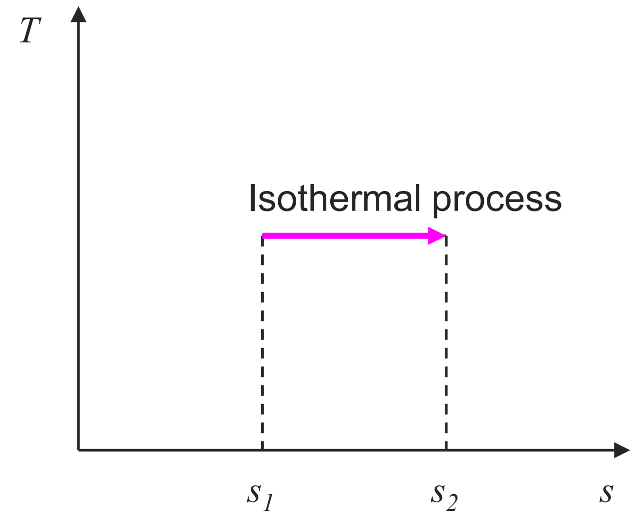
Property Diagrams Involving Entropy

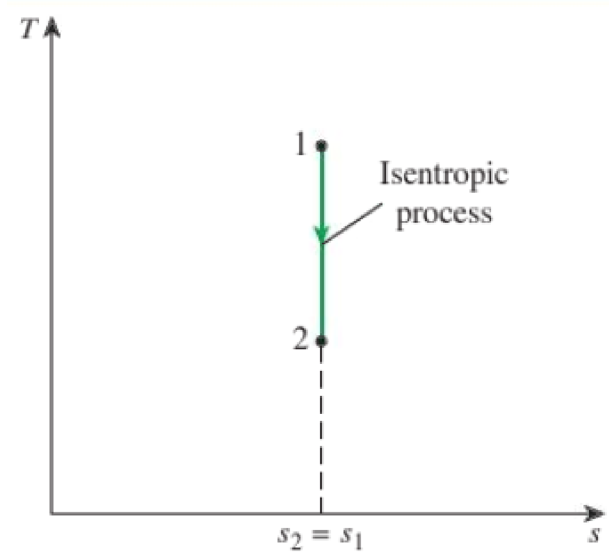
- Internally reversible isothermal process:

$$Q_{\text{int rev}} = T_0 \Delta S \quad \text{(kJ)}$$

$$q_{\text{int rev}} = T_0 \Delta s \quad \text{(kJ/kg)}$$



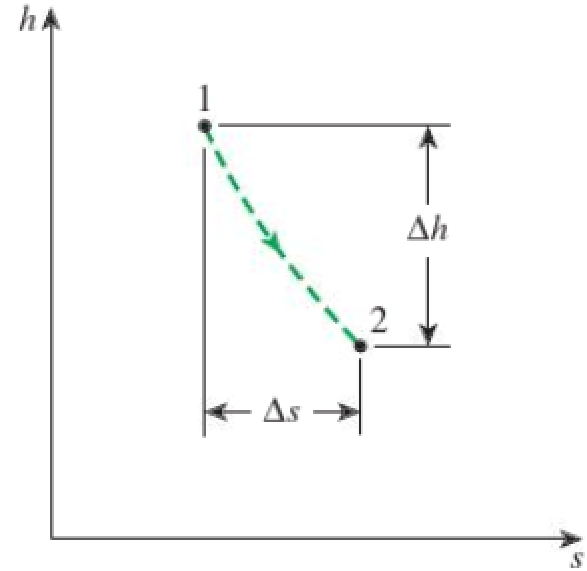
- Isentropic process = constant entropy
  - Vertical line segment



Property Diagrams Involving Entropy

h-s diagram (Mollier diagram):

- Useful in analysis of steady flow devices
  - Enthalpy *h*, primary property in 1st Law analysis
  - Entropy *s*, property that accounts for irreversibilities in adiabatic processes

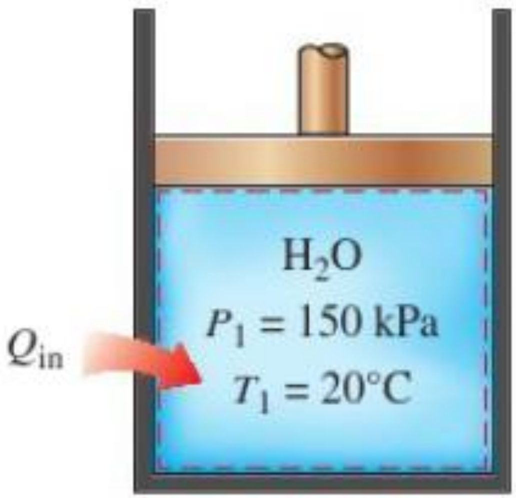


Example 1 – Entropy Change During a Constant Pressure Process

7-4: A piston-cylinder device initially contains 1.5 kg of liquid water at 150 kPa and 20°C. The water is now heated at constant pressure by the addition of 4000 kJ of heat. Determine the entropy change of the water during this process.

Assumptions: Tank is stationary; no change to K.E or P.E; $\Delta E = \Delta U$

It is a closed system with moving boundary work. Heating at constant pressure causes fluid to expand and there is work done W by the fluid.

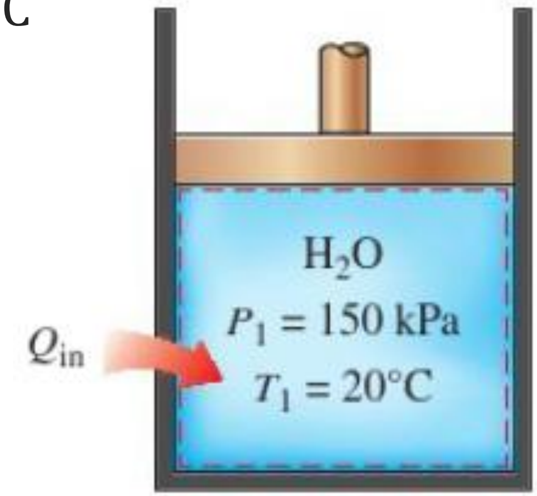


Example 1 – Entropy Change During a Constant Pressure Process

Entropy change: $\Delta S_{sys} = S_2 - S_1 = m(s_2 - s_1)$

The entropy values at the start (state 1) and at the end (state 2) must be known.

State 1 (initial state): $P_1 = 150 \text{ kPa}, \quad T_1 = 20°\text{C}$



From Table A-4:

$$s_1 \cong s_{f@20°\text{C}} = 0.2965 \text{ kJ/kgK}$$

$$h_1 \cong h_{f@20°\text{C}} = 83.915 \text{ kJ/kgK}$$

Why is the enthalpy value needed?

Example 1 – Entropy Change During a Constant Pressure Process

To determine state 2 (end state), use the 1st Law analysis for a closed system in a constant pressure process

$$Q - W = \Delta U$$

$$Q = W + \Delta U$$

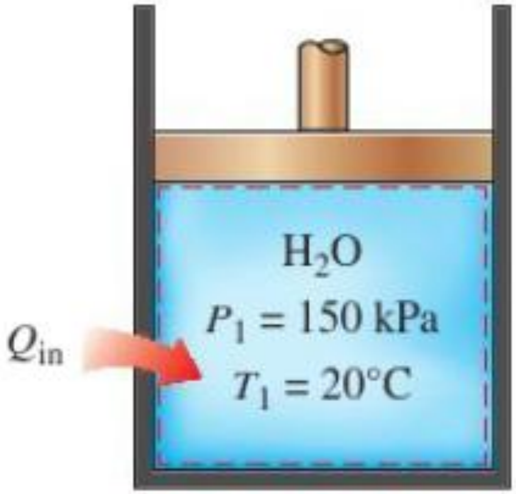
$$Q = P\Delta V + \Delta U = \Delta H \qquad \because \Delta H = P\Delta V + \Delta U$$

$$\therefore Q = m(h_2 - h_1)$$

$$4000 \text{ kJ} = (1.5 \text{ kg})(h_2 - 83.915) \text{ kJ/kg}$$

$$h_2 = 2750.6 \text{ kJ/kg}$$

At the same time, since it is a constant pressure process, $P_2 = P_1 = 150$ kPa



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Example 1 – Entropy Change During a Constant Pressure Process

State 2: $P_2 = 150\ \mathrm{kPa}$ $h_2 = 2750.6\ \mathrm{kJ/kg}$

Looking through the property tables, state 2 is found to be in the superheated state (Table A-6).

By interpolation:

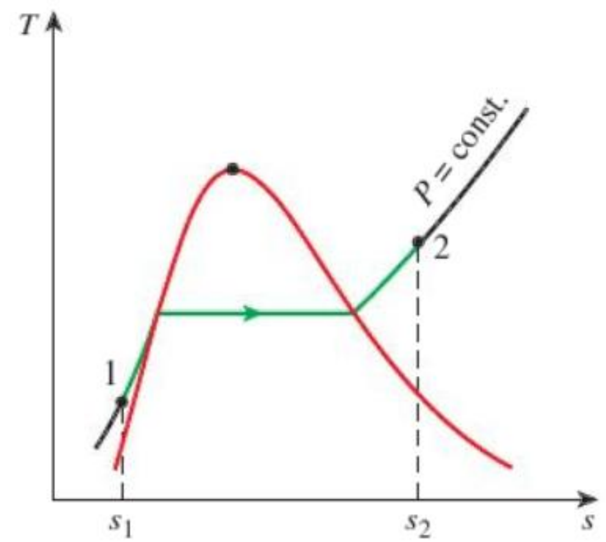
$$s_2 = 7.3926\ \mathrm{kJ/kg \cdot K}$$

Hence, the entropy change is:

$$\Delta S = S_2 - S_1 = m(s_2 - s_1)$$

$$= (1.5\ \mathrm{kg})(7.3926 - 0.2965)\ \mathrm{kJ/kg \cdot K}$$

$$= 10.64\ \mathrm{kJ/K}$$



1st *Tds* Equation (Gibbs Equation)

- Entropy in differential form:

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \quad (\text{kJ/K})$$

$$\delta Q_{\text{int rev}} = TdS \text{ (kJ)} \quad \text{or} \quad \delta q_{\text{int rev}} = Tds \text{ (kJ/kg)}$$

- From 1st Law:

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

- Where,

$$\delta W_{\text{int rev,out}} = PdV$$

- After substitution:

$$TdS - PdV = dU$$

$$TdS = dU + PdV \quad \text{or} \quad \boxed{Tds = du + Pdv}$$

2nd *Tds* Equation

- The 2nd *Tds* equation is obtained by replacing du with the definition of enthalpy: $h = u + Pv$

$$dh = du + Pdv + vdP \qquad \rightarrow du = dh - Pdv - vdP$$

- 1st *Tds* equation:

$$Tds = du + Pdv$$

- After substitution:

$$\boxed{Tds = dh - vdP}$$

- Entropy change can now be related to changes in other properties; these relations are independent of processes
- Alternative forms:

$$\boxed{ds = \frac{du}{T} + \frac{Pdv}{T}} \qquad \boxed{ds = \frac{dh}{T} - \frac{vdP}{T}}$$

Entropy Change for Liquids & Solids

- Liquids and solids can be approximated as incompressible substances, hence $dv \cong 0$

$$ds = \frac{du}{T} + \cancel{\frac{Pdv}{T}} \qquad\qquad ds = \frac{du}{T} = \frac{cdT}{T}$$

- For incompressible substances:

$$c_p = c_v = c \qquad\qquad \rightarrow du = cdT$$

- Integrating:

$$\boxed{s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{avg}\ln\frac{T_2}{T_1} \qquad (\mathrm{kJ/kg \cdot K})}$$

- Entropy change for liquids and solids are only dependent on temperature and independent of pressure
- For isentropic processes,

$$\Delta s = 0, \qquad \Delta T = 0$$

$$R_u = 8.314\ \mathrm{J/mol \cdot K}$$

Entropy Change for Ideal Gases

- For ideal gases:

$$R_{sp} = \frac{R_u}{\text{Molar Mass}}$$

$$Pv = R_{sp}T \qquad du = c_v dT \qquad dh = c_p dT$$

- Substituting into *Tds* relations:

$$\boxed{\begin{aligned} ds &= \frac{du}{T} + \frac{Pdv}{T} \\ ds &= c_v \frac{dT}{T} + R_{sp}\frac{dv}{v} \end{aligned}}$$

$$\boxed{\begin{aligned} ds &= \frac{dh}{T} - \frac{vdP}{T} \\ ds &= c_p \frac{dT}{T} - R_{sp}\frac{dP}{P} \end{aligned}}$$

- Integrating:

$$\boxed{s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R_{sp}\ln\frac{v_2}{v_1}}$$

$$\boxed{s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R_{sp}\ln\frac{P_2}{P_1}}$$

Entropy Change for Ideal Gases

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R_{sp}\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R_{sp}\ln\frac{P_2}{P_1}$$

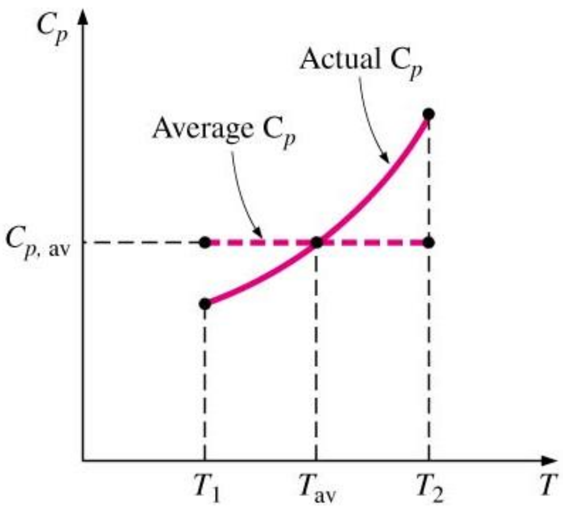
- Assuming constant (avg) specific heats:

$$c_v(T) = c_{v,avg}$$

$$c_p(T) = c_{p,avg}$$

$$s_2 - s_1 = c_{v,avg}\ln\frac{T_2}{T_1} + R_{sp}\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{p,avg}\ln\frac{T_2}{T_1} - R_{sp}\ln\frac{P_2}{P_1}$$



Assumption is sufficiently accurate when temperature range is less than a few hundred degrees

Entropy Change for Ideal Gases

- For isentropic process: $\Delta s = 0$
- From 1st *Tds* equation:

$$s_2 - s_1 = 0 = c_v \ln \frac{T_2}{T_1} + R_{sp} \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = -\frac{R_{sp}}{c_v} \ln \frac{v_2}{v_1} \qquad \rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\frac{R_{sp}}{c_v}}$$

- Since

$$R_{sp} = c_p - c_v \quad \& \quad k = c_p / c_v \qquad \rightarrow \frac{R_{sp}}{c_v} = k - 1$$

- Therefore:

$$\boxed{\left(\frac{T_2}{T_1}\right)_{isen} = \left(\frac{v_1}{v_2}\right)^{k-1}}$$

Entropy Change for Ideal Gases

$$R_{sp} = c_p - c_v$$

$$k = c_p / c_v$$

- For isentropic process: $\Delta s = 0$
- From 2nd *Tds* equation:

$$s_2 - s_1 = 0 = c_p \ln \frac{T_2}{T_1} - R_{sp} \ln \frac{P_2}{P_1}$$

$$\ln \frac{T_2}{T_1} = \frac{R_{sp}}{c_p} \ln \frac{P_2}{P_1} \qquad \rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R_{sp}}{c_p}}$$

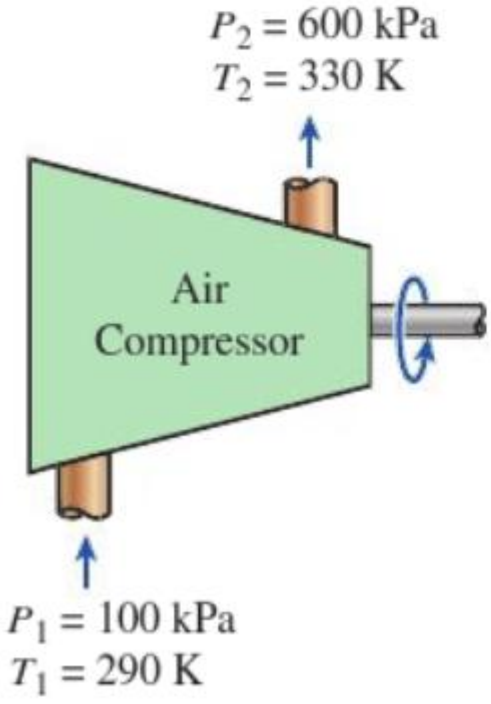
$$\boxed{\left(\frac{T_2}{T_1}\right)_{isen} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}$$

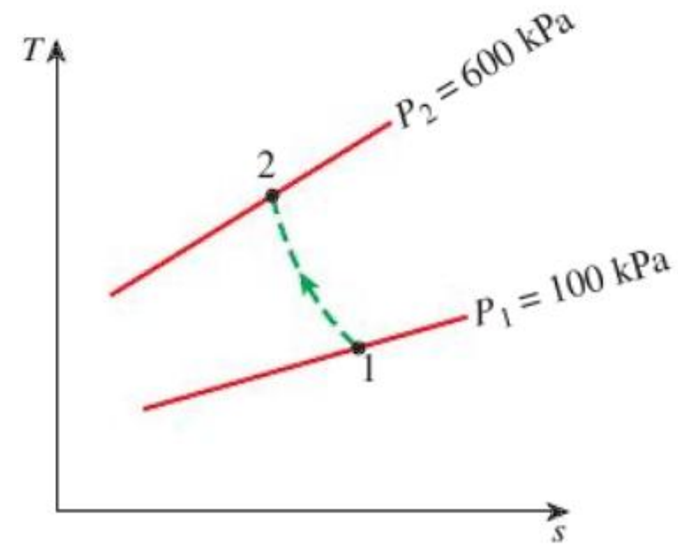
Previously: $\left(\frac{T_2}{T_1}\right)_{isen} = \left(\frac{v_1}{v_2}\right)^{k-1}$

- Therefore: $\boxed{\left(\frac{P_2}{P_1}\right)_{isen} = \left(\frac{v_1}{v_2}\right)^{k}}$ or $\boxed{Pv^k = \text{cnst}}$

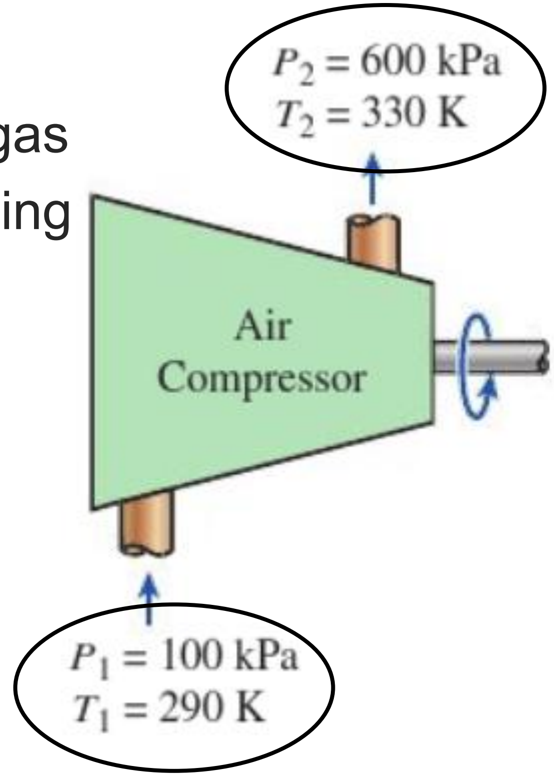
Example 2 – Entropy Change of an Ideal Gas

7-9: Air is compressed from an initial state at 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process by using average specific heats.





