MA3010 – 2nd Law of Thermodynamics

Dr Aw Kuan Thai
ktaw@ntu.edu.sg

Contents

1) Introduction
2) Thermal Energy Reservoirs
3) Heat Engines
4) Kelvin-Planck Statement
5) Refrigerators & Heat Pumps
6) Clausius Statement
7) Perpetual Motion Machines
8) Reversible & Irreversible Processes
9) The Carnot Cycle
10) Carnot Principles
11) Thermodynamic Temperature Scale
12) Carnot Heat Engines, Refrigerators & Heat Pumps

Reference: Thermodynamics Chapter 6

Introduction

- Fulfilling the 1st law of thermodynamics does not guarantee that a process that take place
  - E.g. 1: Supplying light and heat to a filament lightbulb does not generate electric energy
  - E.g. 2: Hot coffee does not get hotter in a cooler room

https://picsart.com/i/gif-light-important-part-of-life-light-gif-bulb-cool-195138003001202

https://giphy.com/gifs/hot-coffee-giudanskycom-2jd7CRuYayGpW

- Processes occur in a certain direction and not in reverse
- A process must satisfy both 1st and 2nd laws of thermodynamics to proceed

Introduction – 2nd Law of Thermodynamics

1. It identifies the direction of processes

2. It determines the theoretical limits for the performance of engineering systems, e.g. heat engines, refrigerators
   - Defines “perfection” for thermodynamic processes
   - Used as a benchmark for real engineering systems

3. It asserts that energy has *quality* as well as quantity – determines degree of degradation of energy during a process
   - Energy at a high temperature has better quality than the same energy at a low temperature

4. Predicts degree of completion for chemical reactions
   - A process is completed when entropy stops increasing

Thermal Energy Reservoirs

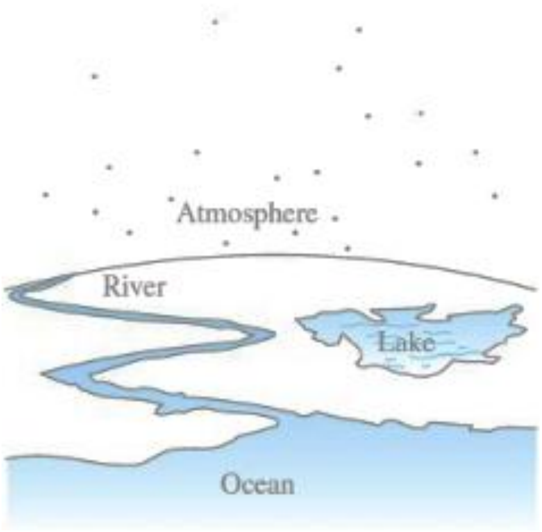
- Hypothetical body with a relatively large *thermal energy capacity* (mass x specific heat) that can absorb/supply finite amounts of heat **without undergoing any change in temperature**

- In practice, large bodies of water (oceans, lakes, rivers) and the atmospheric air can be modelled as thermal energy reservoirs

$$Q = m \times c \times \Delta T$$

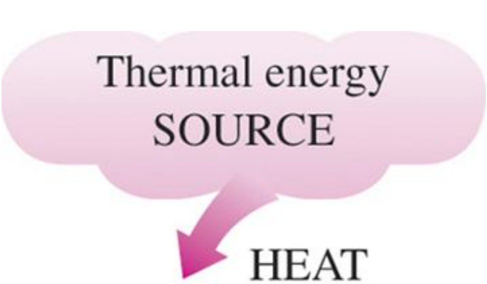
$$\Delta T = \frac{Q}{m \cdot c}$$

Thermal energy capacity



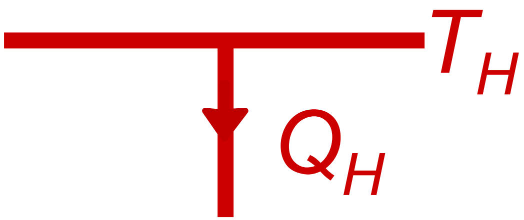
Thermal Energy Reservoirs – Heat Source & Heat Sink

- Source: supplies heat energy
- Sink: absorbs heat energy



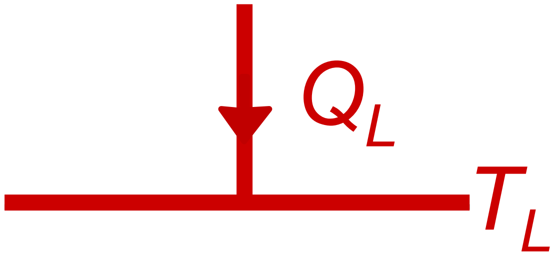
e.g. Sun, furnace, etc

Can be simplified to:



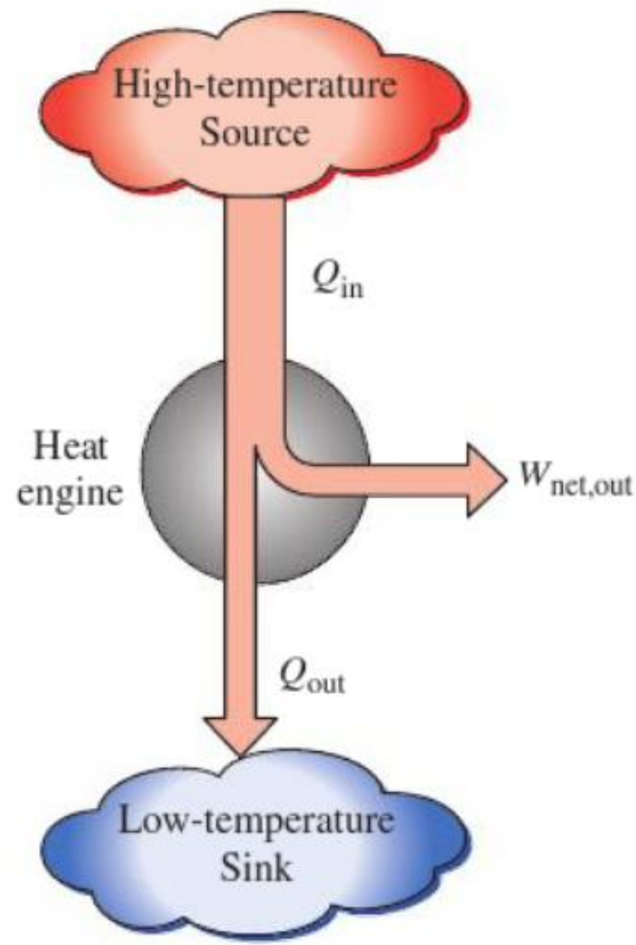
e.g. river, atmosphere, etc

Can be simplified to:



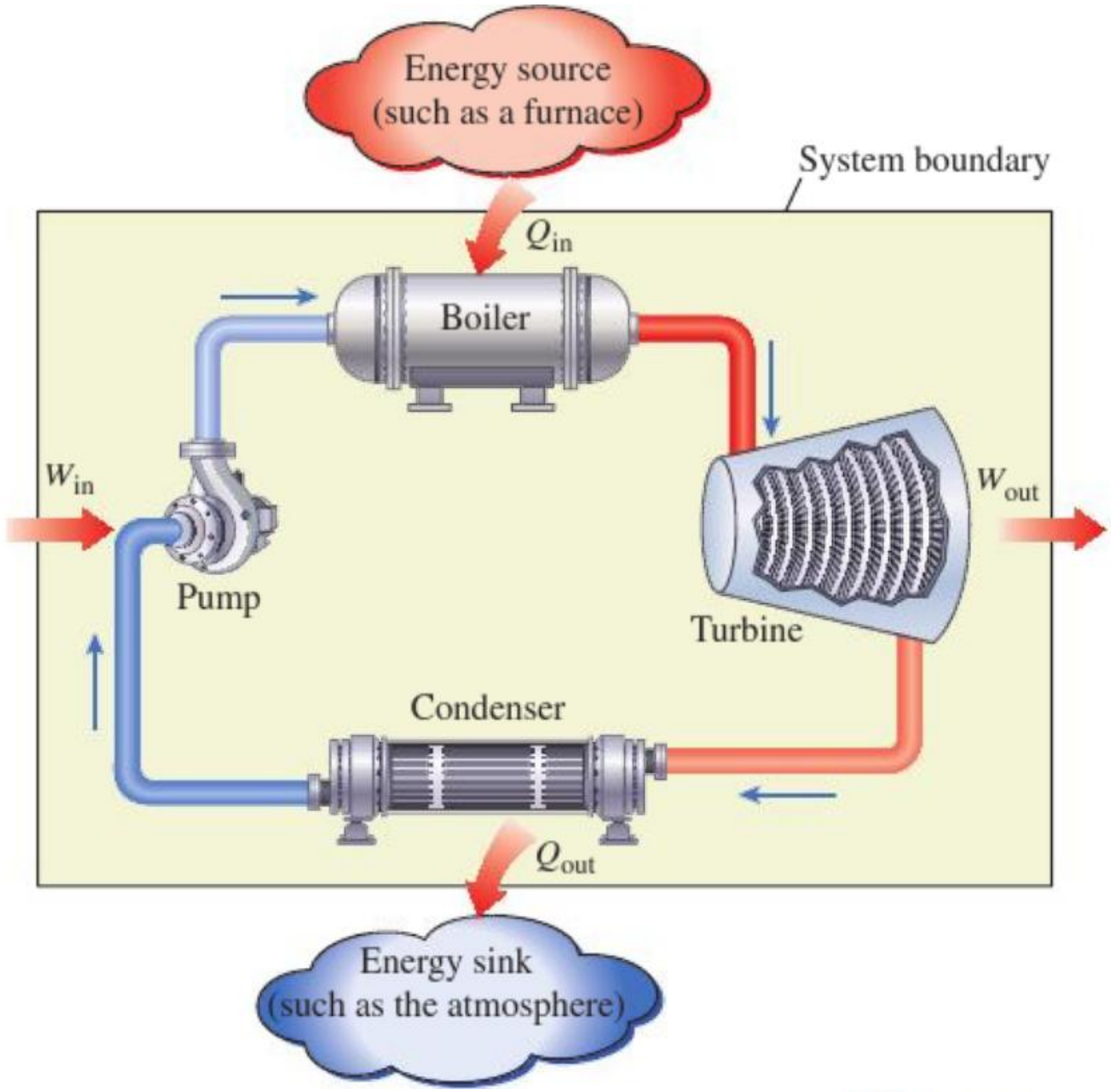
Heat Engines

- Work is *easily* converted into other forms of energy such as heat but the reverse is more difficult
- Heat engines convert heat to work
  - Receive heat from a high-temperature source
  - Convert part of the heat received to work
  - Reject the remaining waste heat to a low-temperature sink
  - Operate on a **cycle**
- Typically uses a fluid to transfer heat, known as the *working fluid*



Heat Engines – Steam Power Plant

- Water/steam as the working fluid
- Q_{in} = heat supplied to steam in boiler
- W_{out} = work extracted from steam in turbine
- Q_{out} = heat rejected by steam in condenser
- W_{in} = work required to pump water into boiler