Example 2 – Entropy Change of an Ideal Gas



Assumption: Air is considered to be an ideal gas

Both initial and final stages are defined. Using Table A-2b, the average specific heat is:

$$T_{avg} = 310\text{K} \quad \rightarrow c_{p,avg} = 1.005\ \text{kJ/kg}\cdot\text{K}$$

Why c_p and not c_v?

$$s_2 - s_1 = c_{p,avg} \ln\frac{T_2}{T_1} - R_{sp} \ln\frac{P_2}{P_1}$$

$$= (1.005\ \text{kJ/kg}\cdot\text{K}) \ln\frac{330\ \text{K}}{290\ \text{K}} - (0.287\ \text{kJ/kg}\cdot\text{K}) \ln\frac{600\ \text{kPa}}{100\ \text{kPa}}$$

$$\Delta s = -0.3842\ \text{kJ/kg}\cdot\text{K}$$

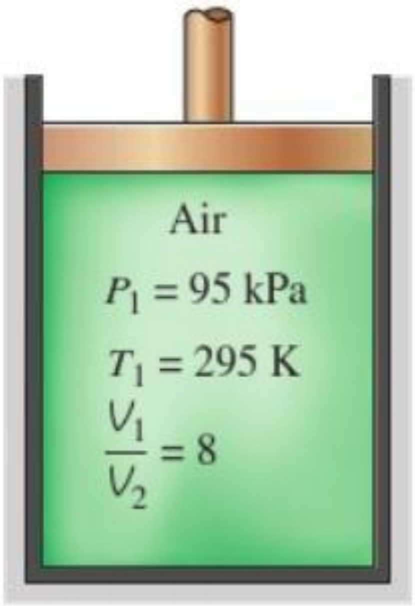
Negative value implies heat is lost to the surroundings

Example 3 – Isentropic Compression of Air in a Car Engine

7-10: Air initially at 22°C and 95 kPa is isentropically compressed in a car engine. If the compression ratio V_1/V_2 of this engine is 8, determine the final temperature of the air.

Assumption: The air is an ideal gas.

Constant specific heat for air



The air in the car engine is considered to be a closed system.

Example 3 – Isentropic Compression of Air in a Car Engine

For closed systems: $$\frac{V_1}{V_2} = \frac{v_1}{v_2} = 8$$

$k = c_p/c_v$ is a function of temperature. How to find k?

For ideal gas undergoing isentropic processes:

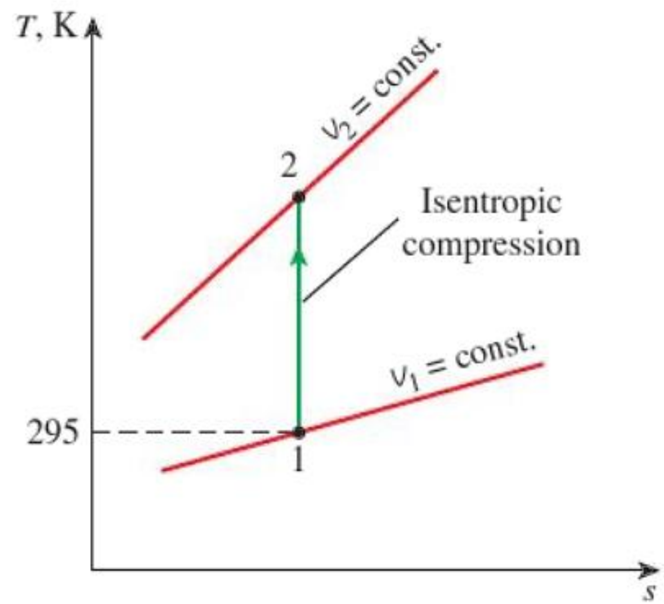
$$\left(\frac{T_2}{T_1}\right)_{isen} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Estimate average temp. to be 475 K.

From Table A-2B, k = 1.389

Final temperature:

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = (295\ \mathrm{K})(8)^{0.389} = 662.4\ \mathrm{K}$$



Check: *calculated* average temp. = 478.7 K

(Result is close enough)

Entropy Change – Reversible Processes

For reversible processes:

- Differential form:

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \quad (\text{kJ/K})$$

- Integrate the differential form to give the change in entropy for reversible processes:

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \quad (\text{kJ/K})$$

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Entropy Change – Reversible Processes

Special case: Internally reversible isothermal heat transfer

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} = \int_1^2 \left(\frac{\delta Q}{T_0}\right)_{\text{int rev}} = \frac{1}{T_0}\int_1^2 (\delta Q)_{\text{int rev}}$$

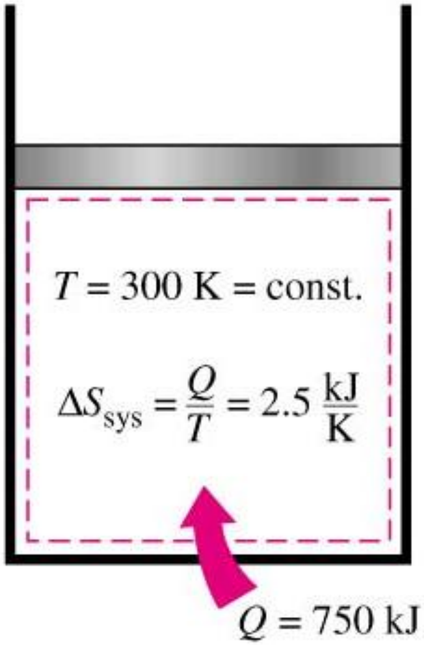
- The above reduces to:

$$\boxed{\Delta S = \frac{Q}{T_0} \quad (\mathrm{kJ/K})}$$

- Where T_0 is the constant temperature (K), Q is the heat transfer for the internally reversible process
- Commonly used to determine the entropy change of thermal energy reservoirs that supply/absorb heat indefinitely at constant temperature

Example 4 – Entropy Change During an Isothermal Process

7-1: A piston-cylinder device contains a liquid-vapour mixture of water at 300 K. During a constant pressure process, 750 kJ of heat is transferred to the water, vaporising part of the liquid. Determine the entropy change of water during this process.



Assumptions:

- Take the entire contents as a closed system
- System undergoes an internally reversible, isothermal process

$$\Delta S_{sys,isothermal} = \frac{Q}{T_{sys}} = \frac{750\text{ kJ}}{300\text{ K}} = 2.5\text{ kJ/K}$$

Entropy change is positive since heat is transferred into the system

Entropy Change – Irreversible Processes

Reversible Vs Irreversible Processes

$$\Delta S_{sys} = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \quad (\text{kJ/K})$$

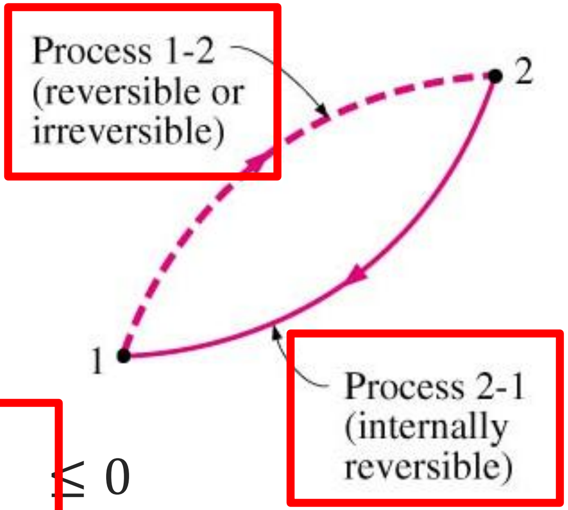
- The integral will give the value of entropy change only if the integration is carried out along an internally reversible path between two states
- For irreversible process, the entropy change can be determined along an imaginary internally reversible path

Entropy Change – Irreversible Processes

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \qquad (\text{kJ/K})$$

For Irreversible Processes:

- Consider a cycle where:
  - Process 1 to 2 is arbitrary
  - Process 2 to 1 is reversible



- Clausius inequality: $$\oint \left(\frac{\delta Q}{T}\right)_{\text{int rev}} \leq 0$$
- Integrate for the cycle: $$\int_1^2 \frac{\delta Q}{T} + \boxed{\int_2^1 \left(\frac{\delta Q}{T}\right)_{\text{int rev}}} \leq 0$$
- 2nd term is entropy change:

$$\int_1^2 \frac{\delta Q}{T} + \boxed{(S_1 - S_2)} \leq 0 \qquad \rightarrow S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

- Differential form: $$dS \geq \frac{\delta Q}{T}$$

Entropy change of a **closed system** for an **irreversible process** is **greater than** $\int \delta Q/T$ evaluated for that process!

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Entropy Change – Irreversible Processes

For Irreversible Processes:

$$S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

- The "additional" entropy change for irreversible processes is due to entropy generation due to the presence of irreversibilities

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

$$\Delta S_{sys} = \int_1^2 \frac{\delta Q}{T} + S_{gen}$$

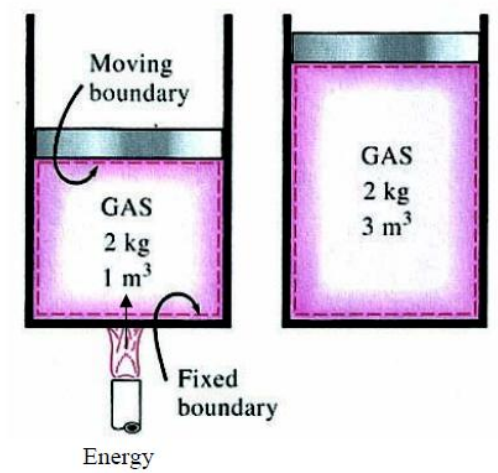
Entropy change of a closed system	=	Entropy transfer due to heat transfer	+	Entropy generation due to irreversibilities

- Entropy generation S_{gen}:
  - Always positive (irreversible) or zero (reversible)
  - Not a property; value is process dependent

Recap: Systems

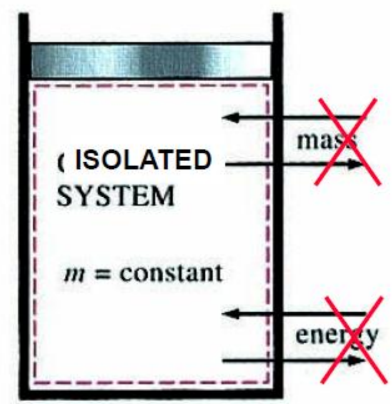
1. Closed Systems

- Fixed mass
- Energy can be transferred through boundary by heat or work
- Boundary can be movable



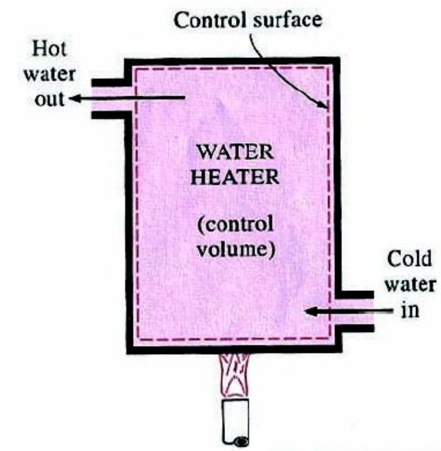
Isolated

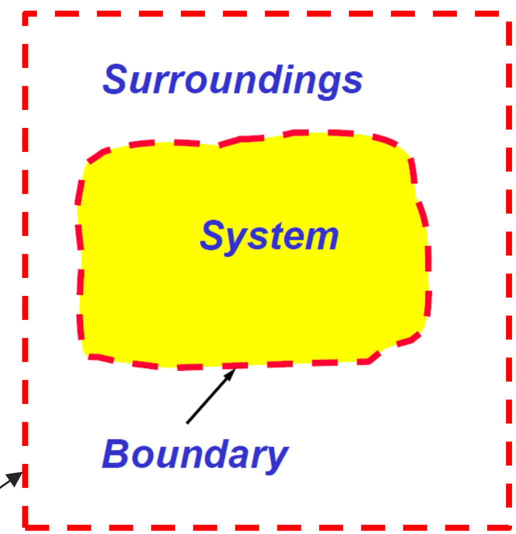
No mass flow, no energy transferred through boundary



E.g. A system of interest and its surroundings constitutes an isolated system

2. Flow Systems

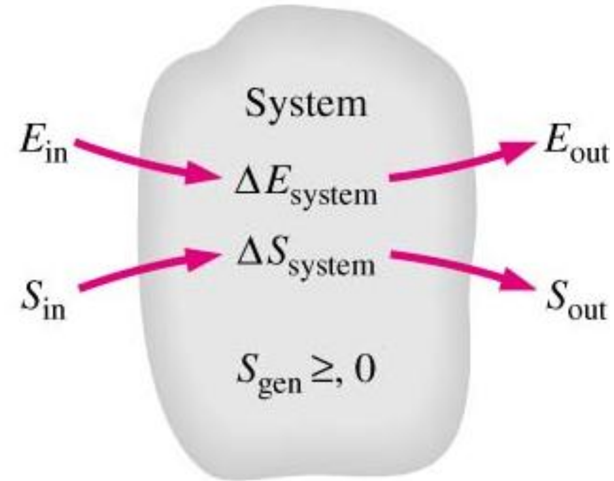




Isolated system boundary

Entropy Change – Systems

Entropy change of a system during a process is equal to the net entropy transfer through the system boundary and entropy generated within the system



$$\Delta E_{system} = E_{in} - E_{out}$$
$$\Delta S_{system} = S_{in} - S_{out} + S_{gen}$$

$$\boxed{\Delta S_{system} = S_{in} - S_{out} + S_{gen}}$$

Change in total entropy of the system = Total entropy entering − Total entropy leaving + Total entropy generated

Entropy Change: $\boxed{\boxed{\Delta S_{system}} = S_{in} - S_{out} + S_{gen}}$

- Entropy changes when the state of the system changes

$$\Delta S_{system} = S_{final} - S_{initial}$$

- If the entropy at each state is known, the entropy change between the states is simply:

$$\Delta S_{system} = S_{final} - S_{initial} = S_2 - S_1$$

System at time t_1: S_1

System at time t_2: S_2

- Entropy change is zero in steady flow devices during steady operation (turbines, compressors, pumps, nozzles, diffusers, heat exchangers etc.)

$$\Delta S_{system} = S_{final} - S_{initial} = 0$$

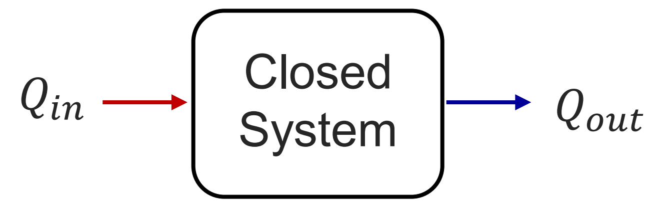
Entropy Change: $\Delta S_{system} = \boxed{S_{in} - S_{out}} + S_{gen}$

- Entropy transfer (S_{in} or S_{out}) does not transfer through work

$$S_{work} = 0$$

- For closed systems, entropy is transferred only by heat

$$S_{in} - S_{out} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$



Where,

- Q_k = heat transfer
- T_k = boundary temperature through which heat is transferred

Entropy Change: $\Delta S_{system} = S_{in} - S_{out} + \boxed{S_{gen}}$

Entropy generation (S_{gen})

- A measure of entropy created by irreversibilities during a process (friction, heat transfer via finite temperature difference, mixing etc.)
- Entropy generation is zero for reversible processes

$$S_{gen} \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$$

Entropy Change – Closed Systems

$$\boxed{\Delta S_{system} = S_{in} - S_{out} + S_{gen}}$$

For closed systems:

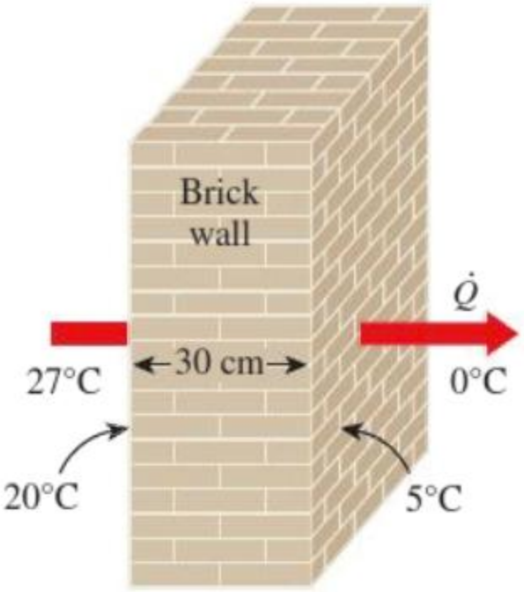
$$\Delta S_{closed} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

Where,

- Q_k = heat transfer
- T_k = boundary temperature through which heat is transferred

Example 5 – Entropy Change for Closed Systems

7-17: Consider steady heat transfer through a 5-m x 7-m brick wall of a house of thickness 30 cm. On a day when the temperature of the outdoors is 0°C, the house is maintained at 27°C. The temperatures of the inner and outer surfaces of the wall are measured to be 20°C and 5°C respectively, and the heat transfer rate is 1035 W. Determine the rate of entropy generation in the wall.



Assumptions: Steady-state heat transfer
One-dimensional heat transfer

The wall is taken to be a closed system.

$$\Delta S_{closed} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

Example 5 – Entropy Change for Closed Systems

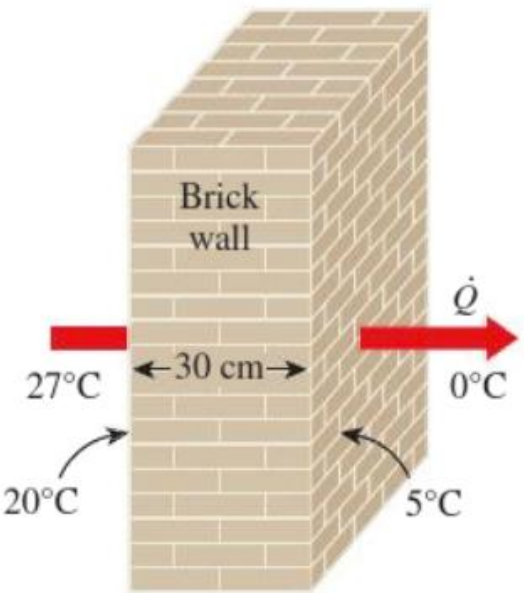
Rate form: $$\frac{dS_{sys}}{dt} = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

Steady-state: $$\cancel{\frac{dS_{sys}}{dt}} = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$\left(\frac{\dot{Q}}{T}\right)_{in} - \left(\frac{\dot{Q}}{T}\right)_{out} + \dot{S}_{gen} = 0$$

$$\frac{1035\text{ W}}{293\text{ K}} - \frac{1035\text{ W}}{278\text{ K}} + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = 0.191\text{ W/K}$$



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Closed Systems – Special cases

Adiabatic closed systems (no heat transfer):

$$\Delta S_{closed} = S_2 - S_1 = \cancel{\sum \frac{Q_k}{T_k}} + S_{gen}$$

$$\Delta S_{adiabatic\ system} = S_{gen} > 0$$

- In adiabatic closed systems, the change in entropy is only due to entropy generation within the system boundaries
- Since entropy generation for irreversible processes can never be less than zero, the entropy in an adiabatic closed system always increases

Closed Systems – Special cases

Isentropic Process (closed systems):

- If the process is $\boxed{\text{adiabatic}}$ (no heat transfer) and $\boxed{\text{reversible}}$:

$$\Delta S_{isen} = S_2 - S_1 = \cancel{\sum \frac{Q_k}{T_k}} + \cancel{S_{gen}}$$

$$\Delta S_{isen} = 0 \qquad \text{or} \qquad S_2 = S_1$$

- A reversible adiabatic process is also known as an isentropic process

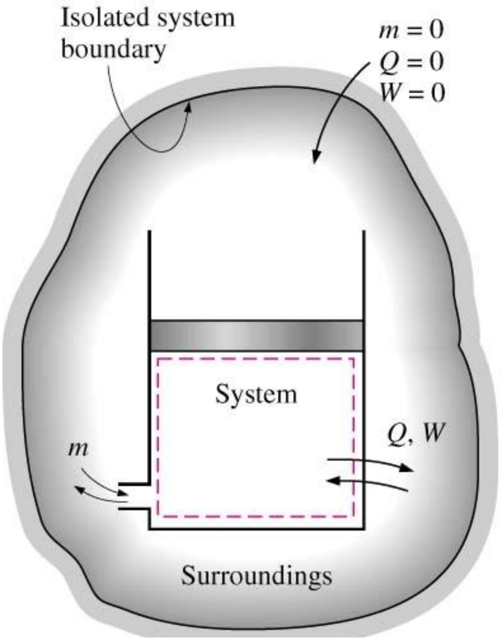
Closed Systems – Special cases

Isolated systems:

- No mass, heat or work transfer for an isolated system

$$\Delta S_{isolated} = S_2 - S_1 = \cancel{\sum \frac{Q_k}{T_k}} + S_{gen}$$

$$\Delta S_{isolated} = S_{gen} > 0$$



- The entropy for an isolated system must always increase: <u>Increase of Entropy Principle</u>

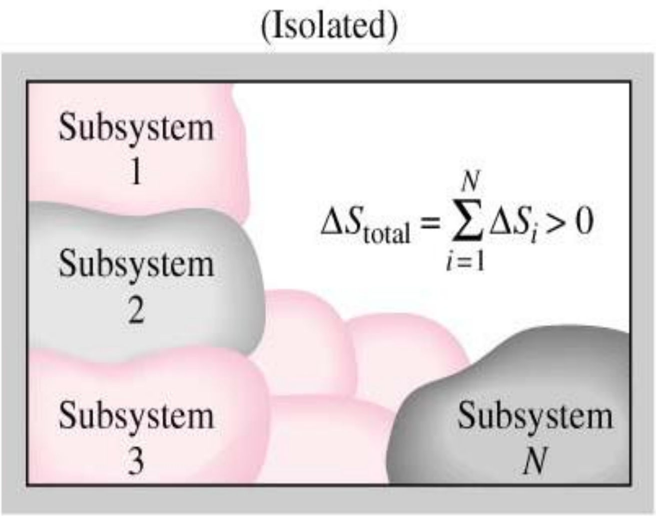
- *Lower limit only achieved through reversible processes, in which entropy remains constant*

Closed Systems – Special cases

Isolated systems:

- An isolated systems may consist of any number of subsystems
- Entropy is an extensive property; total entropy = sum of its parts

$$\Delta S_{total} = \sum_{i=1}^{N} \Delta S_i > 0$$



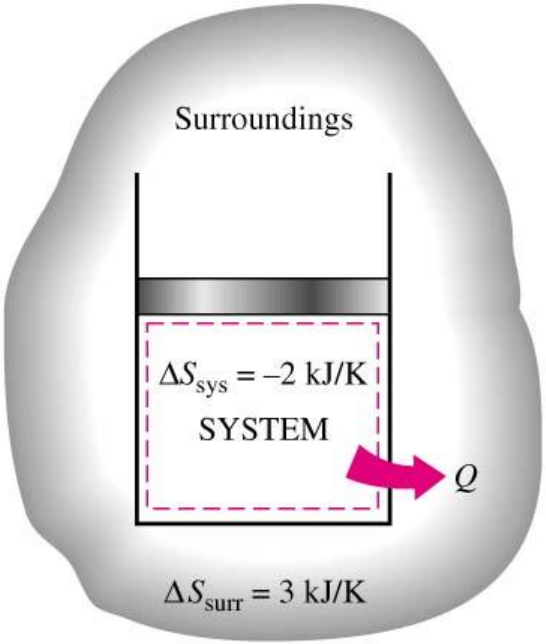
- E.g. A closed system and its surroundings constitutes an isolated system:

$$\Delta S_{sys} + \Delta S_{surr} = \Delta S_{isolated} = S_{gen} > 0$$

Closed Systems – Special cases

Closed system & its surroundings:

$$\Delta S_{sys} + \Delta S_{surr} = \Delta S_{isolated} = S_{gen} > 0$$



- For a closed system:

$$\Delta S_{sys} = S_2 - S_1$$

- Consider the surroundings as a thermal energy reservoir (no entropy generation) at temperature T_{surr} receiving heat Q from the system:

$$\Delta S_{surr} = \frac{Q}{T_{surr}}$$

$$S_2 - S_1 + \frac{Q}{T_{surr}} = S_{gen} > 0$$

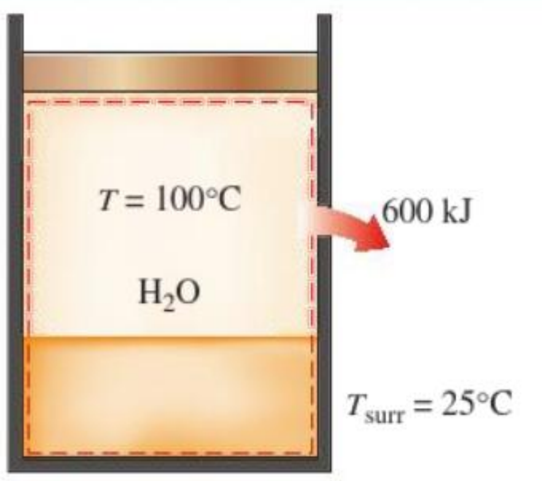
Having higher entropy generation means the process is more irreversible

Example 6 – Entropy Change for Closed Systems & Surroundings

7-21: A frictionless piston-cylinder device contains a saturated liquid-vapour mixture of water at 100°C. During a constant pressure process, 600 kJ of heat is transferred to the surrounding air at 25°C. As a result, part of the water vapour in the cylinder condenses. Determine (a) the entropy change of the water and (b) the total entropy change during this heat transfer process.

Assumptions: No irreversibilities within the system boundaries; process is internally reversible

Water temperature is constant everywhere including the boundaries



Example 6 – Entropy Change for Closed Systems & Surroundings

a) Water in the cylinder is a closed system:

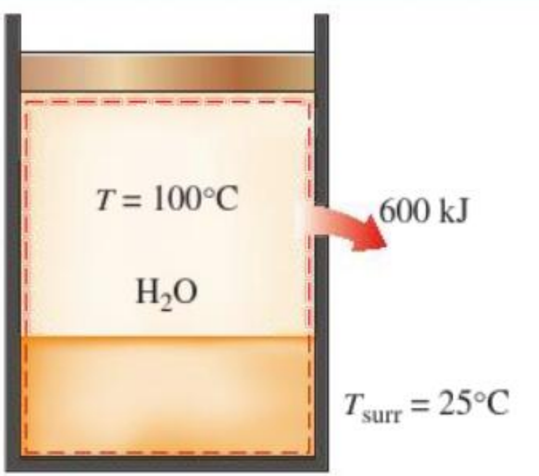
$$\Delta S_{sys} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

Entropy change for reversible process:

$$\Delta S_{sys} = \sum \frac{Q_k}{T_k} + \cancel{S_{gen}}$$

$$= \left(\frac{Q}{T_{sys}}\right) = \frac{-600\ \text{kJ}}{373\ \text{K}}$$

$$\Delta S_{sys} = -1.61\ \text{kJ/K}$$



Example 6 – Entropy Change for Closed Systems & Surroundings

b) Total entropy change: $\Delta S_{tot} = \Delta S_{sys} + \Delta S_{surr}$

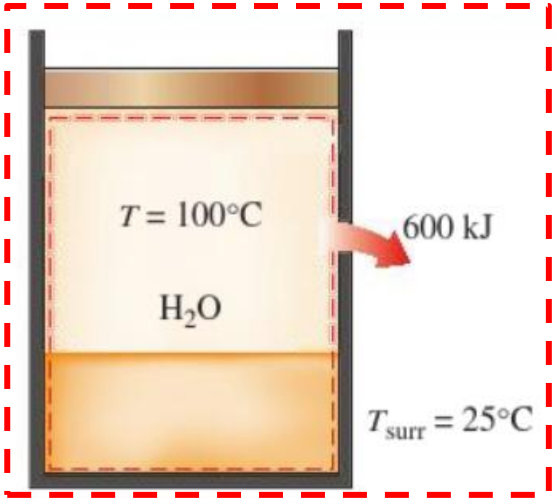
$$\Delta S_{surr} = \frac{Q}{T_{surr}}$$

$$= \frac{Q_{out}}{T_b} = \frac{600}{298}$$

$$= 2.01 \text{ kJ/K}$$

$$S_{tot} = \Delta S_{sys} + \Delta S_{surr}$$

$$= -1.61 + 2.01 = 0.40 \text{ kJ/K}$$

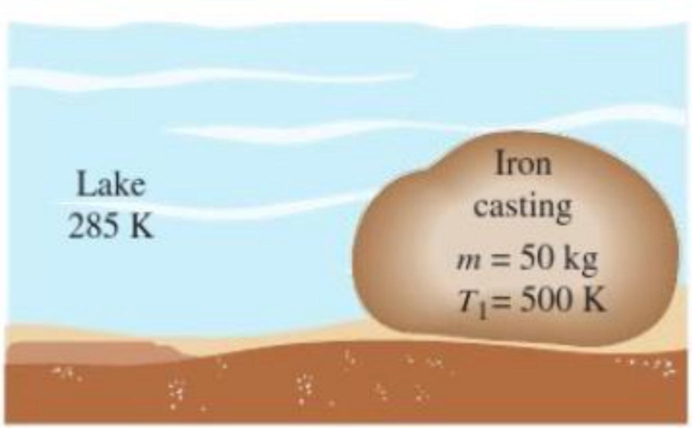


Example 7 – Entropy Generation

7-19: A 50-kg block of iron casting at 500 K is thrown into a large lake that is at a temperature of 285 K. The iron block eventually reaches thermal equilibrium with the lake water. Assuming an average specific heat of 0.45 kJ/kg.K for the iron, determine (a) the entropy change change of the iron block, (b) the entropy change of the lake water, and (c) the entropy generated during this process.

Assumptions:

- Both the water and iron block are incompressible
- Constant specific heats can be used for the water and iron
- K.E and P.E changes for the iron block are negligible; $\Delta E = \Delta U$



Example 7 – Entropy Generation

a) The iron casting block is a closed system.

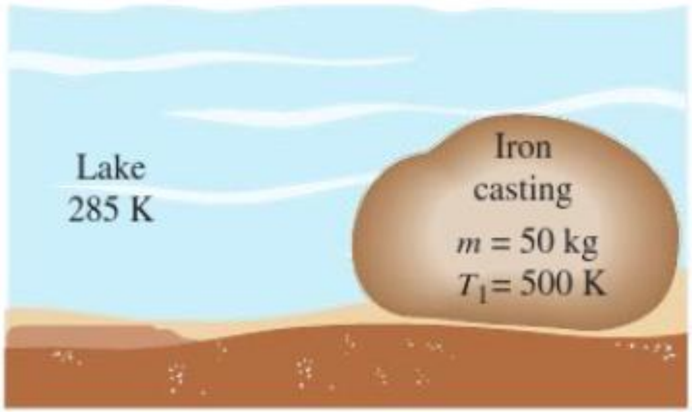
$$s_2 - s_1 = c_{avg} \ln \frac{T_2}{T_1}$$

$$\Delta S_{sys} = m(s_2 - s_1) = mc_{avg} \ln \frac{T_2}{T_1}$$

$$= (50 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K}) \ln \frac{285}{500}$$

$$= -12.65 \text{ kJ/K}$$

Can the same equation be used for entropy change in the lake?



Example 7 – Entropy Generation

b) The lake is considered to be a thermal energy reservoir: $\Delta S_{surr} = \dfrac{Q}{T_{surr}}$

Q is the heat transfer from the iron block as it cools to equilibrium:

$$Q = mc_{avg}(T_2 - T_1) = (50 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K})(285 \text{ K} - 500 \text{ K})$$

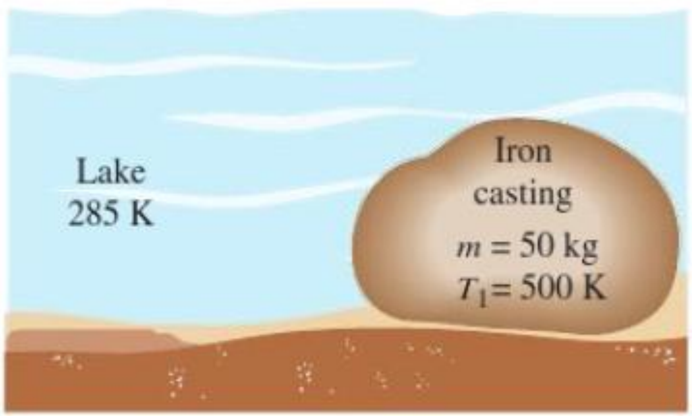
$$= -4838 \text{ kJ}$$ (Negative; heat is lost from the block)

Entropy change for the lake:

$$\Delta S_{lake} = \frac{Q_{lake}}{T_{lake}}$$

$$= \frac{4838 \text{ kJ}}{285 \text{ K}}$$ (Positive; heat is gained by the lake)

$$= 16.97 \text{ kJ/K}$$



Example 7 – Entropy Generation

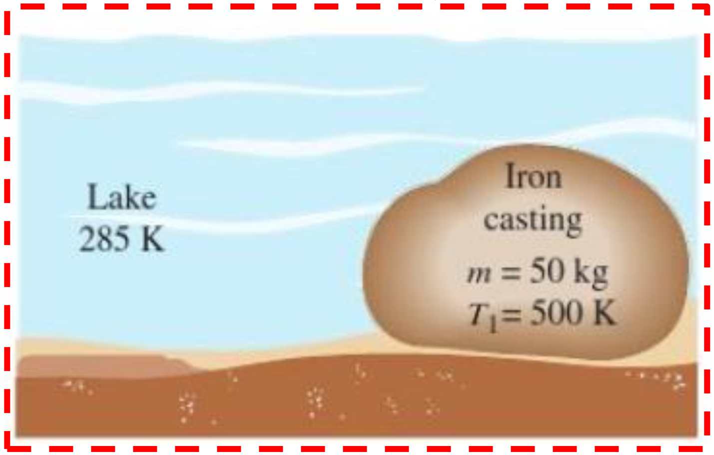
c) Entropy generation for the whole process:

$$\Delta S_{sys} + \Delta S_{surr} = \Delta S_{isolated} = S_{gen} > 0$$

$$S_{gen} = \Delta S_{sys} + \Delta S_{lake}$$

$$= -12.65 + 16.97 = 4.32 \text{ kJ/K}$$

Isolated system boundary



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Entropy Change – Remarks

- Entropy change of a **closed system** may be negative (e.g. heat removal) but entropy generation can never be negative
  - when a system loses heat to the surroundings, entropy change in the surroundings is positive
- Entropy change of an **isolated system** can never be negative
  - E.g. Heat transfer between system and surroundings usually involve a finite temperature difference which is an irreversibility. Hence, there is entropy generation

$$\Delta S_{iso} = S_{gen}$$

$$\Delta S_{iso} = \Delta S_{closed} + \Delta S_{surr} = S_{gen}$$

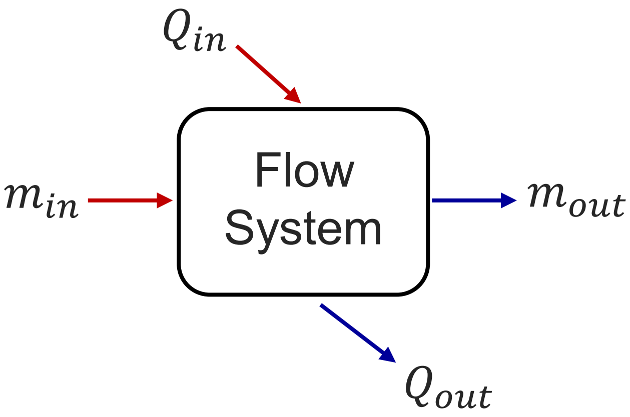
Entropy Balance – Flow Control Volumes

Entropy Transfer by Mass Flow

- Flow control volumes involve mass inflow, mass outflow, or both
- Mass contains entropy as well as energy, in proportion to the mass and the state condition
- Hence,

$$S_{in,\,mass} = m_i s_i$$

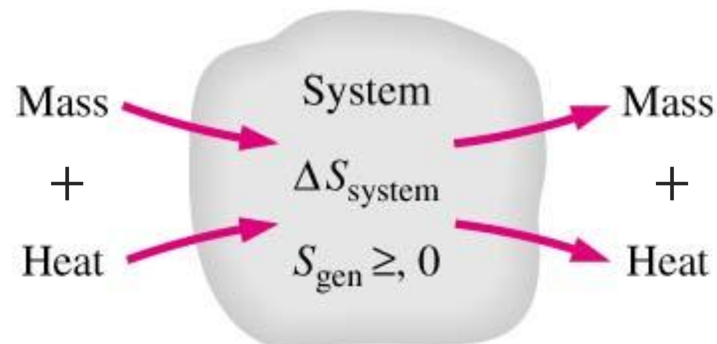
$$S_{out,\,mass} = m_e s_e$$



Entropy Balance – Flow Control Volumes

Entropy Change for Flow Control Volume:

$$\overset{\text{Change in entropy}}{\Delta S_{CV}} = \overbrace{S_{in} - S_{out}}^{\text{Net entropy transfer by heat and mass flow}} + \overset{\text{Entropy generation}}{S_{gen}}$$