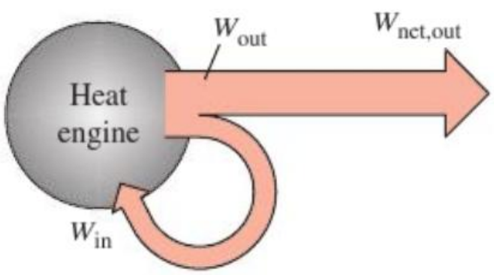


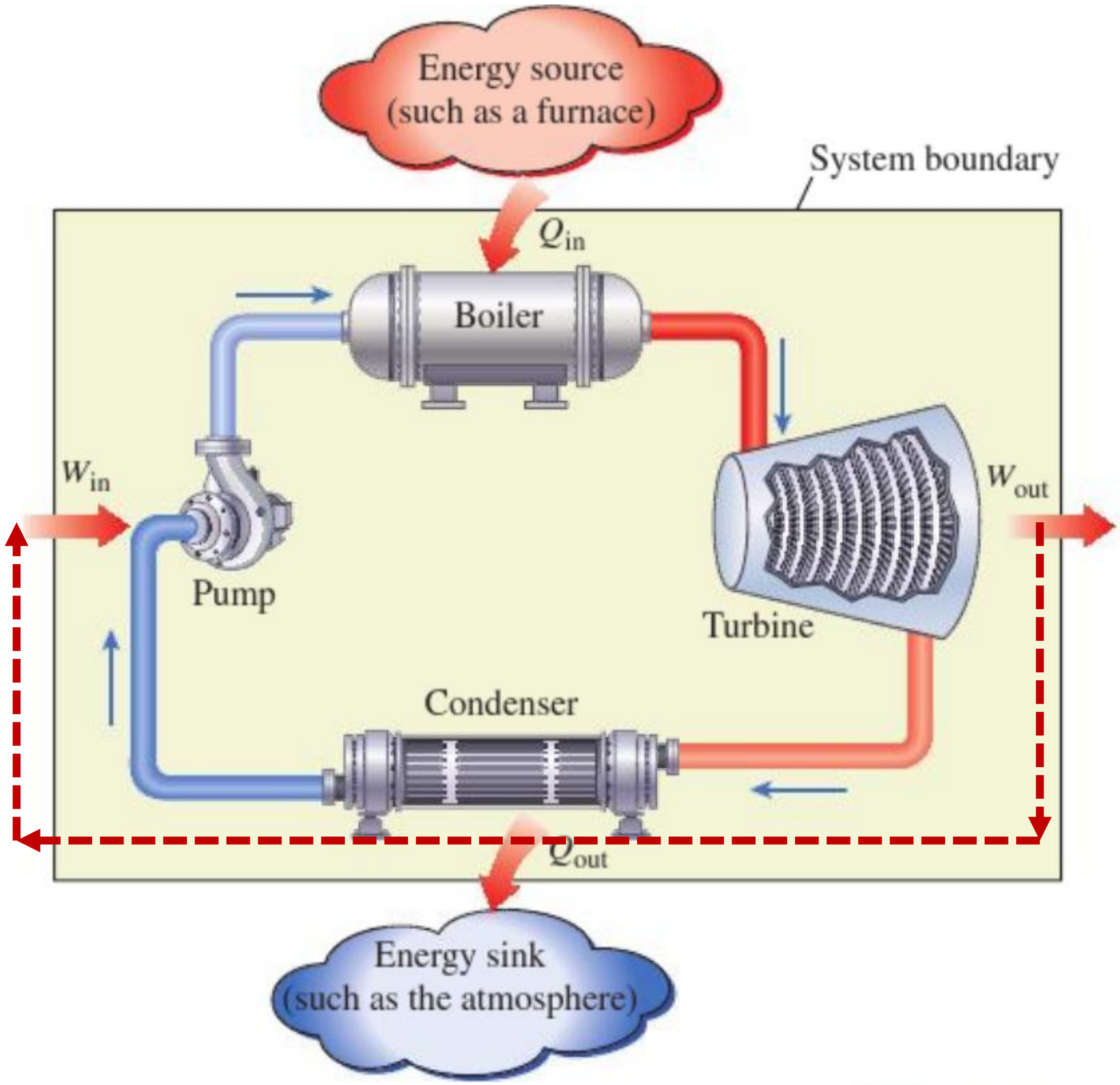
Heat Engines

- Part of work output is used to drive the pump

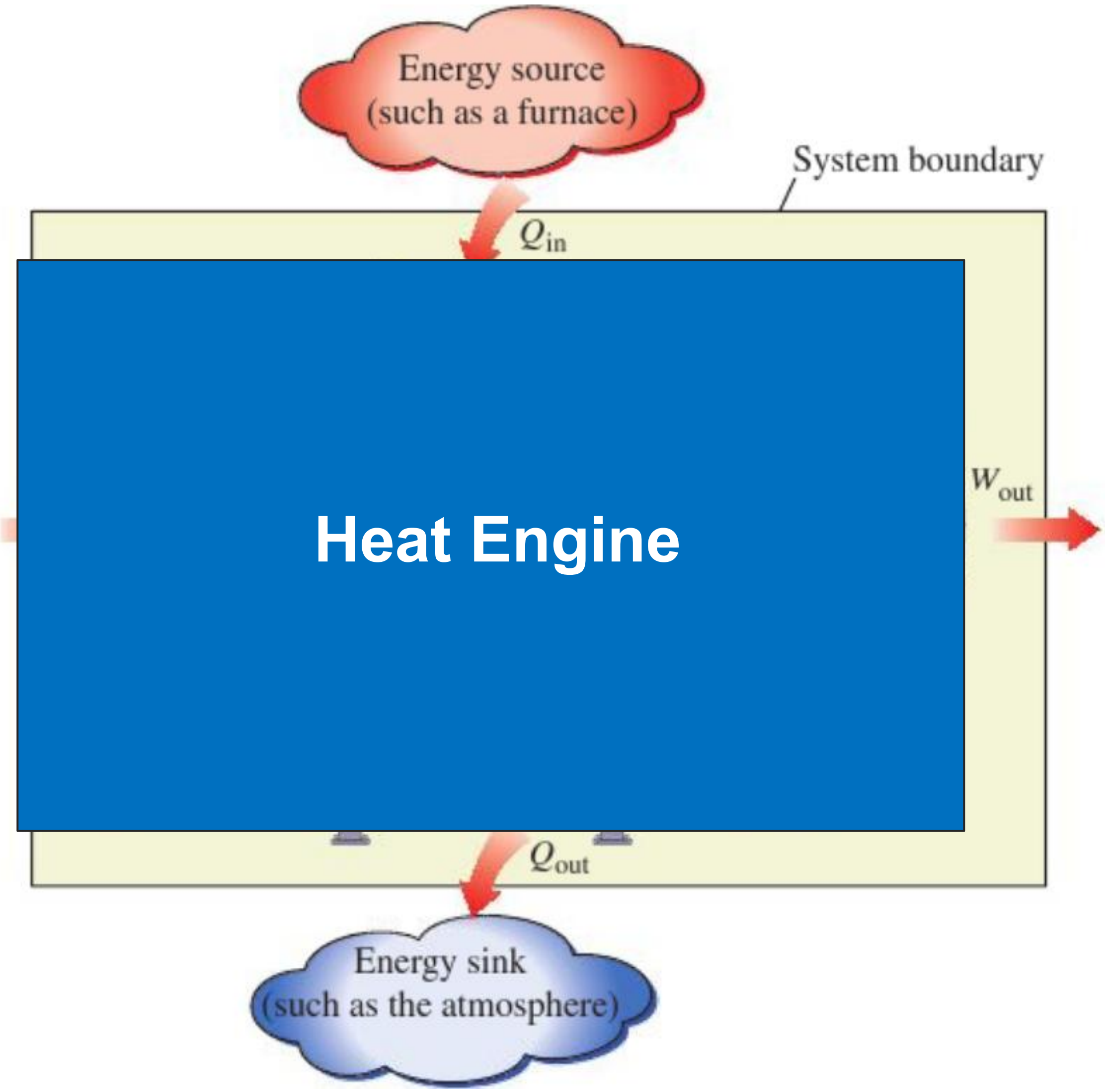
- Net output from heat engine:

$$W_{net,out} = W_{out} - W_{in}$$

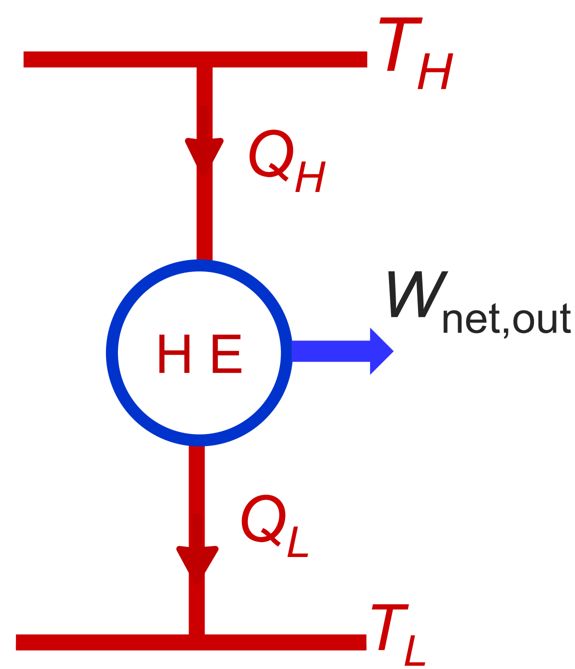




Heat Engines



Simplified:



Heat Engines – Energy Balance

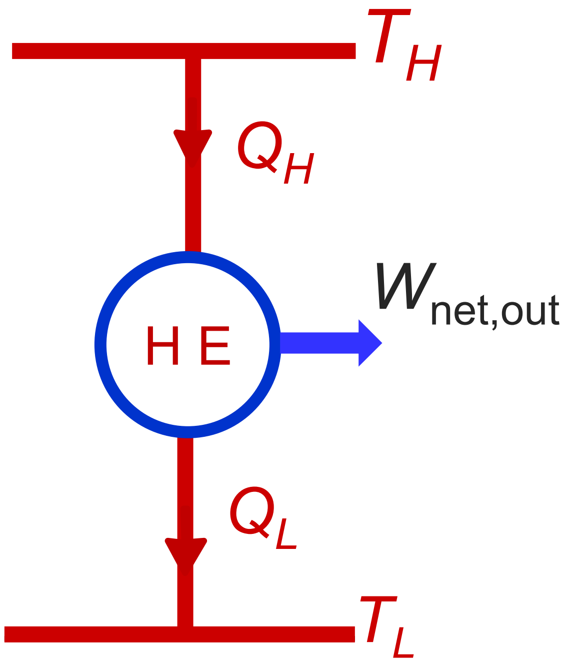
From 1st Law: $Q - W = \Delta U$

Sign convention: arrow going into system = +ve

For a cycle: $Q - W = 0$

$$Q_H + (-Q_L) - W_{net,out} = 0$$

$$W_{net,out} = Q_H - Q_L$$



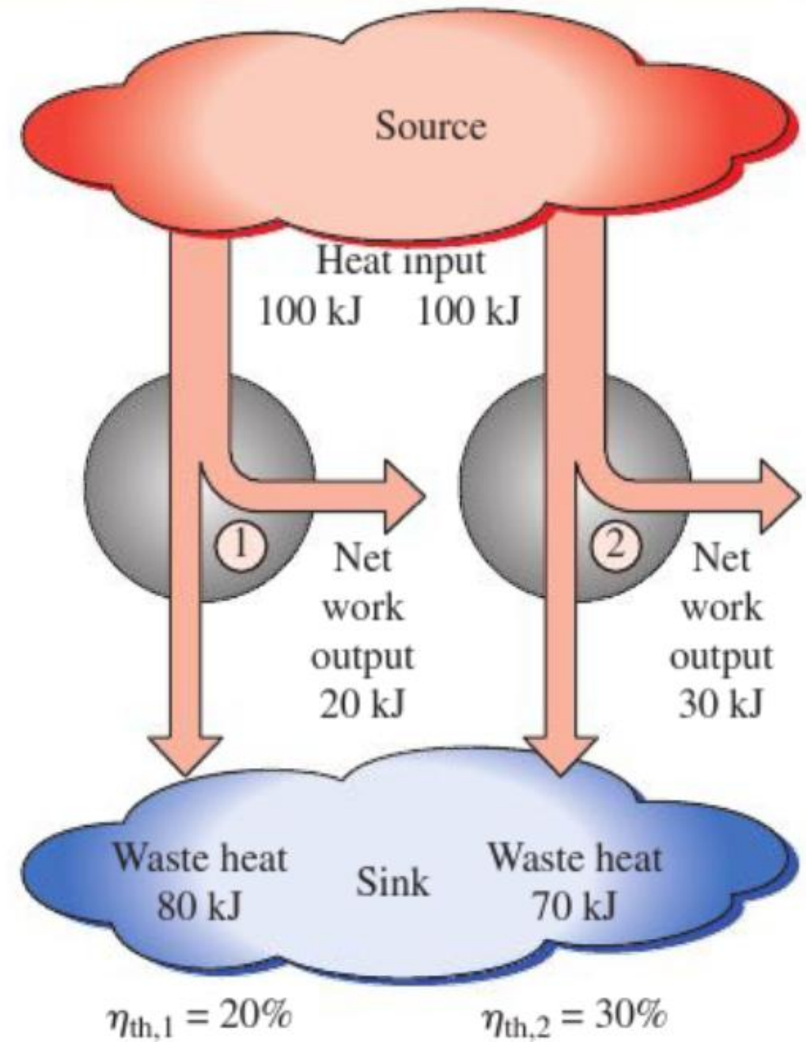
Heat Engines – Thermal Efficiency

- A measure of how well a heat engine converts heat input into useful work

$$\boxed{\text{Thermal Efficiency} = \frac{\text{Net work output}}{\text{Total heat input}}}$$

$$\eta_{\text{th}} = \frac{W_{net,out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

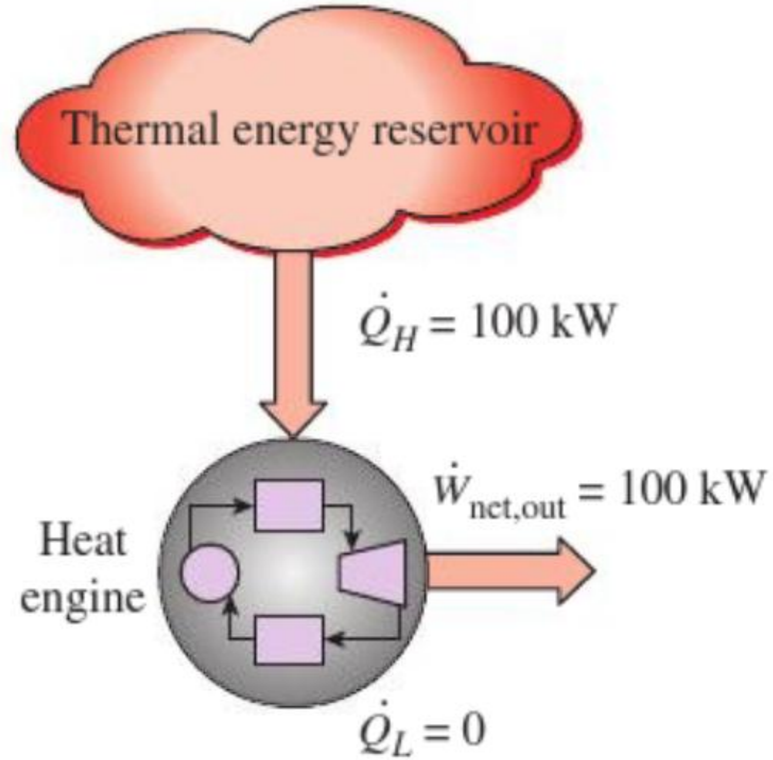
$$\eta_{\text{th}} = \frac{W_{net,out}}{Q_H} = \boxed{1 - \frac{Q_L}{Q_H}}$$



Is it possible to have 100% efficiency?

2nd Law of Thermodynamics – Kelvin-Planck Statement

$$\eta_{\text{th}} = 1 - \frac{Q_L}{Q_H}$$



- If $Q_L = 0$, heat engine will have 100% efficiency
- However, there is always waste heat produced
- The cycle cannot be completed without rejecting heat to a low-temperature sink

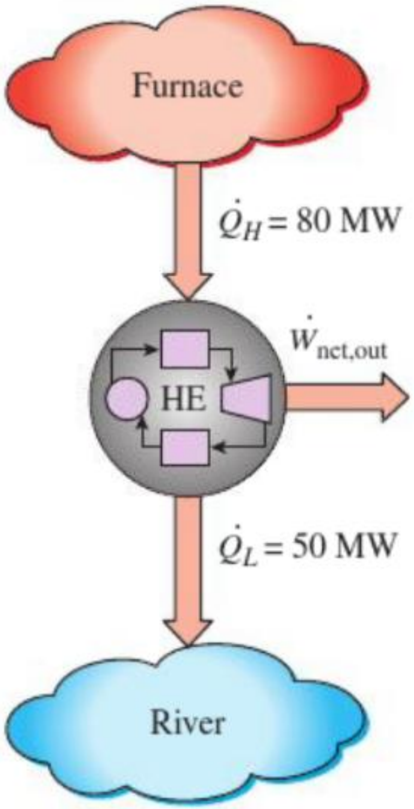
Kelvin-Planck Statement:
It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.
Even *theoretically perfect* heat engines do not have an efficiency of 100%!

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Example 1 – Net Power Production of a Heat Engine

Heat is transferred to a heat engine from a furnace at a rate of 80 MW. If the rate of waste heat rejection to a nearby river is 50 MW, determine the net power output and the thermal efficiency for this heat engine.

Assumptions: Negligible heat losses through pipes & other components



$$\dot{W}_{net,out} = \dot{Q}_H - \dot{Q}_L$$

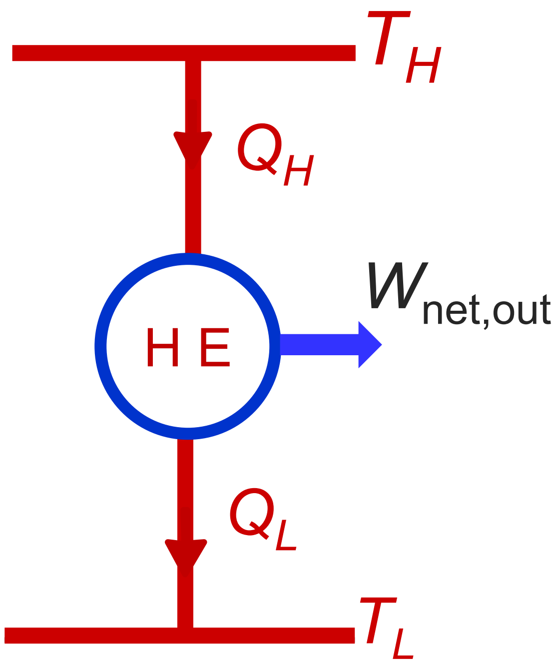
$$= 80 - 50 = 30 \text{ MW}$$

$$\eta_{th} = \frac{\dot{W}_{net,out}}{\dot{Q}_H}$$

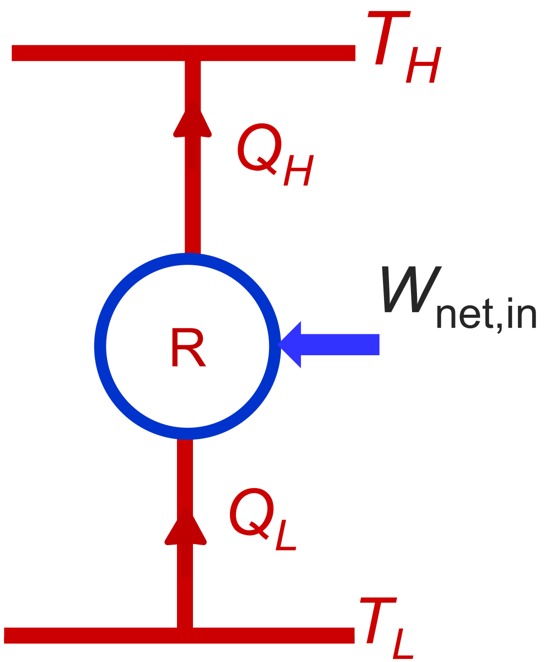
$$= \frac{30 \text{ MW}}{80 \text{ MW}} = 0.375$$

Reverse Heat Engines

Heat Engine:



Reverse Heat Engine:

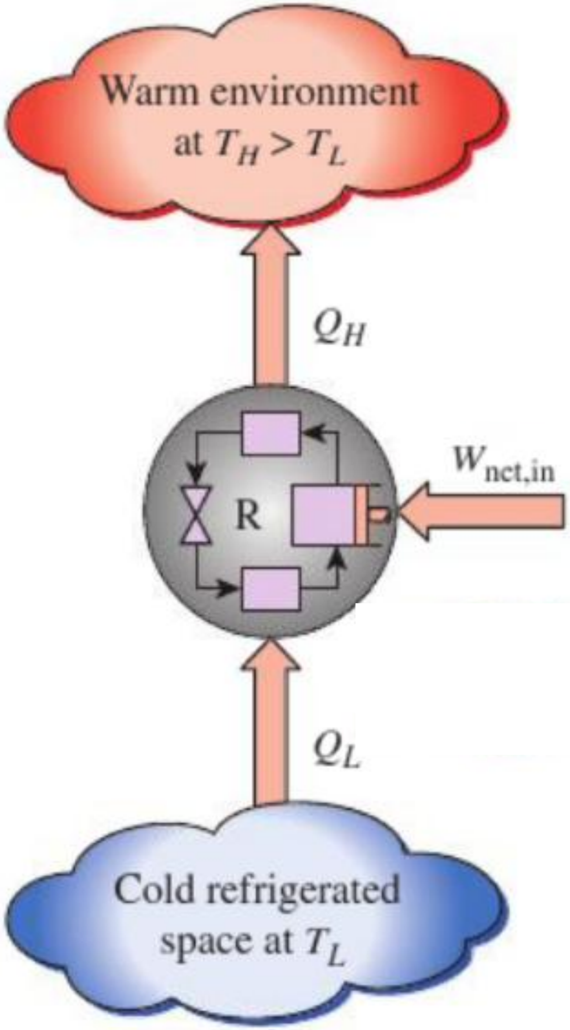


Refrigerator/Heat Pump

Refrigerators and Heat Pumps

- Heat transfer from high temperatures to low temperatures by nature
- The opposite can only be achieved using refrigerators and heat pumps