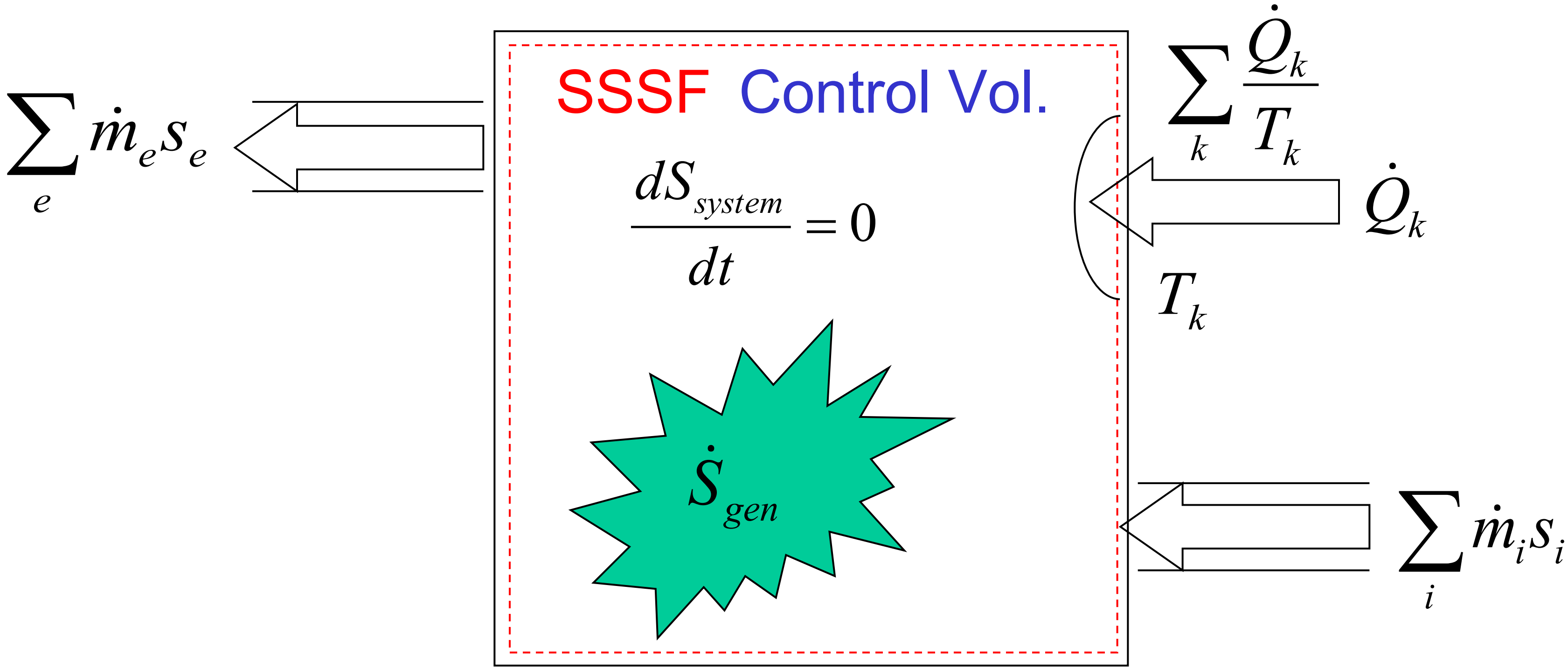
$$(S_2 - S_1)_{CV} = \sum m_i s_i - \sum m_e s_e + \sum \frac{Q_k}{T_k} + S_{gen}$$

Rate form:

$$\dot{S}_{CV} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

Entropy Balance – Flow Control Volumes

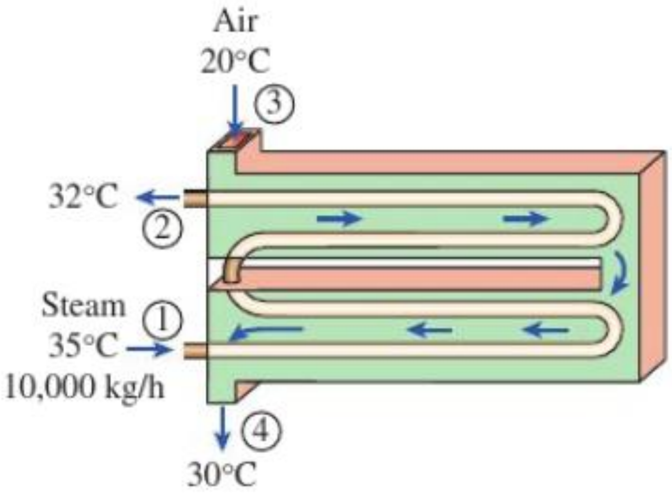
Consider a Steady State; Steady Flow Control Volume:



$$\sum_k \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e = 0$$

Example 8 – Entropy Generation in a Heat Exchanger

Air in a large building is kept warm by heating it with steam in a heat exchanger. Saturated water vapour enters this unit at 35°C at a rate of 10,000 kg/h and leaves as a saturated liquid at 32°C. Air at 1 atm pressure enters the unit at 20°C and leaves at 30°C at about the same pressure. Determine the rate of entropy generation associated with this process.



Assumption:
- SS control volume
- No heat loss to surroundings
- Negligible ΔK.E and ΔP.E.
- Air is an ideal gas
- Air pressure is constant

Example 8 – Entropy Generation in a Heat Exchanger

Entropy change for SSSF:

$$\sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \cancel{\sum \frac{\dot{Q}_k}{T_k}} + \dot{S}_{gen} = 0$$

$$\dot{m}_w s_1 + \dot{m}_a s_3 - \dot{m}_w s_2 - \dot{m}_a s_4 + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \dot{m}_w (s_2 - s_1) + \dot{m}_a (s_4 - s_3)$$

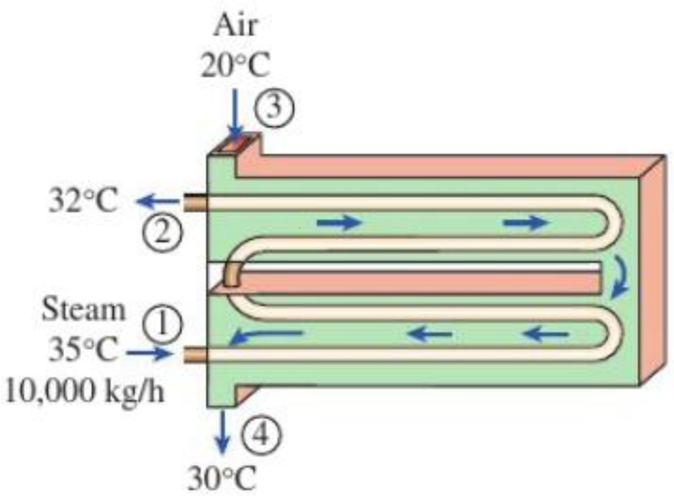
Properties for steam/water (Table A4):

$$T_1 = 35°\mathrm{C}, h_1 = 2564.6\,\mathrm{kJ/kg}\,, s_1 = 8.3517\,\mathrm{kJ/kg \cdot K}$$

$$T_2 = 32°\mathrm{C}, h_2 = 134.1\,\mathrm{kJ/kg}\,, s_2 = 0.4641\,\mathrm{kJ/kg \cdot K}$$

Properties for air (Table A2):

$$c_p = 1.005\,\mathrm{kJ/kg \cdot K}$$



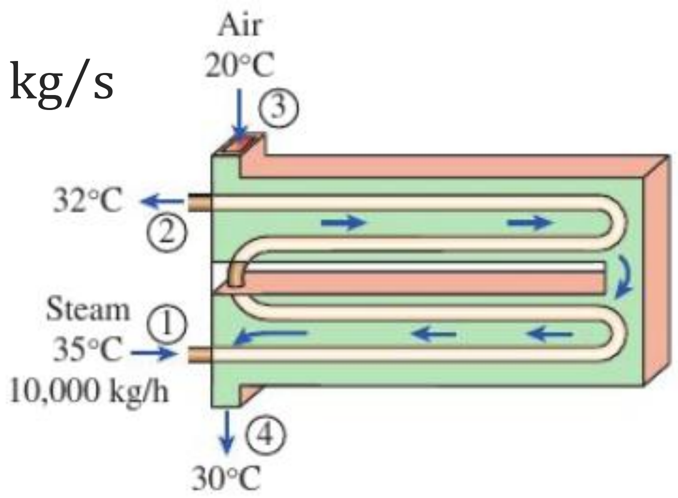
Example 8 – Entropy Generation in a Mixing Chamber

Energy balance for heat exchanger (flow control volume):

$$\sum \dot{m}_i h_i - \sum \dot{m}_e h_e + \dot{Q} - \dot{W} = 0$$

$$\dot{m}_w(h_1 - h_2) + \dot{m}_a c_p(T_3 - T_4) = 0$$

$$\dot{m}_a = \frac{\dot{m}_w(h_1 - h_2)}{c_p(T_4 - T_3)} = \frac{6751}{1.005(30 - 20)} = 671.7\,\mathrm{kg/s}$$



Entropy balance:

$$\dot{S}_{gen} = \dot{m}_w(s_2 - s_1) + \dot{m}_a(s_4 - s_3)$$

$$= \dot{m}_w(s_2 - s_1) + \dot{m}_a\left(c_p \ln\frac{T_4}{T_3} - R_{sp} \ln\frac{P_4}{P_3}\right)$$

$$= \left(\frac{10000}{3600}\right)(0.4641 - 8.3517) + 671.7\left(1.005 \ln\frac{30 + 273}{20 + 273}\right) = 0.745\,\mathrm{kW/K}$$

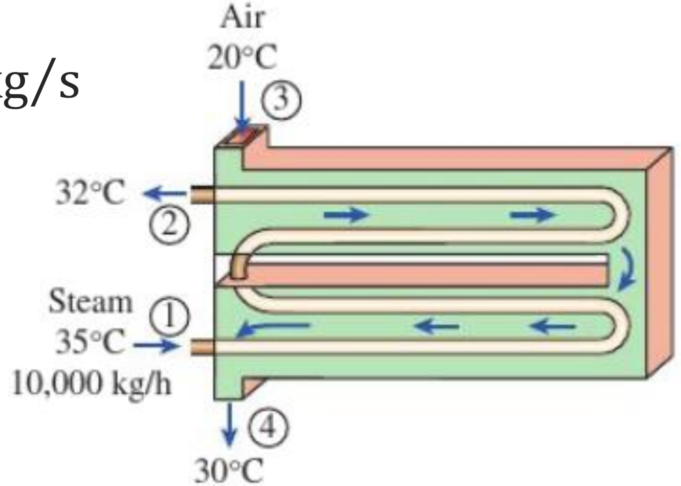
Example 8 – Entropy Generation in a Mixing Chamber

Heat transfer between steam and air:

$$\dot{Q} = \dot{m}_w(h_1 - h_2) = \left(\frac{10000}{3600}\right)(2564.6 - 134.1)\ \mathrm{kJ/kg} = 6751\ \mathrm{kW}$$

$$\dot{Q} = \dot{m}_a c_p(T_4 - T_3)$$

$$\dot{m}_a = \frac{\dot{Q}}{c_p(T_4 - T_3)} = \frac{6751}{1.005(30 - 20)} = 671.7\ \mathrm{kg/s}$$



Entropy balance:

$$\dot{S}_{gen} = \dot{m}_w(s_2 - s_1) + \dot{m}_a(s_4 - s_3)$$

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$$= \left(\frac{10000}{3600}\right)(0.4641 - 8.3517) + 671.7\left(1.005\ln\frac{30 + 273}{20 + 273}\right) = 0.745\ \mathrm{kW/K}$$

Flow Control Volumes – Special Cases

$$\dot{S}_{CV} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

- For steady state; steady flow process: $\dot{S}_{CV} = 0$

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

- For single stream, steady flow process:

$$\dot{m}_i = \dot{m}_e = \dot{m}$$

$$\dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

Flow Control Volumes – Special Cases

- For single stream, steady flow, and adiabatic process:

$$\dot{m}(s_e - s_i) = \cancel{\sum \frac{\dot{Q}_k}{T_k}} + \dot{S}_{gen}$$

$$\dot{S}_{gen} \geq 0 \qquad \rightarrow s_e \geq s_i$$

  – Entropy of fluid will increase as it flows through an adiabatic device

- If the device/process is adiabatic and reversible:

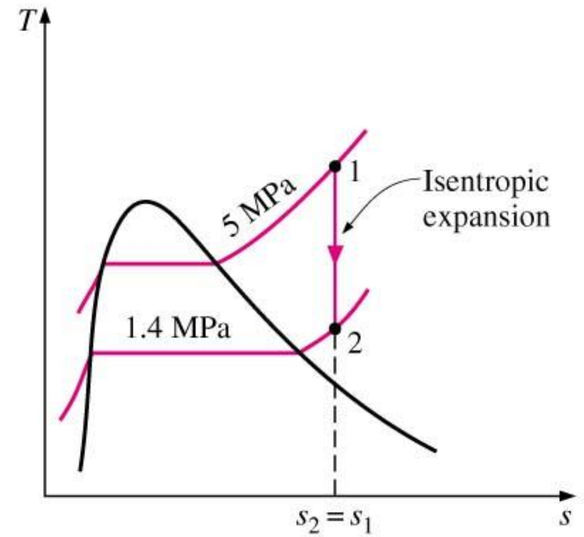
$$\dot{m}(s_e - s_i) = \cancel{\sum \frac{\dot{Q}_k}{T_k}} + \cancel{\dot{S}_{gen}}$$

$$\rightarrow s_e = s_i$$

  – Isentropic flow (constant entropy)

Example 9 – Isentropic Expansion of Steam in a Turbine

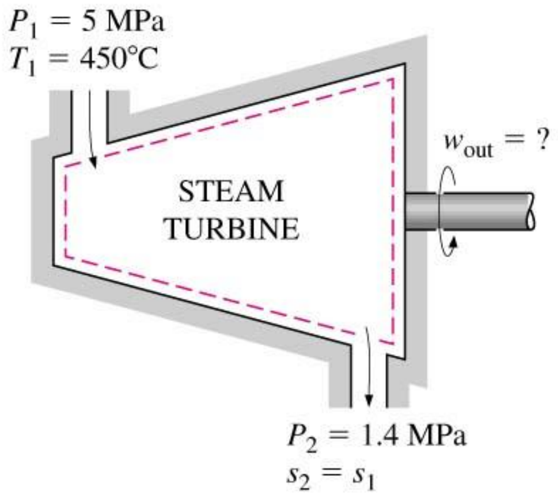
Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.



- Turbine as control volume; negligible change in P.E & K.E
- State 1 (refer to property tables)

$$P_1 = 5\,\text{MPa}, \quad T_1 = 450^\circ C,$$
$$s_1 = 6.8186\,\text{kJ/kgK}$$
$$h_1 = 3316.2\,\text{kJ/kg}$$

- State 2?



Example 9 – Isentropic Expansion of Steam in a Turbine

State 2:

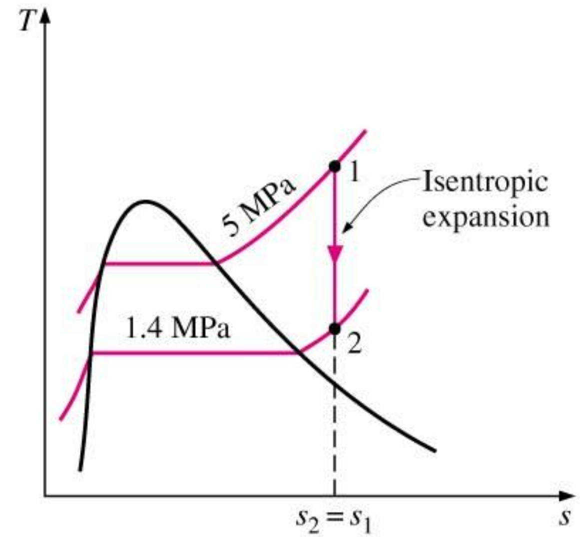
$$P_2 = 1.40 \text{ MPa}$$

Reversible adiabatic:

$$s_2 = s_1 = 6.8186 \, \text{kJ/(kg} \cdot \text{K)}$$

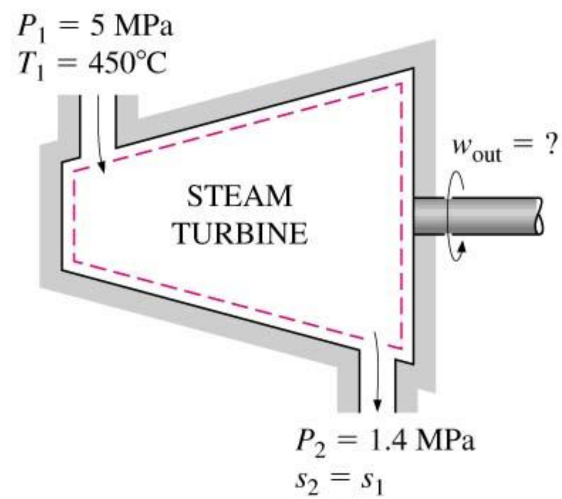
From property table:

$$h_2 = 2966.6 \ \text{kJ/kg}$$



For work output, use 1st Law for steady-state control volume:

$$\cancel{\dot{Q}} - \dot{W} = \dot{m}(h_2 - h_1)$$



Work output per unit mass:

$$\dot{w} = h_1 - h_2$$

$$= 3316.2 - 2966.6 = 349.6 \, \text{kJ/kg}$$

Reversible Steady Flow Work

- Consider the differential energy balance equation for a steady state, steady flow device undergoing an internally reversible process:

$$\delta q_{rev} - \delta w_{rev} = dh + d(\mathrm{ke}) + d(\mathrm{pe})$$

- From *Tds* equations:

$$\delta q_{rev} = Tds \;\; \& \;\; Tds = dh - vdP \qquad \therefore \delta q_{rev} = dh - vdP$$

- Substituting:

$$\cancel{dh} - vdP - \delta w_{rev} = \cancel{dh} + d(\mathrm{ke}) + d(\mathrm{pe})$$

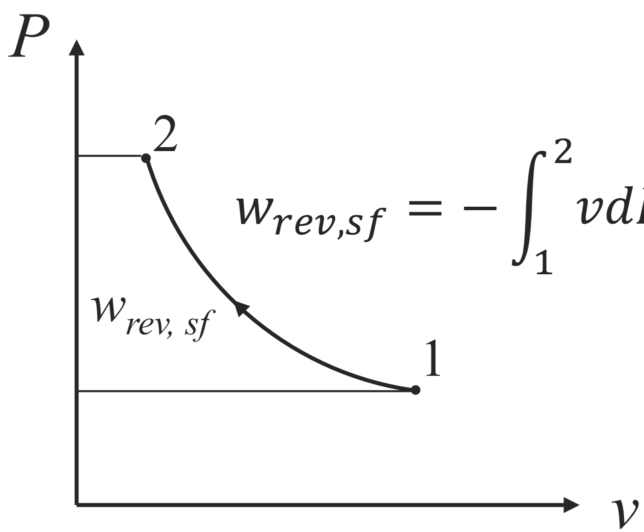
$$-\delta w_{rev} = vdP + d(\mathrm{ke}) + d(\mathrm{pe})$$

- Integrating: $w_{rev} = -\int_i^e vdP - \Delta\mathrm{ke} - \Delta\mathrm{pe}$

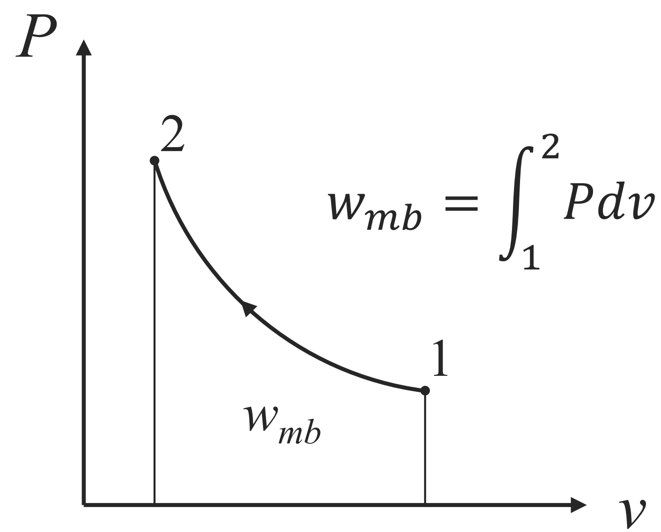
- When $\Delta\mathrm{ke} \cong 0$ & $\Delta\mathrm{pe} \cong 0$: $\boxed{w_{rev} = -\int_i^e vdP}$

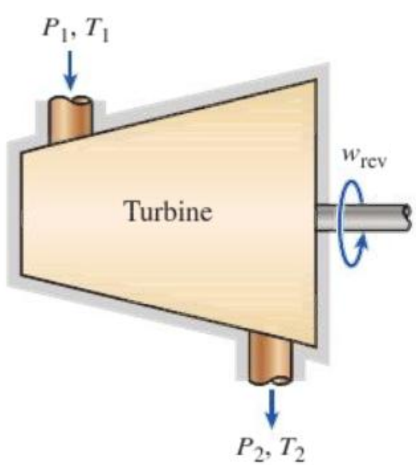
Reversible Steady Flow Work

- Meaning of $\int_1^2 v\,dP$?

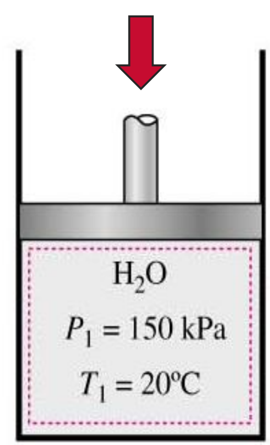


$\neq$





Reversible, steady flow work



Moving boundary work

Reversible Steady Flow Work – Special Cases

$$w_{rev} = -\int_1^2 v\,dP$$

- For incompressible fluids (constant v) and $\Delta \mathrm{ke} \cong 0$ & $\Delta \mathrm{pe} \cong 0$ (e.g. pumps):

$$w_{rev} = -v(P_2 - P_1)$$

- In flow devices where it does no work and fluid is (e.g. nozzles or pipes)

$$0 = -v(P_2 - P_1) - \frac{\mathcal{V}_2^2 - \mathcal{V}_1^2}{2} - g(z_2 - z_1)$$

Bernoulli Equation

Reversible Steady Flow Work – Fluid Types

- Examining: $\boxed{w_{rev} = -\int_1^2 v dP}$

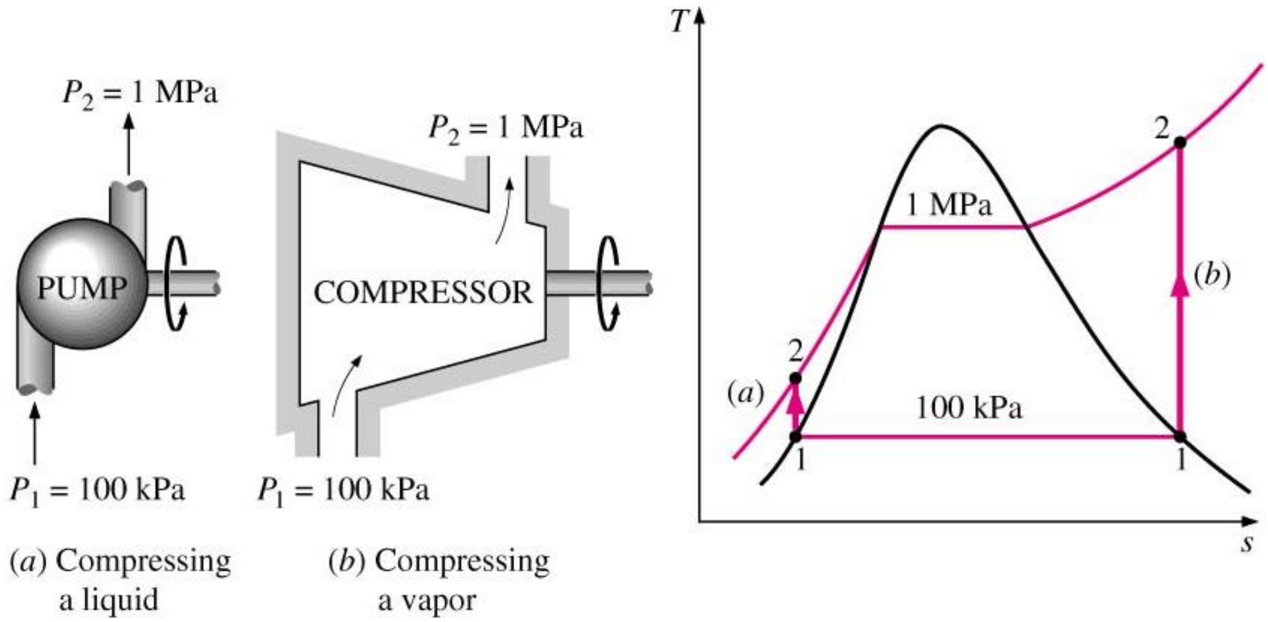
- Reversible steady work flow is closely associated to specific volume v of the fluid flow through the device.
  - Larger specific volume = larger work produced/consumed

- Pumps handle liquids (small specific volume) and hence consumes less power

- Compressors handle gases (large specific volume) and tends to consume more power

Example 10 – Compressing a Liquid VS Compressing a Gas

Determine the work input to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (a) saturated liquid at the inlet; and (b) saturated vapour at the inlet.



Example 10 – Compressing a Liquid VS Compressing a Gas

a) Pump ($v \approx$ constant)

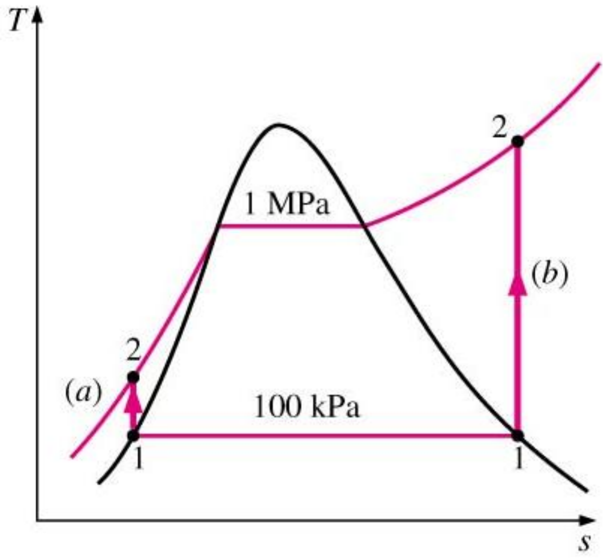
From property tables:

$$v_1 \approx v_{f,@100\ \mathrm{kPa}} = 0.001043\ \mathrm{m^3/kg}$$

$$w_{rev} = -\int_1^2 v dP = -v_1(P_2 - P_1)$$

$$= -(0.001043\ \mathrm{m^3/kg})[(1000 - 100)\mathrm{kPa}]$$

$$= -0.94\ \mathrm{kJ/kg}$$



Example 10 – Compressing a Liquid VS Compressing a Gas

b) Compressors (for gases)

From *Tds* relations:

$$Tds = dh - vdP$$

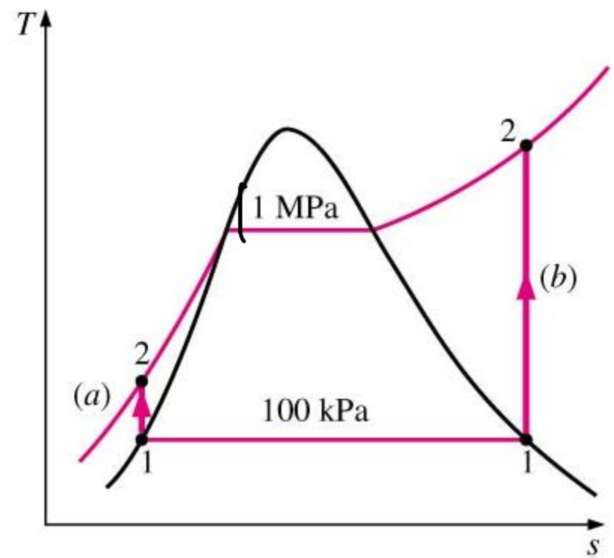
For isentropic process:

$$ds = 0 \qquad dh = vdP$$

$$w_{rev} = -\int_1^2 vdP = -\int_1^2 dh$$

$$= -(h_2 - h_1)\ \mathrm{kJ/kg}$$

Need to find properties at State 2



Example 10 – Compressing a Liquid VS Compressing a Gas

b) Compressors (for gases)

State 1:

$$P_1 = 100\,\text{kPa}, \quad \text{sat. vapour}$$
$$s_1 = 7.3594\,\text{J/kgK}$$
$$h_1 = 2675.5\,\text{kJ/kg}$$

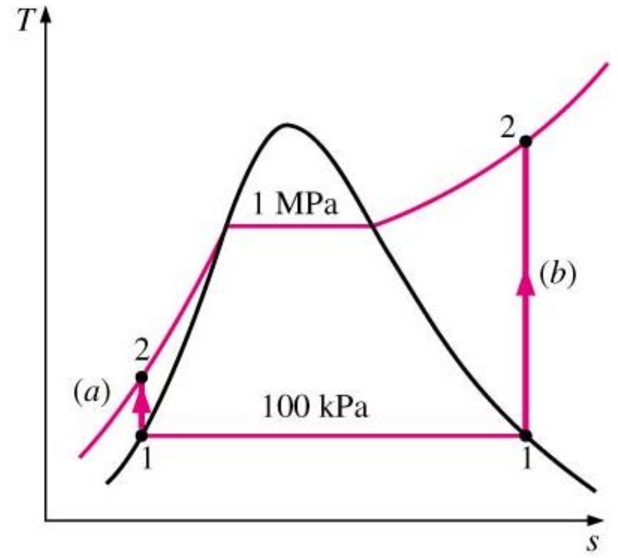
State 2:

$P_2 = 1$ MPa

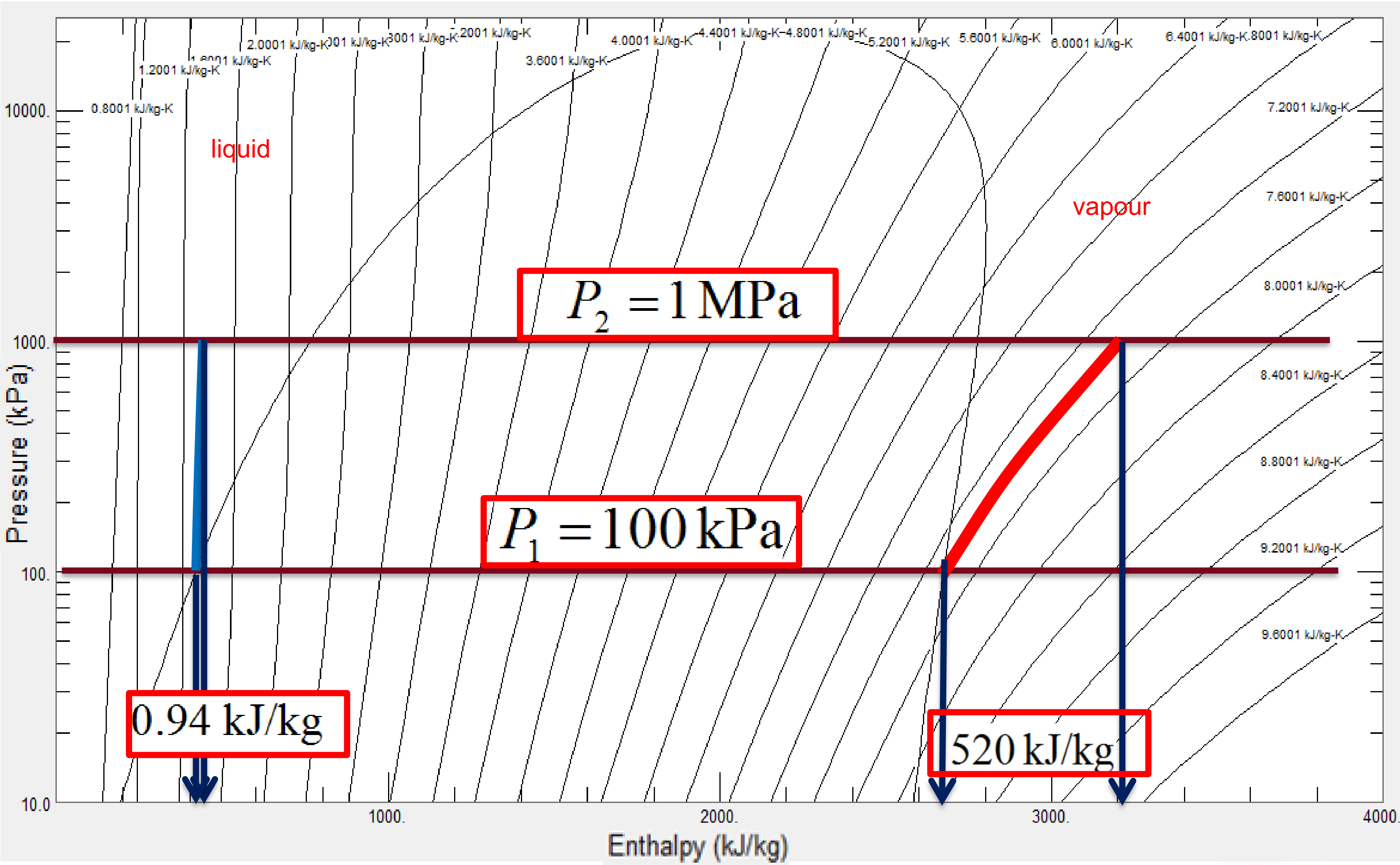
$s_2 = s_1 = 7.3594$ J/kgK

$$w_{rev} = -(h_2 - h_1)\,\text{kJ/kg}$$
$$= -(3195.5 - 2675.5)\,\text{kJ/kg}$$
$$w_{rev} = -520\,\text{kJ/kg}$$

More work input required for compressor than pump



P-h Diagram for Water



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Polytropic Work in Steady Flow Devices

- For a polytropic process:

$$Pv^n = \text{const.}$$

- Work done when flow through the device is polytropic:

$$w_{poly} = -\int_1^2 v\,dP = -\int_1^2 \left(\frac{C}{P}\right)^{1/n} dP$$

$$= -\frac{n}{n-1}(P_2v_2 - P_1v_1)$$

- Not to be confused with polytropic work in a closed system:

$$w_{poly} = \int_1^2 P\,dv = -\frac{1}{n-1}(P_2v_2 - P_1v_1)$$

- For an ideal gas:

$$Pv = R_{sp}T$$

$$w_{poly} = -\frac{nR_{sp}(T_2 - T_1)}{n-1} = -\frac{nR_{sp}T_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