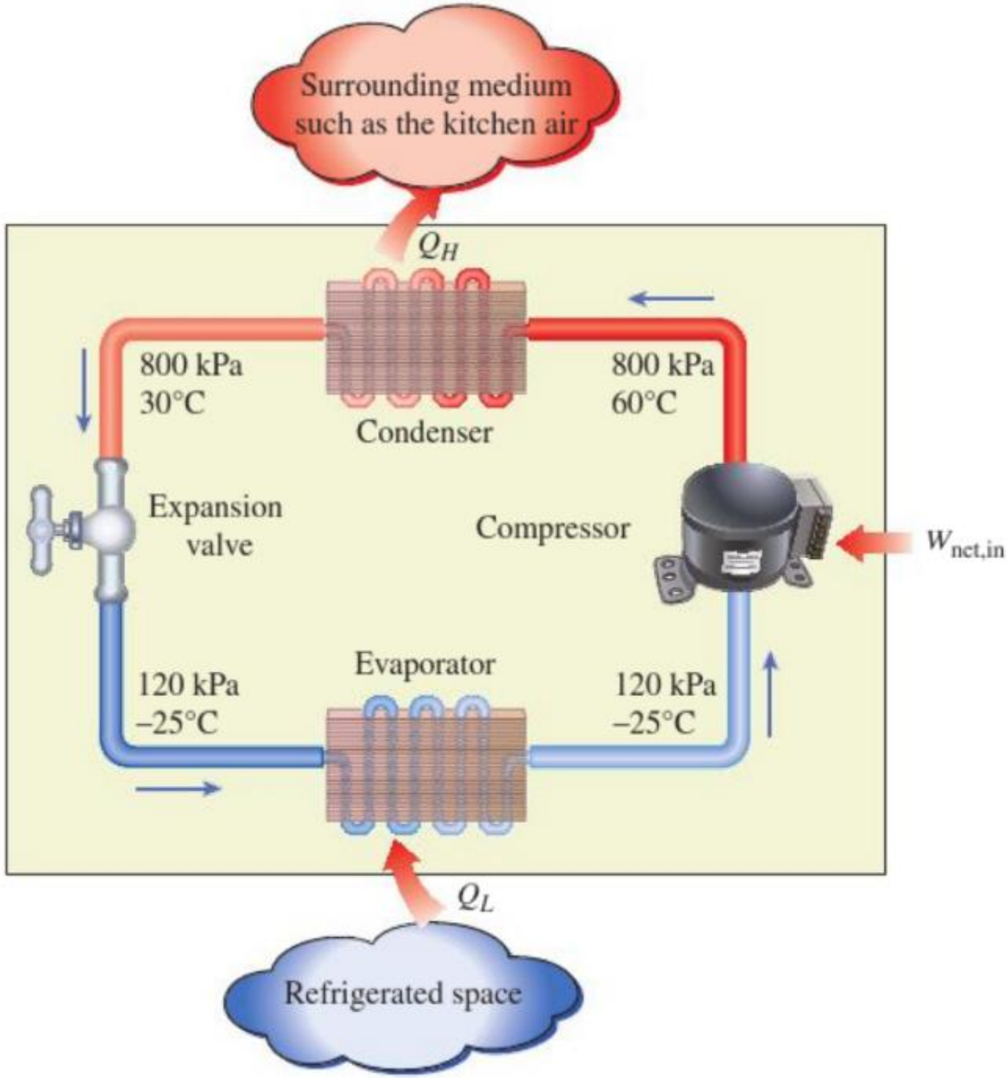
- Refrigerators and heat pumps are simply “reverse heat engines”
- Operate in a cycle
- Working fluid is called a refrigerant



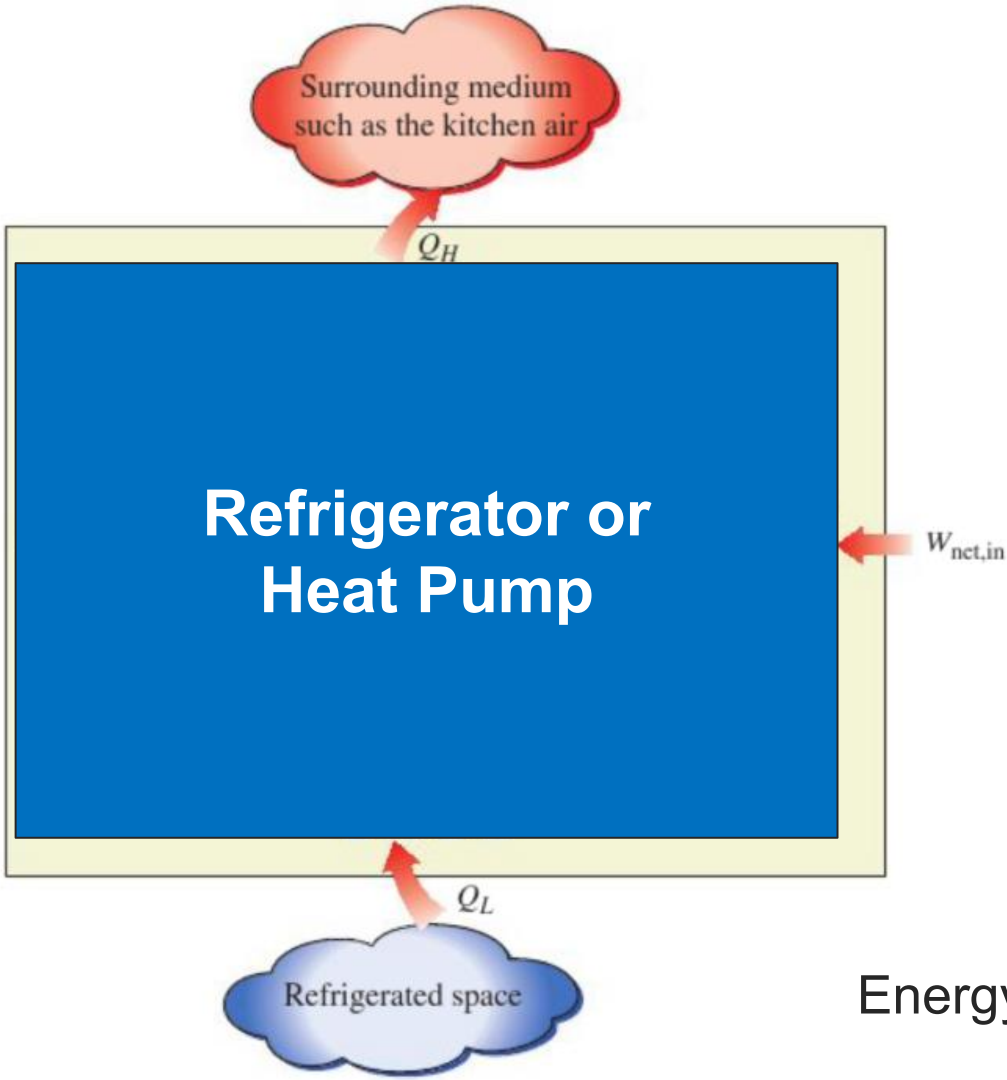
Refrigerators and Heat Pumps

- Vapour-compression refrigeration system is the most commonly used cycle

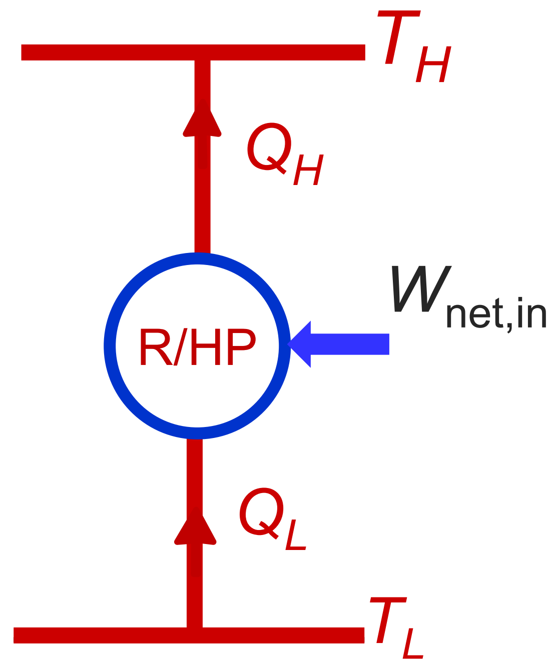
- W_{in} = work input to compressor to compress refrigerant from low to high pressure
- Q_H = heat rejected by refrigerant in condenser
- Q_L = heat absorbed by refrigerant in evaporator