Polytropic Work in Steady Flow Devices – Special Cases

$$Pv^n = \text{cnst}$$

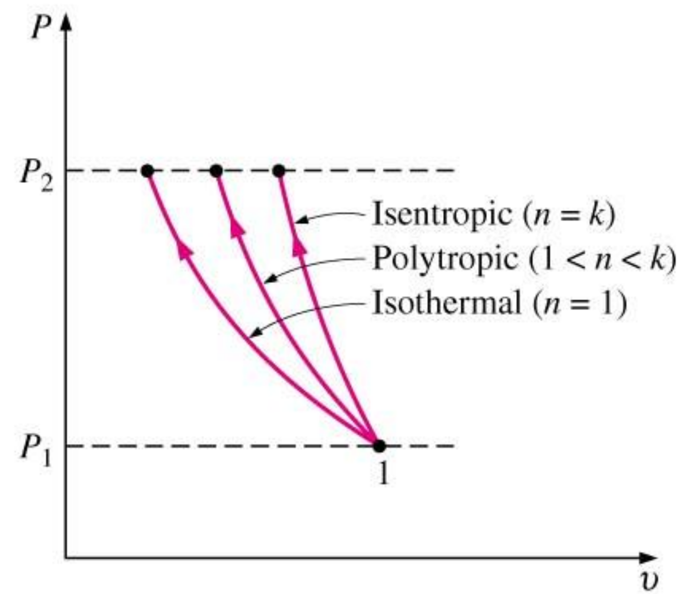
- $n = k$ (isentropic) for ideal gas

$$w_{s=\text{cnst}} = -\frac{kR_{sp}(T_2 - T_1)}{k-1} = -\frac{kR_{sp}T_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

- $n = 1$ (isothermal)

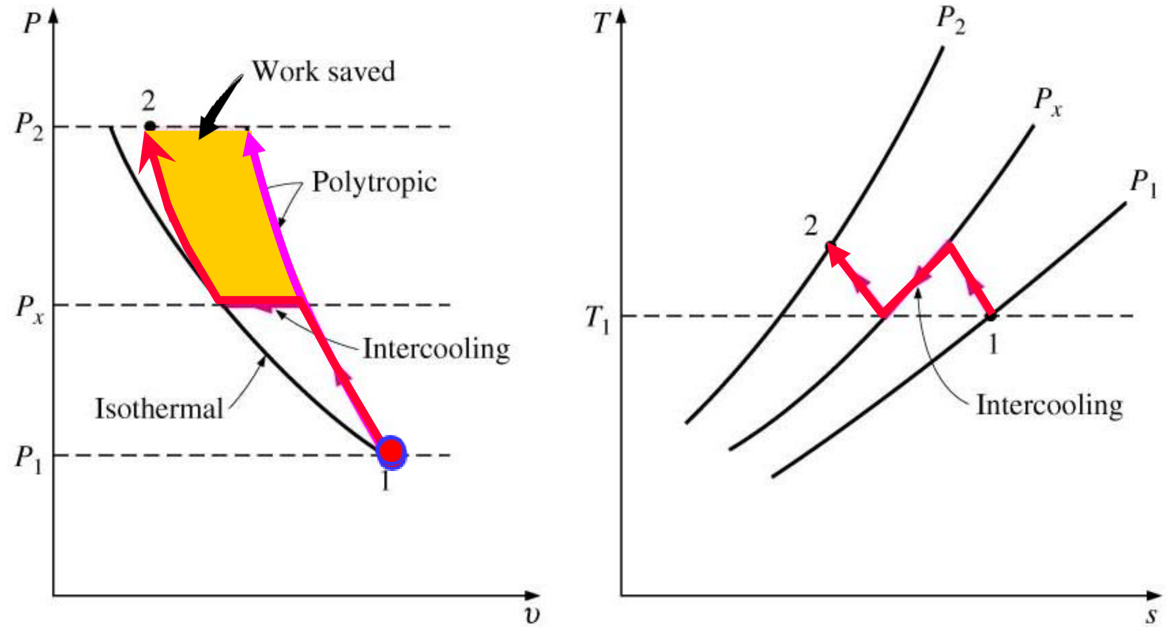
$$w_{T=cnst} = -R_{sp}T\ln\frac{P_2}{P_1}$$

- When compressing an ideal gas between the same pressure ratio, cooling the gas helps reduce the power consumption



Polytropic Work in Steady Flow Devices – Min. Compressor Work

- Multistage compression with intercooling:
- Work in two-stage compression with intercooling :



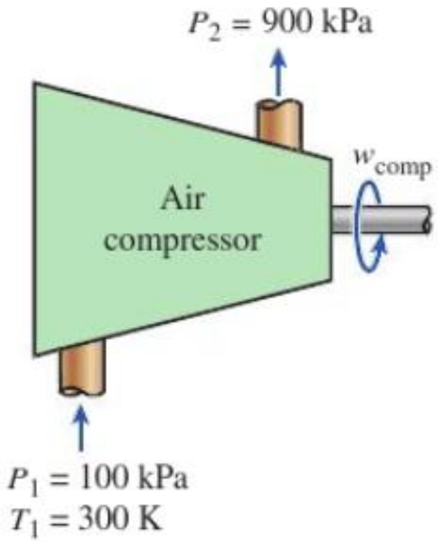
$$w_{comp} = w_{comp,\mathrm{I}} + w_{comp,\mathrm{II}} = \frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_x}\right)^{\frac{n-1}{n}} - 1\right]$$

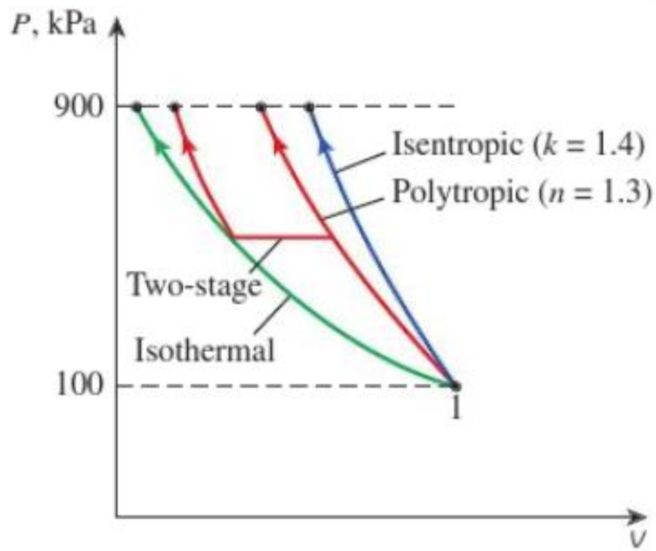
- Minimum total work when pressure ratio across each stage is same:

$$P_x = \sqrt{P_1 P_2} \Rightarrow \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

Example 11 – Work Input for Various Compression Processes

7-13: Air is compressed steadily by a reversible compressor from an inlet state of 100 kPa and 300 K to an exit pressure of 900 kPa. Determine the compressor work per unit mass for (a) isentropic compression with k = 1.4, (b) polytropic compression with n = 1.3, (c) isothermal compression, (d) ideal two-stage compression with intercooling and polytropic exponent of 1.3. The specific gas constant of air is 287 J/kg·K





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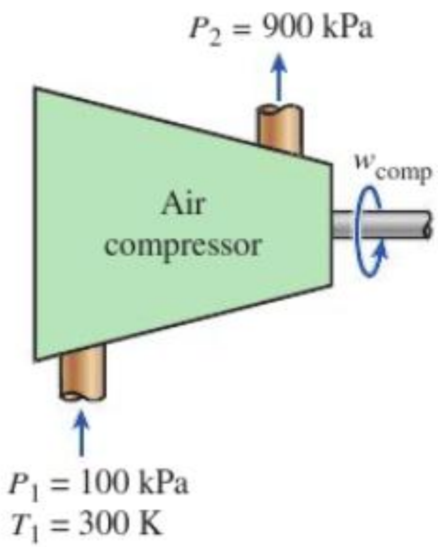
Example 11 – Work Input for Various Compression Processes



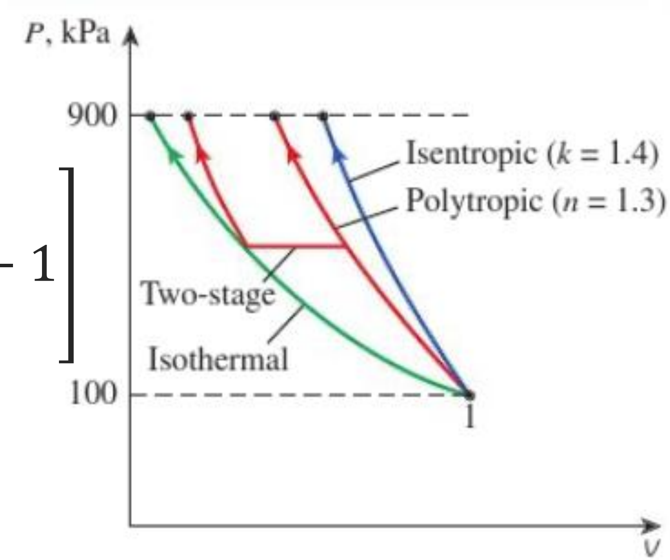
Assumptions: Steady-state, steady flow
Air as an ideal gas
Negligible KE & PE

a) Isentropic compression with k = 1.4:

$$w_{comp} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

$$= \frac{(1.4)(0.287)(300)}{1.4-1}\left[\left(\frac{900}{100}\right)^{(1.4-1)/1.4} - 1\right]$$

$$= 263.2\ \mathrm{kJ/kg}$$



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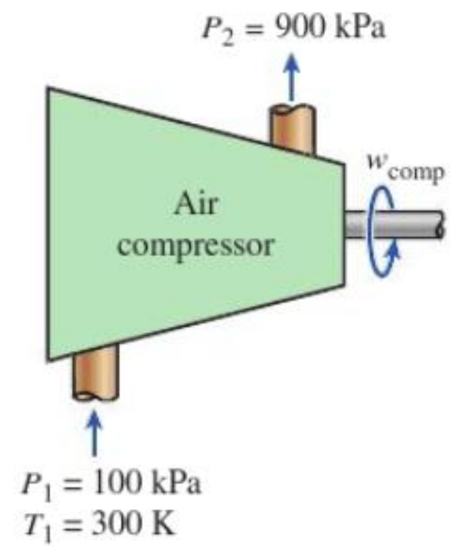
Example 11 – Work Input for Various Compression Processes

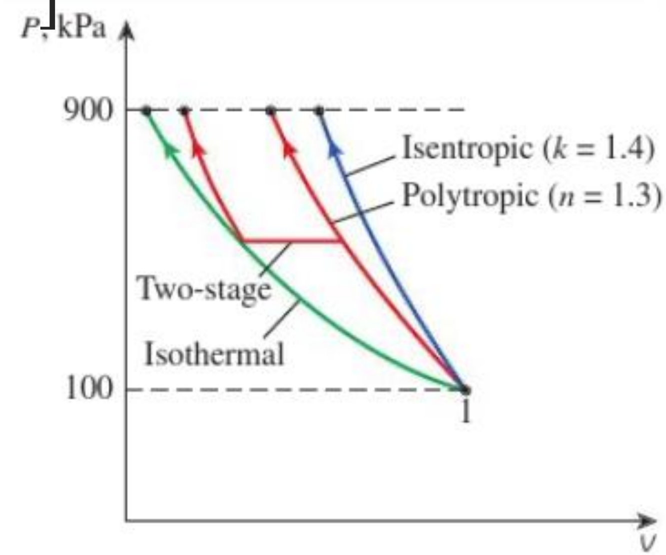


b) Polytropic compression with n = 1.3:

$$w_{comp} = \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

$$= \frac{(1.3)(0.287)(300)}{1.3-1}\left[\left(\frac{900}{100}\right)^{(1.3-1)/1.3} - 1\right]$$

$$= 246.4\ \mathrm{kJ/kg}$$



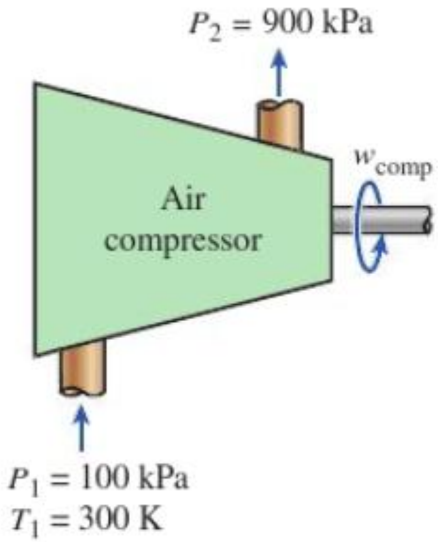
c) Isothermal compression:

$$w_{comp} = RT\ln\frac{P_2}{P_1}$$

$$= (0.287)(300)\ln\left(\frac{900}{100}\right)$$

$$= 189.2\ \mathrm{kJ/kg}$$

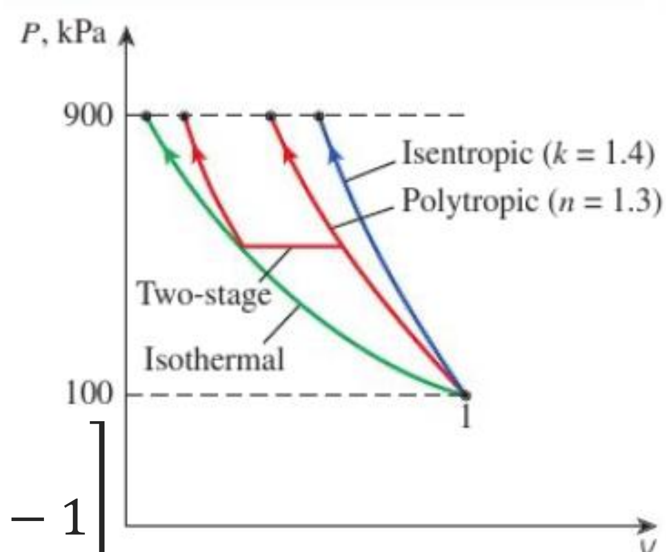
Example 11 – Work Input for Various Compression Processes



d) For ideal two-stage compression with intercooling, pressure ratio is same across each stage. Intermediate pressure ratio:

$$P_x = (P_1 P_2)^{0.5} = \sqrt{(100)(900)}$$
$$= 300 \text{ kPa}$$

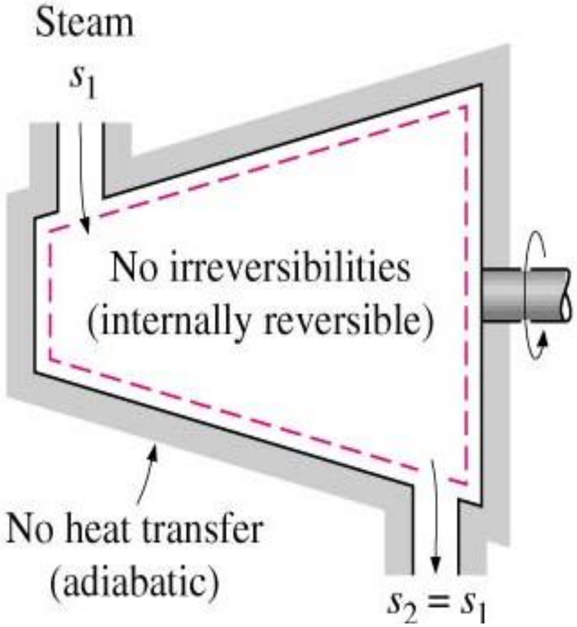
Polytropic work across each stage is the same.



$$w_{comp} = 2 \times w_{comp,\mathrm{I}} = 2 \times w_{comp,\mathrm{II}}$$
$$= 2 \frac{nRT_1}{n-1}\left[\left(\frac{P_x}{P_1}\right)^{(n-1)/n} - 1\right]$$
$$= \frac{2(1.3)(0.287)(300)}{1.3-1}\left[\left(\frac{300}{100}\right)^{(1.3-1)/1.3} - 1\right]$$
$$= 215.3 \text{ kJ/kg}$$

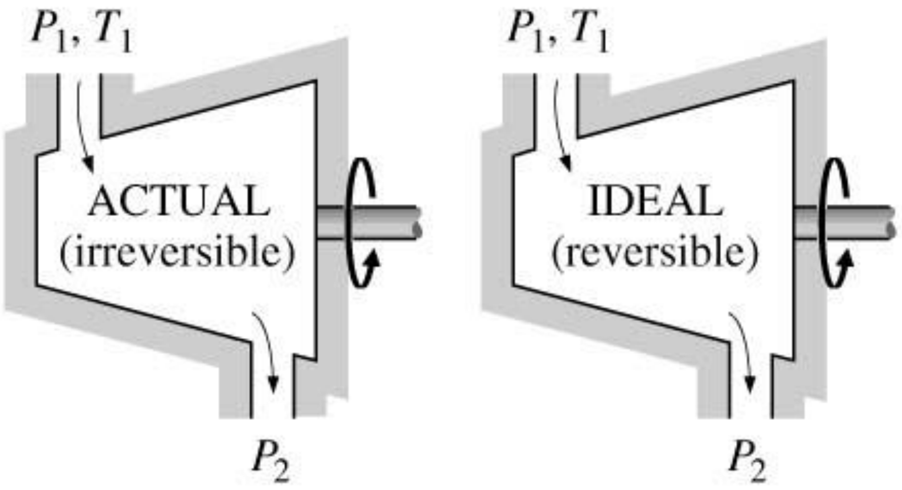
Steady State Control Volumes

- Many SSCV devices are adiabatic or close to adiabatic during operation
- Such devices work best when irreversibilities are minimized
- Isentropic processes would serve as the ideal models for these devices
  - Turbines (extract work from fluid)
    - Enthalpy decreases
  - Pumps; compressors (do work on fluid)
    - Enthalpy increases
  - Nozzles (accelerates the fluid)
    - Enthalpy decreases; KE increases



Steady-flow devices – Isentropic Efficiencies

- Isentropic processes would serve as the ideal process for such adiabatic steady flow devices
- Isentropic efficiency is the measure of the deviation of the actual (adiabatic) process from the idealized one.
- Isentropic efficiencies of turbines, compressors/pump, nozzles serve to compare the actual performance of these (adiabatic) devices to isentropic conditions at the same inlet state and exit pressure.



Isentropic Efficiency of Turbines

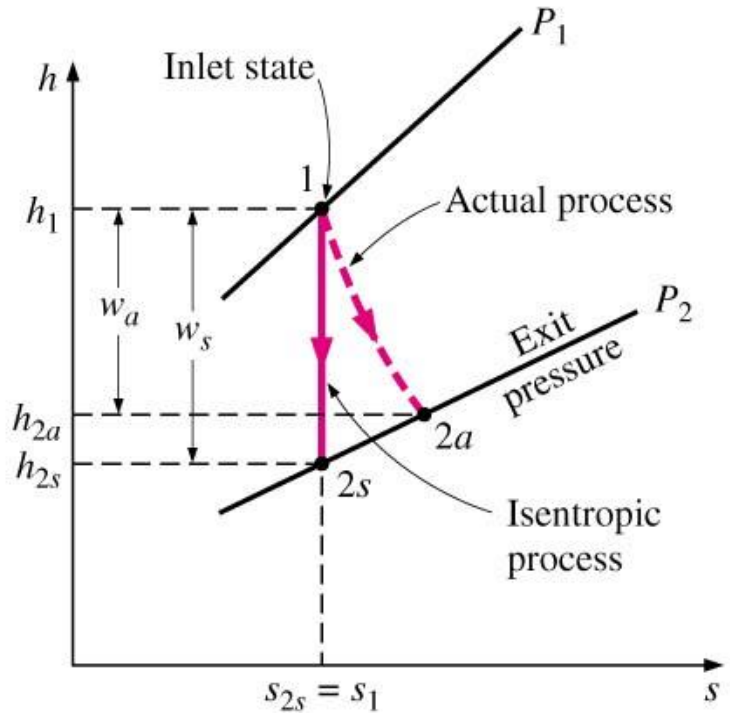
$$w_T = h_1 - h_2$$

- Ratio of actual work output to the ideal work output
  - Ideal process is isentropic (max. work output)
  - Actual process is irreversible (less work output)

$$\eta_T = \frac{\text{Actual work}}{\text{Isentropic work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

- η_T ~ 90% for large turbines
- η_T ~ 70% for small turbines



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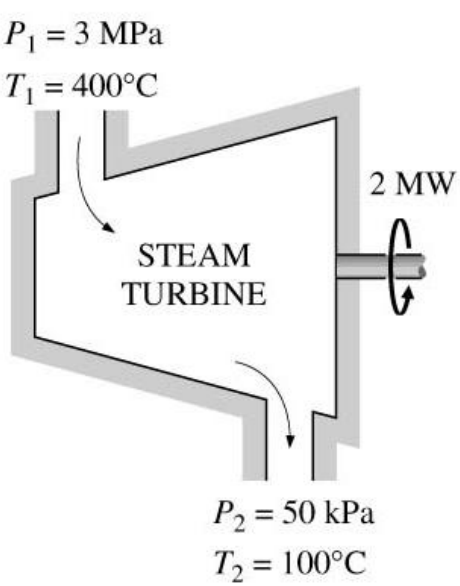
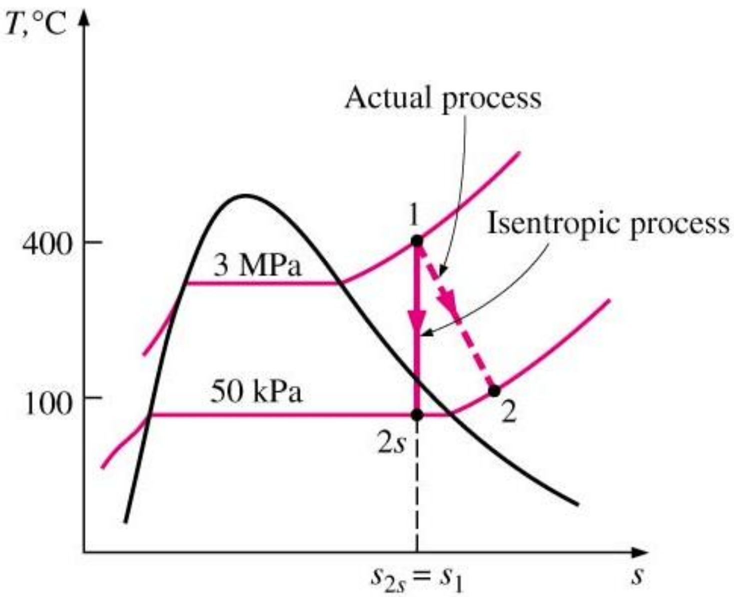
Example 12 – Isentropic Efficiency of a Steam Turbine

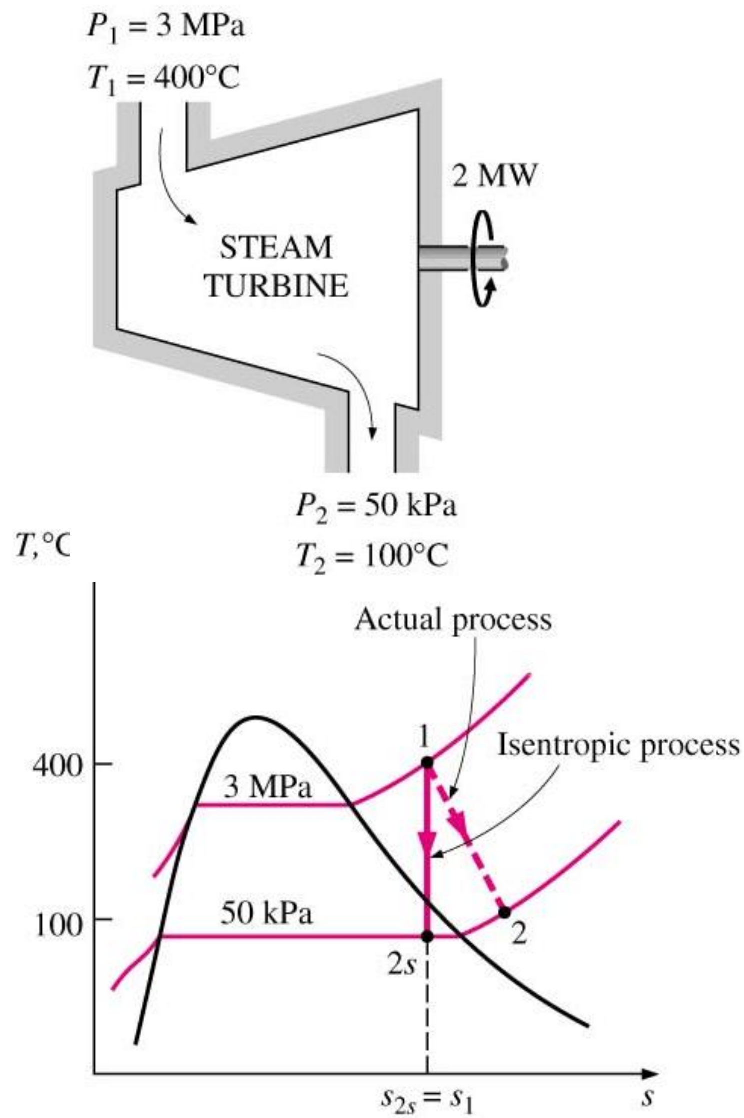
Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine (a) the isentropic efficiency of the turbine and (b) the mass flow rate of the stream flowing through the turbine.





Example 12 – Isentropic Efficiency of a Steam Turbine

Turbine is a steady-state steady flow CV



State 1 (Table A-6)

$$P_1 = 3\,\text{MPa},\ T_1 = 400\ ^{\text{o}}\text{C}$$

$$s_1 = 6.9212\,\text{J/kgK}$$

$$h_1 = 3230.9\,\text{kJ/kg}$$

State 2a (Table A-5)

$$P_{2a} = 50\,\text{kPa},\ T_{2a} = 100\ ^{\text{o}}\text{C}$$

$$h_{2a} = 2682.5\,\text{kJ/kg}$$

State 2s?

$$s_{2s} = s_1$$

Example 12 – Isentropic Efficiency of a Steam Turbine

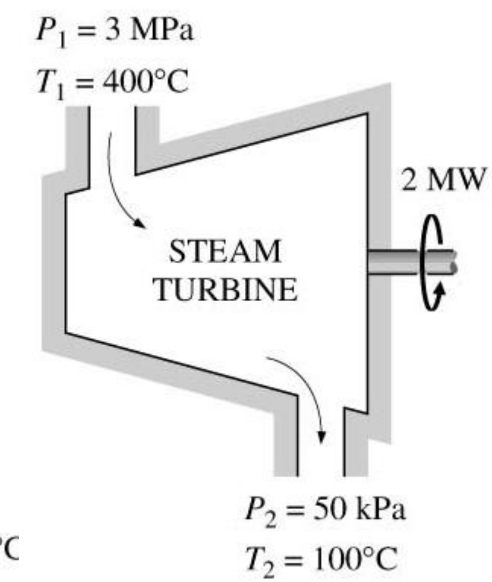
State 2s (Table A-5)

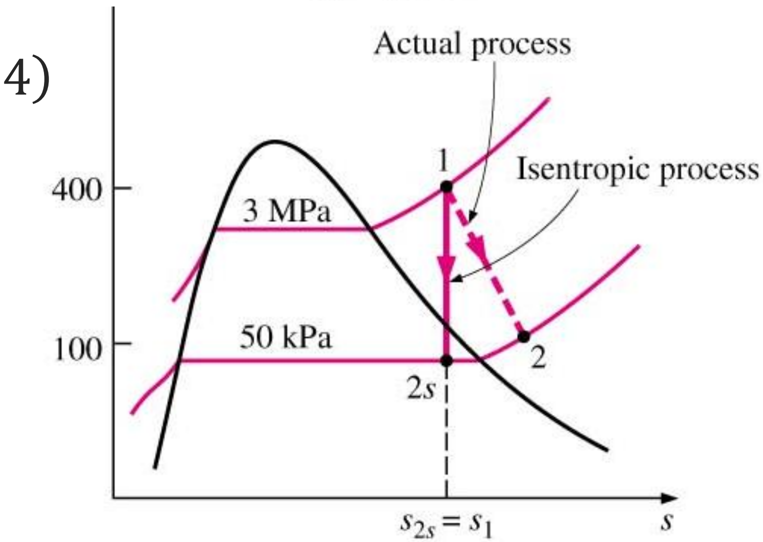
$$P_{2s} = 50\,\text{kPa}, \quad s_{2s} = s_1 = 6.9212\,\text{kJ/kgK}$$

$$s_f = 1.0910\,\text{kJ/kgK}, \quad s_g = 7.5939\,\text{kJ/kgK}$$

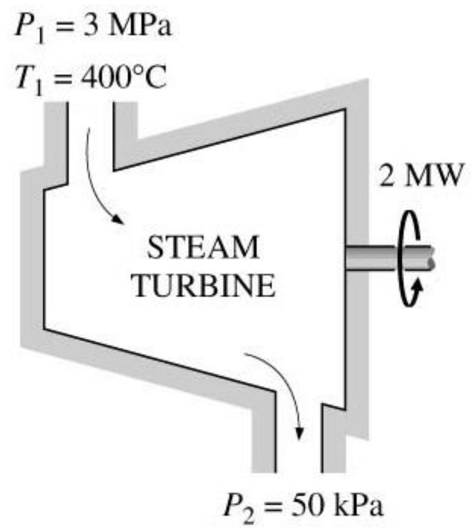
$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9212 - 1.0910}{6.5029} = 0.897$$

$$h_{2s} = h_f + x_{2s} h_{fg} = 340.49 + (0.897)(2305.4)$$
$$= 2407.4\,\text{kJ/kg}$$





Example 12 – Isentropic Efficiency of a Steam Turbine



Isentropic efficiency:

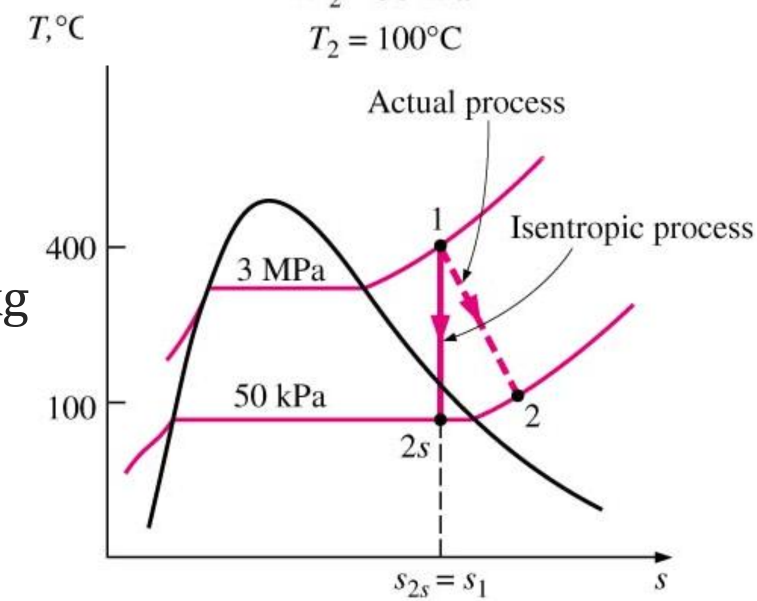
$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3230.9 - 2682.5}{3230.9 - 2407.4} = 0.666$$

Mass flow rate:

$$\dot{W} = \dot{m}(h_1 - h_{2a})$$

$$2000 \text{ kW} = (\dot{m} \text{ kg/s})(3230.9 - 2682.5) \text{ kJ/kg}$$

$$\dot{m} = 3.65 \text{ kg/s}$$



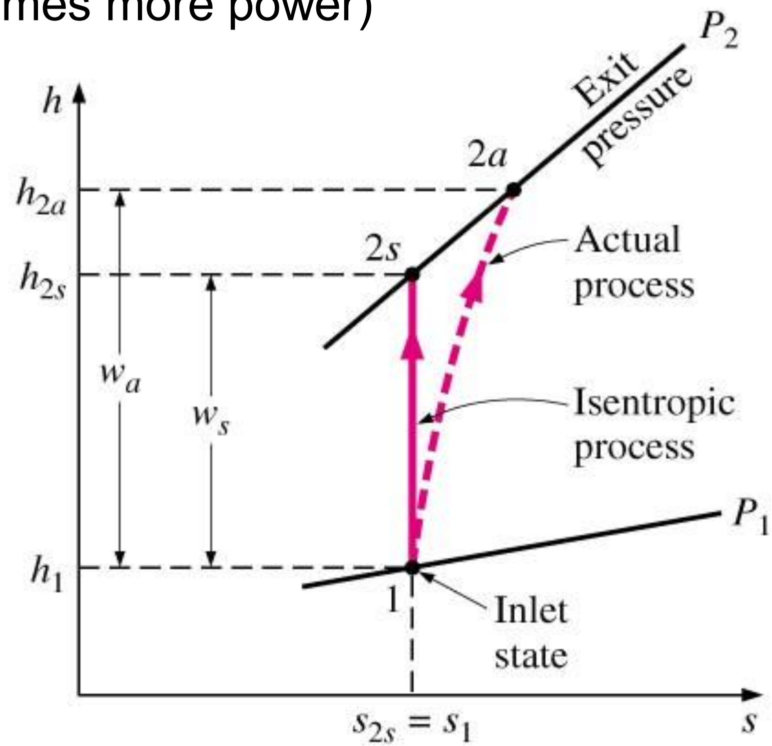
Isentropic Efficiency of Compressors

$$w_C = -(h_1 - h_2)$$

- Ratio of required ideal work input to actual work input
  - Ideal process is isentropic (consumes min. power)
  - Actual process is irreversible (consumes more power)

$$\eta_C = \frac{\text{Isentropic work}}{\text{Actual work}} = \frac{w_s}{w_a}$$

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$



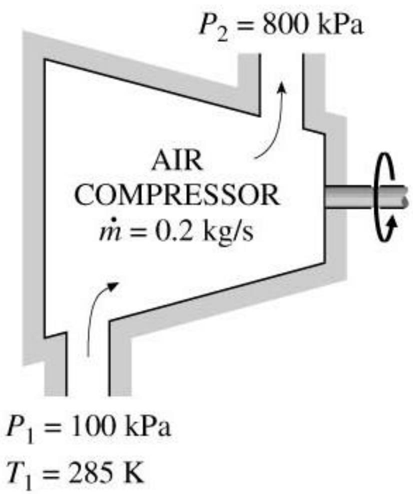
- η_C ~ 75 – 85% for well-designed devices

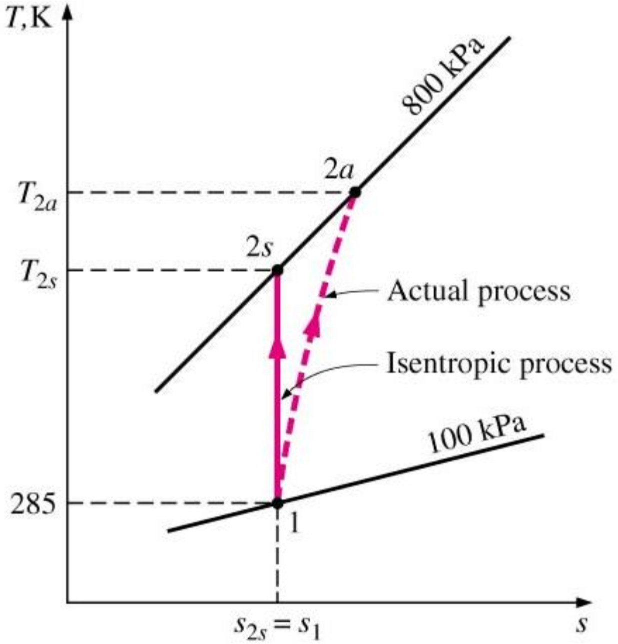
- Isentropic efficiency for pumps:

$$\eta_P = \frac{w_s}{w_a} \cong \frac{v(P_2 - P_1)}{h_{2a} - h_1}$$

Example 13 – Isentropic Efficiency of a Compressor

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80%, determine (a) the exit temperature of air, and (b) the required power input to the compressor.