Refrigerators and Heat Pumps



Simplified:



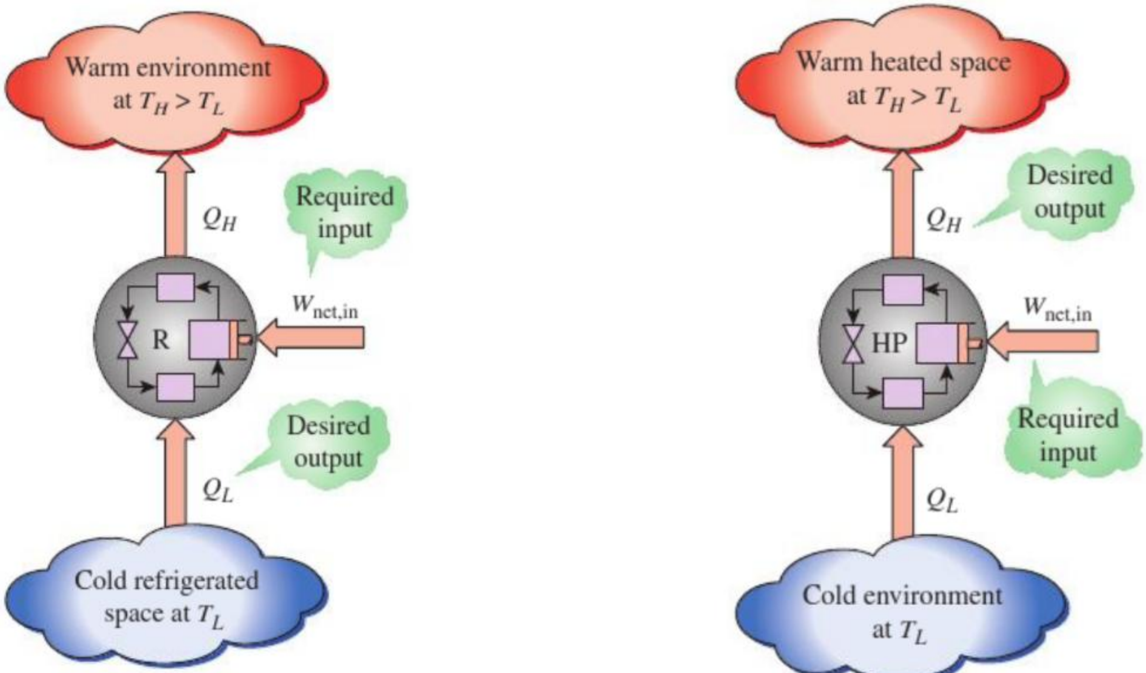
Energy balance: $W_{net,in} = Q_H - Q_L$

Coefficient of Performance

- Efficiency of refrigerators and heat pumps is expressed in terms of **coefficient of performance**

$$\boxed{\text{Coefficient of Performance} = \frac{\text{Desired output}}{\text{Required input}}}$$

- Formula depends on the function of the machine:



NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Coefficient of Performance – Refrigerator

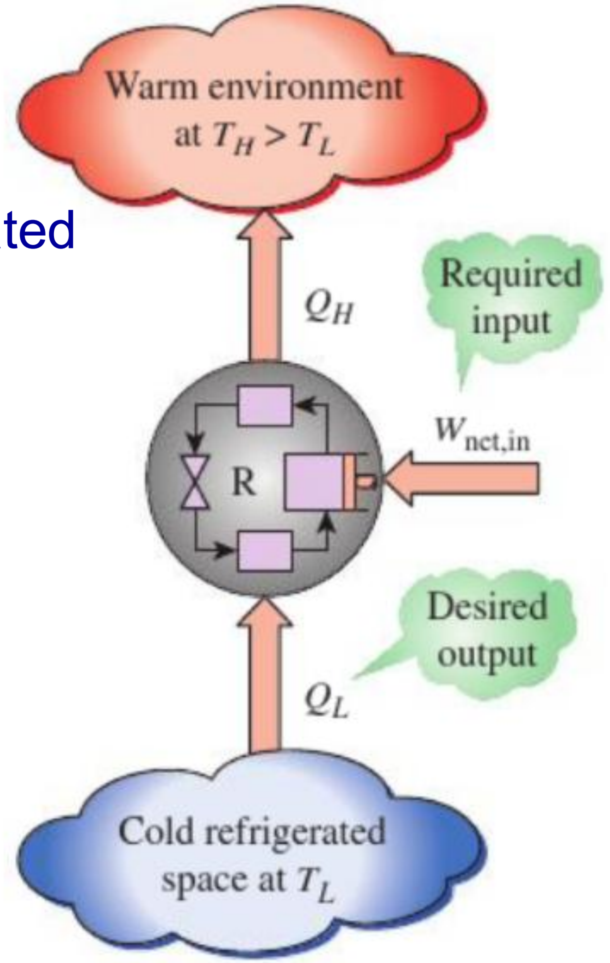
$$\text{Coefficient of Performance} = \frac{\text{Desired output}}{\text{Required input}}$$

Function of a refrigerator is to cool the refrigerated space:

$$\text{COP}_\text{R} = \frac{Q_L}{W_\text{net,in}}$$

Energy balance: $W_{net,in} = Q_H - Q_L$

$$\boxed{\text{COP}_\text{R} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1}}$$



Is it possible to have $\text{COP}_\text{R} > 1$*?*

Coefficient of Performance – Heat Pump

$$\text{Coefficient of Performance} = \frac{\text{Desired output}}{\text{Required input}}$$

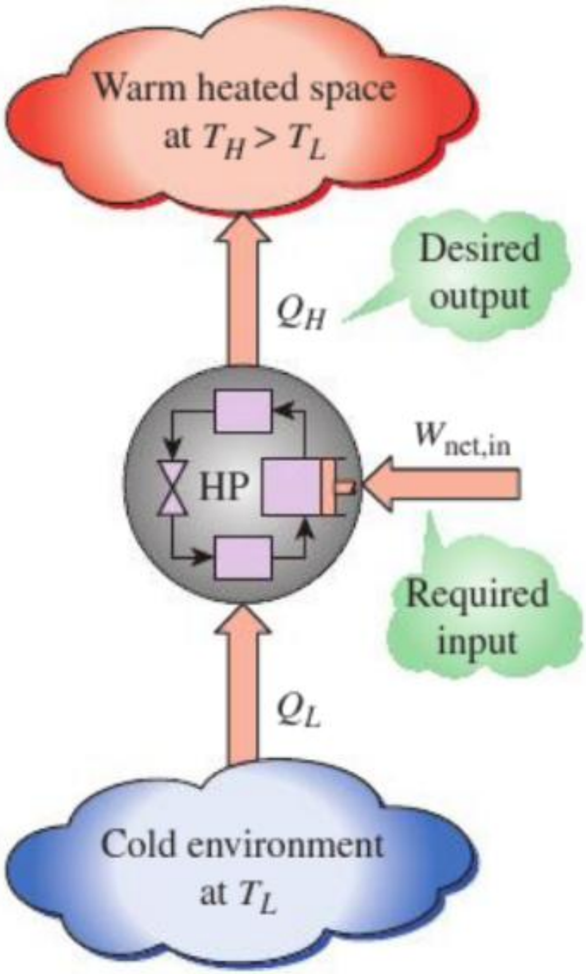
Function of a heat pump is to warm the heated space:

$$\text{COP}_{\text{HP}} = \frac{Q_H}{W_{\text{net,in}}}$$

Energy balance: $W_{net,in} = Q_H - Q_L$

$$\boxed{\text{COP}_{\text{HP}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L/Q_H}}$$

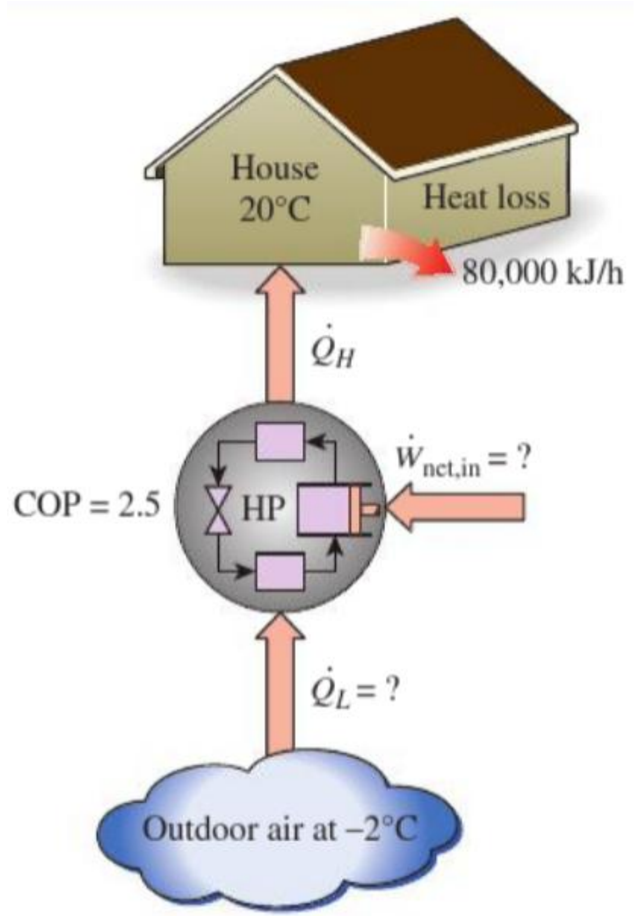
For the same Q_H & Q_L, $\text{COP}_{\text{HP}} = \text{COP}_{\text{R}} + 1$



Is it possible to have $\text{COP}_{\text{HP}} < 1$*?*

Example 2 – Heating a House with a Heat Pump

A heat pump is used to meet the heating requirements of a house and maintain it at 20°C. On a day when the outdoor air temperature drops to –2°C, the house is estimated to lose heat at a rate of 80,000 kJ/h. If the heat pump under these conditions has a COP of 2.5, determine the power consumed by the heat pump and the rate at which heat is absorbed from the cold outdoor air.



Assumptions:

Steady-state operating conditions

Example 2 – Heating a House with a Heat Pump

Heat pump delivers the same rate of heat as the heat loss of the house to maintain the indoor temperature:

$$\dot{Q}_H = 80{,}000 \text{ kJ/h}$$

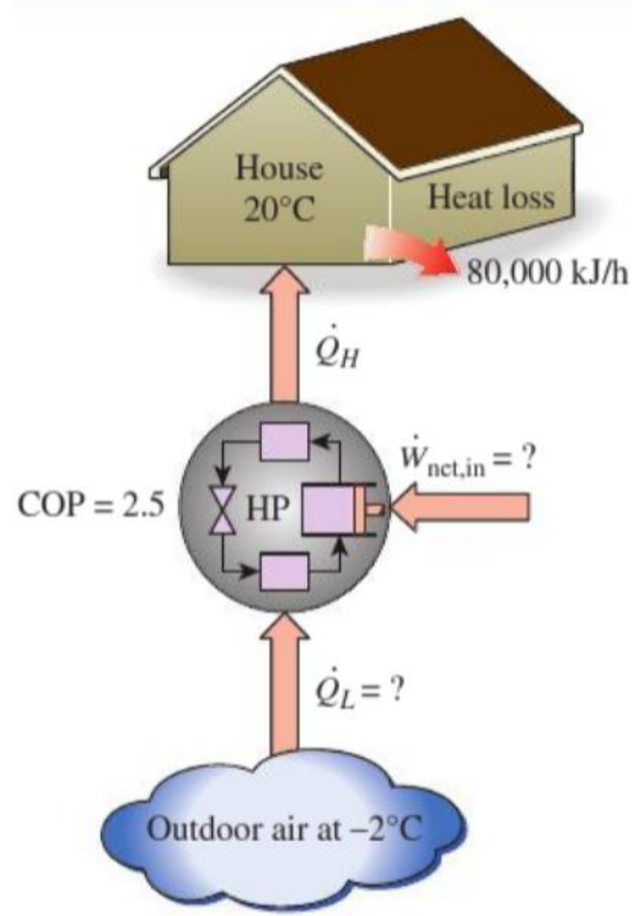
$$\text{COP}_{\text{HP}} = \frac{Q_H}{W_{\text{net,in}}}$$

$$\dot{W}_{\text{net,in}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP}}}$$

$$= \frac{80{,}000 \text{ kJ/h}}{2.5} = 32{,}000 \text{ kJ/h}$$

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{net,in}}$$

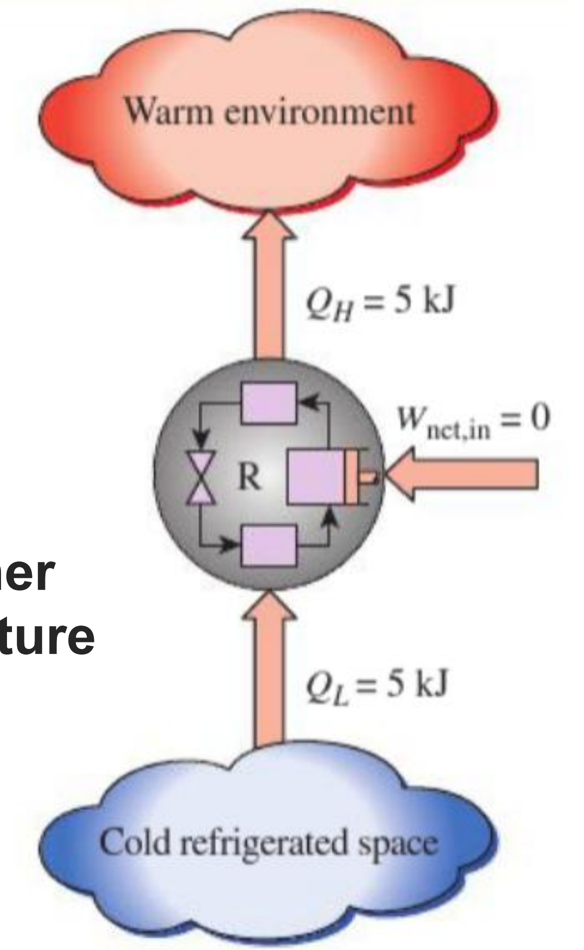
$$= 80{,}000 - 32{,}000 = 48{,}000 \text{ kJ/h}$$



2nd Law of Thermodynamics – Clausius Statement

- Heat is never transferred from a cold medium to a warmer one in nature
- Impossible to have a working refrigerator/heat pump that requires no power input

Clausius Statement:
It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body.



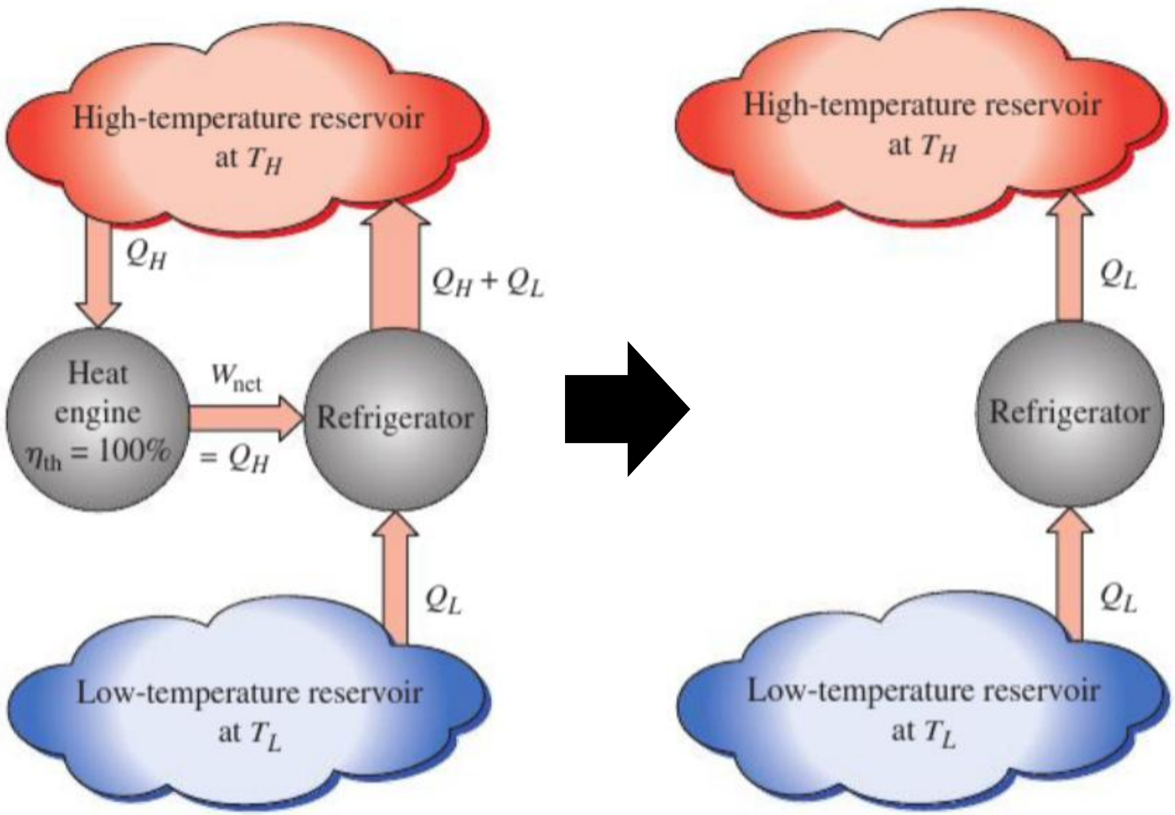
NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Equivalence of the Two Statements