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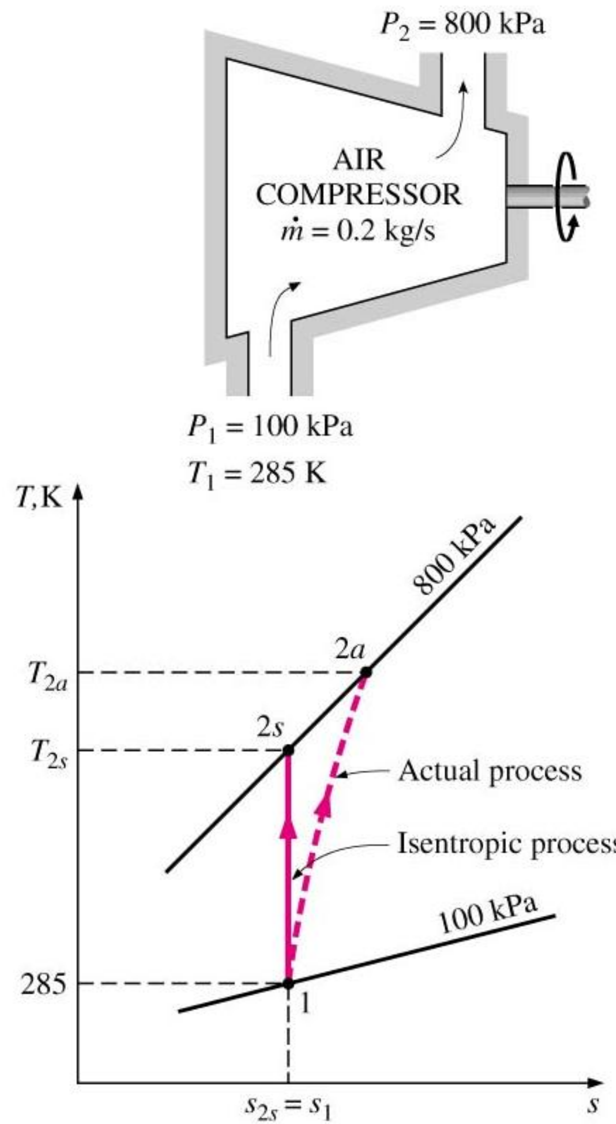
Example 13 – Isentropic Efficiency of a Compressor



Compressor is a SSSF CV

Assumptions: Air as an ideal gas

$T_{avg} = 400$ K

From Table A-2:

$$k = 1.395 \qquad c_p = 1.013\ \mathrm{kJ/kg \cdot K}$$

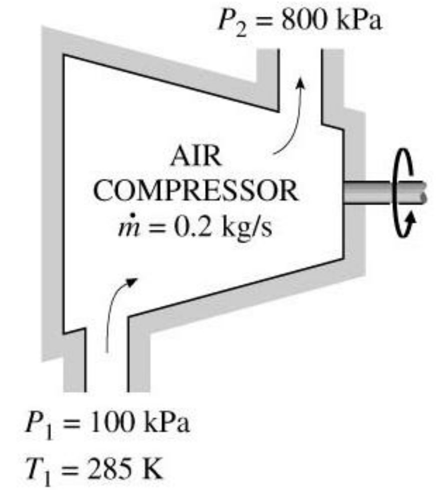
Isentropic relation for ideal gas:

$$T_{2s} = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$= (285\mathrm{K}) \left(\frac{800\ \mathrm{kPa}}{100\ \mathrm{kPa}}\right)^{0.395/1.395} = 513.52\ \mathrm{K}$$

Check: $T_{avg} = 399$ K

Example 13 – Isentropic Efficiency of a Compressor

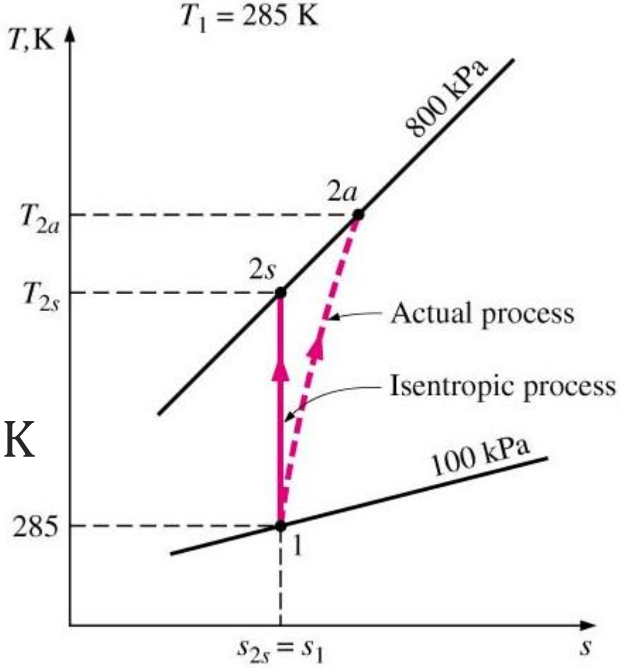


Isentropic efficiency:

$$\eta_C = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_{2a} - T_1)}$$

$$0.8 = \frac{513.52 - 285}{T_{2a} - 285}$$

$$T_{2a} = 570.65 \text{ K}$$



Required power input:

$$\dot{W} = \dot{m}(h_{2a} - h_1) = \dot{m}c_p(T_{2a} - T_1)$$

$$= (0.2\text{kg/s})(1.395\text{kJ/kg} \cdot \text{K})(570.65 - 285)\text{K}$$

$$= 57.87 \text{ kW}$$

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Isentropic Efficiency of Nozzles

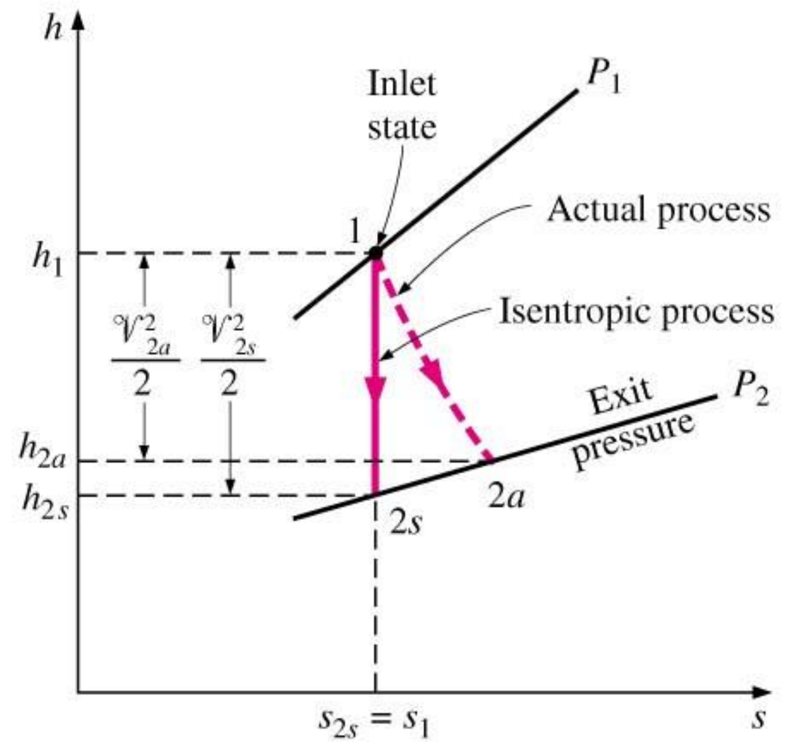
Enthalpy converted to KE

- Ratio of actual KE to the ideal KE at the exit
  - Ideal KE after isentropic process (max. speed)
  - Actual KE (lower speed)

$$\eta_N = \frac{\text{Actual KE @ exit}}{\text{Isen. KE @ exit}} = \frac{\mathcal{V}_{2a}^2}{\mathcal{V}_{2s}^2}$$

$$h_1 - h_{2a} = \frac{\mathcal{V}_{2a}^2}{2}$$

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$



- Typical η_N ~ 90 – 95%

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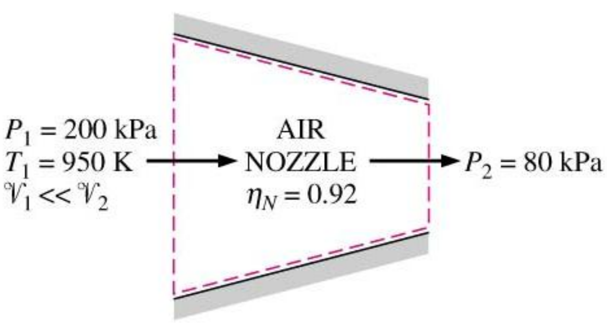
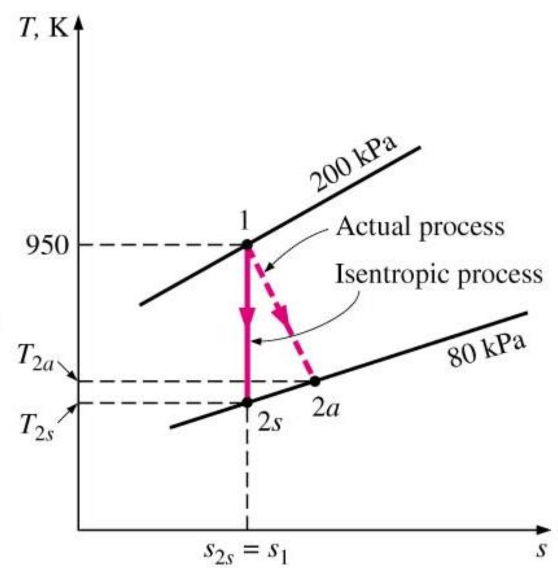
Example 14 – Isentropic Efficiency of a Nozzle

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharge at a pressure of 80 kPa. If the isentropic efficiency of the nozzle is 92%, determine (a) the maximum possible exit velocity, (b) the exit temperature, and (c) the actual velocity of the air. Assume constant specific heats for the air:

k = 1.354, C_p = 1.099 kJ/kg·K



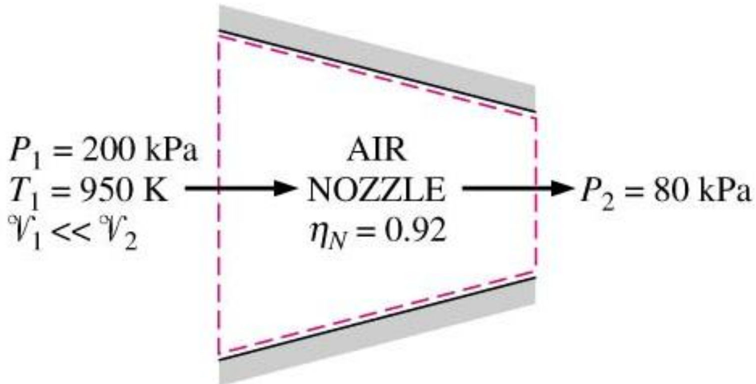


Example 14 – Isentropic Efficiency of a Nozzle

Nozzle is a SSSF control volume

Assumptions: Constant specific heats

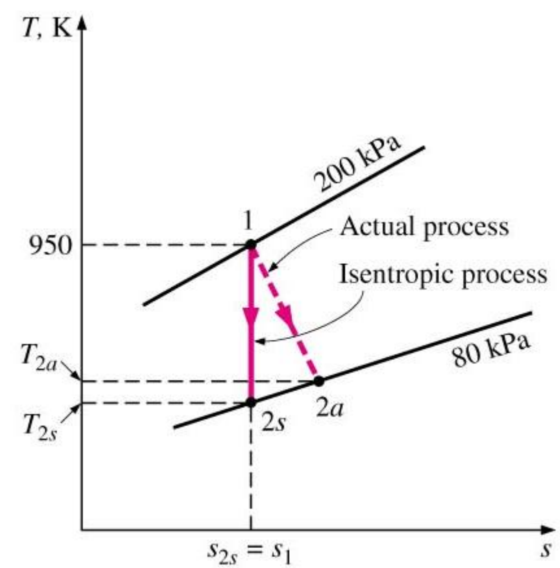
Air as an ideal gas



a) Isentropic process for ideal gas:

$$T_{2s} = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$= (950\text{K}) \left(\frac{80 \text{ kPa}}{200 \text{ kPa}}\right)^{0.354/1.354} = 748 \text{ K}$$



Energy balance for isentropic exit velocity:

$$h_1 + \cancel{\frac{v_1^2}{2}} = h_{2s} + \frac{v_{2s}^2}{2} \qquad \because v_1 \ll v_2$$

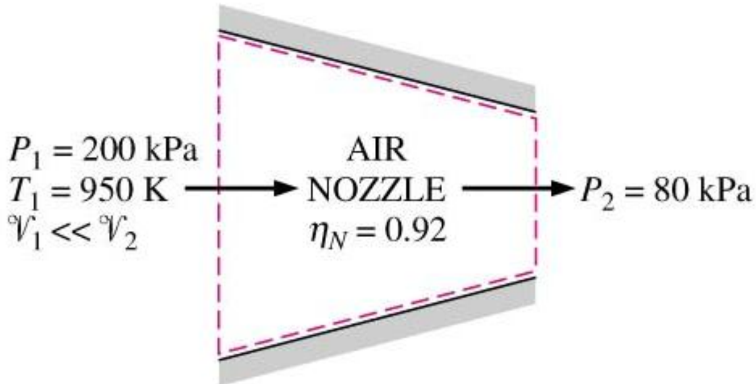
$$\& \; v_2 < v_{2s}$$

Example 14 – Isentropic Efficiency of a Nozzle

$$\mathcal{V}_{2s}^2 = 2(h_1 - h_{2s}) = 2C_{p,avg}(T_1 - T_{2s})$$

$$\mathcal{V}_{2s}^2 = 2(1.099 \times 10^3)(950 - 748)$$

$$\mathcal{V}_{2s} = 666\,\mathrm{m/s}$$



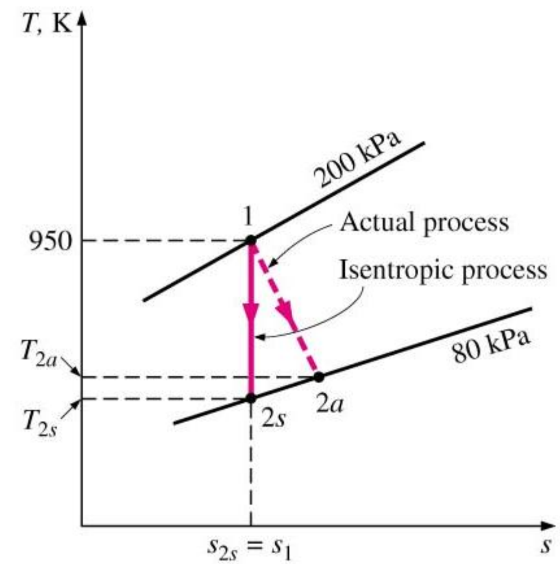
b) Actual exit temp. would be higher than isen. exit temp

Nozzle isentropic efficiency:

$$\eta_N = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{c_{p,avg}(T_1 - T_{2a})}{c_{p,avg}(T_1 - T_{2s})}$$

$$0.92 = \frac{950 - T_{2a}}{950 - 748}$$

$$T_{2a} = 764\ \mathrm{K}$$



Example 14 – Isentropic Efficiency of a Nozzle

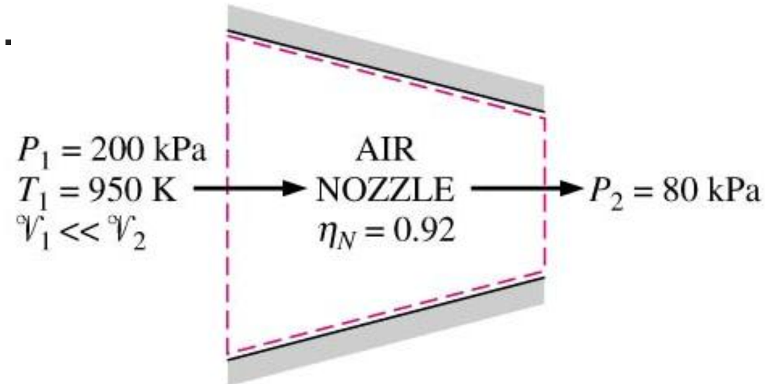
c) Actual exit velocity is lower than isen. exit velocity

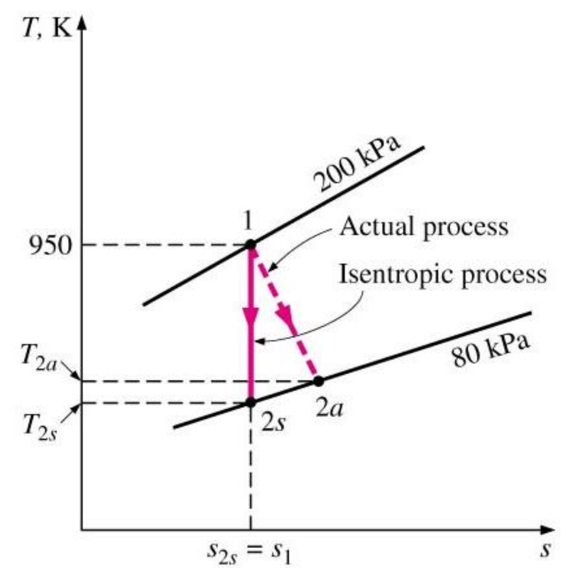
Nozzle isentropic efficiency:

$$\eta_N = \frac{\mathcal{V}_{2a}^2}{\mathcal{V}_{2s}^2} = 0.92$$

$$\mathcal{V}_{2a} = \mathcal{V}_{2s}\sqrt{\eta_N}$$

$$= 666\sqrt{0.92} = 639\,\mathrm{m/s}$$





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Summary

- Entropy is a property
- Entropy can only be transferred by heat but not work
- Area under an internally reversible process curve in a T-s diagram represents the heat transfer
- Entropy is not conserved, it is generated when irreversibilities are present in the process but it cannot be destroyed
- Entropy generation is zero for reversible processes
- Entropy change for a closed system can be negative but entropy generation can never be negative
- Entropy change of an isolated system can never be negative

Summary

- Entropy of a closed system can be *increased or decreased* by heat transfer (Q/T) and *increased* by irreversibilities (S_{gen})

$$\Delta S_{closed} = S_2 - S_1 = \sum \frac{Q_k}{T_k} + S_{gen}$$

- Entropy of isolated systems can only increase as the change in entropy is the equivalent to entropy generation within the isolated system.

$$\Delta S_{iso} = S_{gen}$$

- Q/T and S_{gen} are process dependent. Entropy change (ΔS) is dependent on initial and final states only
- Entropy change can also be obtained from property tables (S_1, S_2), or for ideal gases:

$$s_2 - s_1 = c_{v,avg} \ln \frac{T_2}{T_1} + R_{sp} \ln \frac{v_2}{v_1} \qquad s_2 - s_1 = c_{p,avg} \ln \frac{T_2}{T_1} - R_{sp} \ln \frac{P_2}{P_1}$$

Summary

- In steady-state, steady-flow control volumes, entropy due to heat transfer (Q/T) and irreversibilities (S_{gen}) are balanced by entropy inflow and outflow with the mass

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{Q_k}{T_k} + S_{gen}$$

- Reversible ($S_{gen} = 0$) and adiabatic ($Q = 0$) processes are also known as isentropic processes ($S_1 = S_2$, $s_1 = s_2$)
- For reversible steady-flow through a work device (e.g. pumps, compressors)

$$w_{rev} = -\int_1^2 v dP$$

- Cooling the gas during a polytropic work (Pv^n = cnst) process for compressors reduces the work consumption
- For real flow devices (e.g. nozzles, turbines), deviation from isentropic flow is computed using isentropic efficiencies