- Kelvin-Planck statement and the Clausius statement are equivalent
- Any device that contradicts either statement, would contradict the other statement as well

Refrigerator powered by 100% efficient heat engine

Contradiction of Kelvin-Planck statement



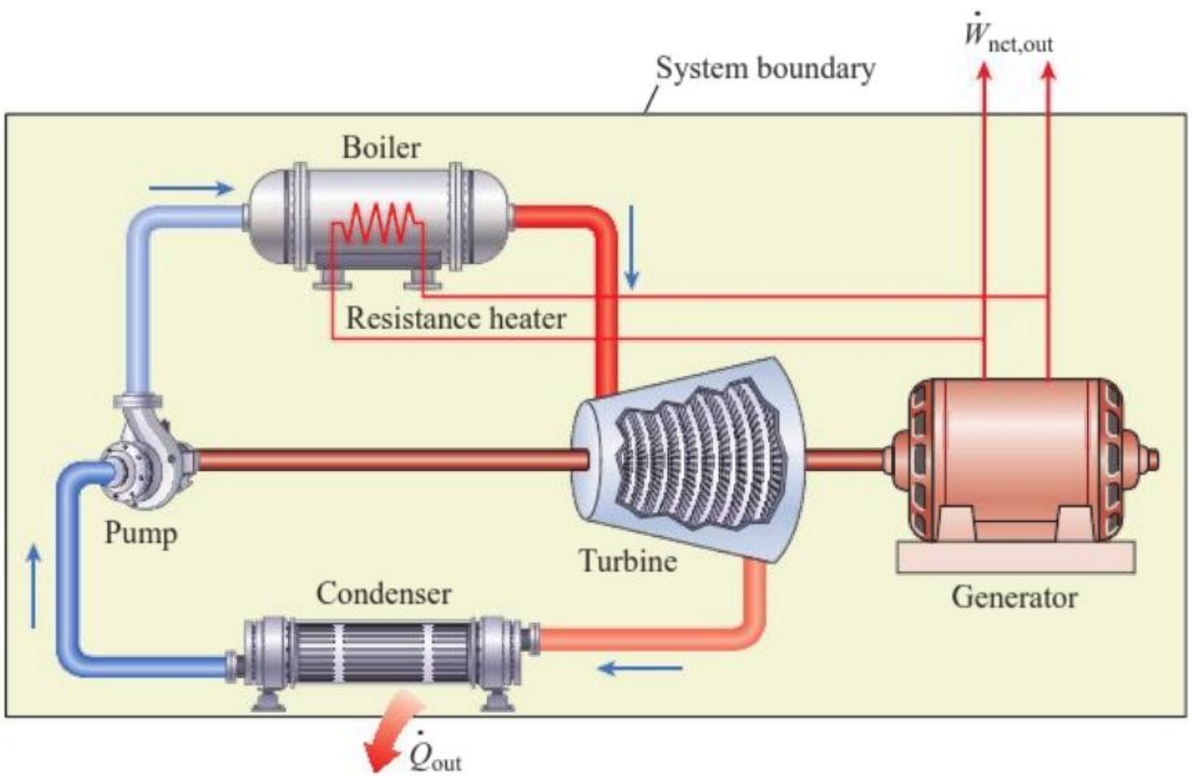
Equivalent refrigerator

Contradiction of Clausius statement

Perpetual Motion Machines (PMM)

- A device that contradicts either the 1st law or 2nd law of thermodynamics
  - Contradiction of the 1st law: perpetual motion machine of the first kind (PMM1)

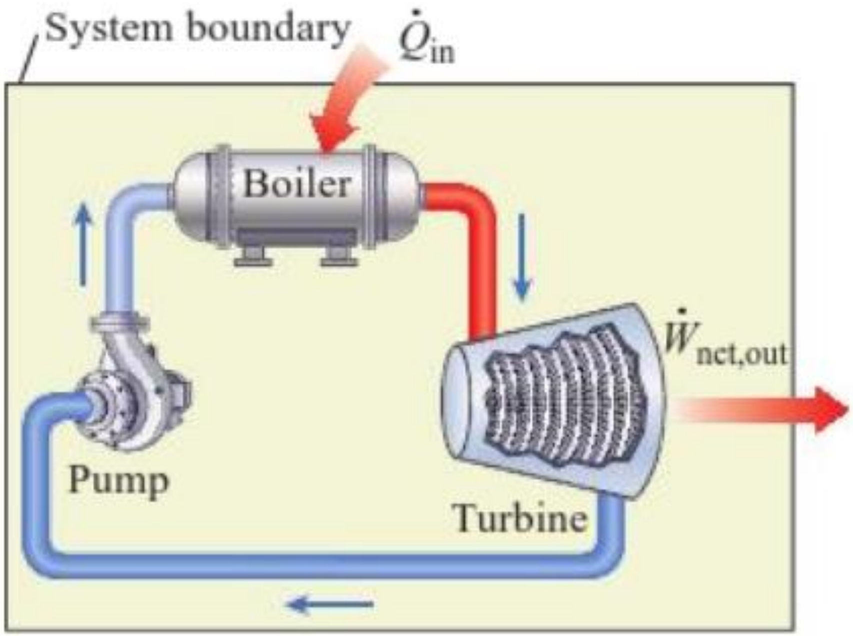
PMM1:



Perpetual Motion Machines (PMM)

- A device that violates either the 1st law or 2nd law of thermodynamics
  - Contradiction of the 2nd law: perpetual motion machine of the second kind (PMM2)

PMM2:

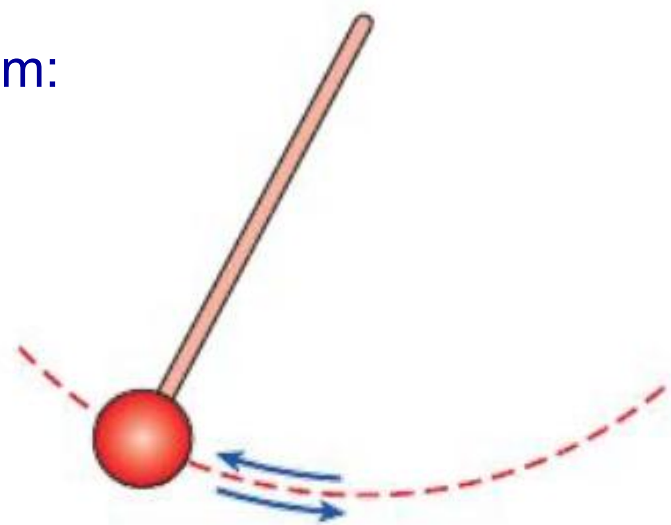


NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Reversible & Irreversible Processes

Recall: 2^{nd} law can determine the *theoretical limits* for performance of engineering systems/processes

Consider a pendulum:



Process in which there is continuous conversion between potential and kinetic energy of the mass

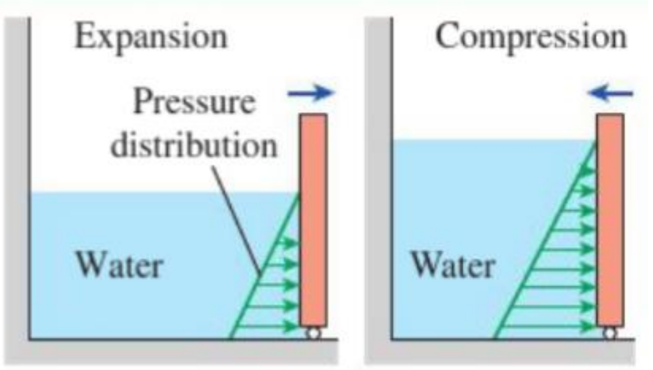
What is its theoretical limit?

Reversible & Irreversible Processes

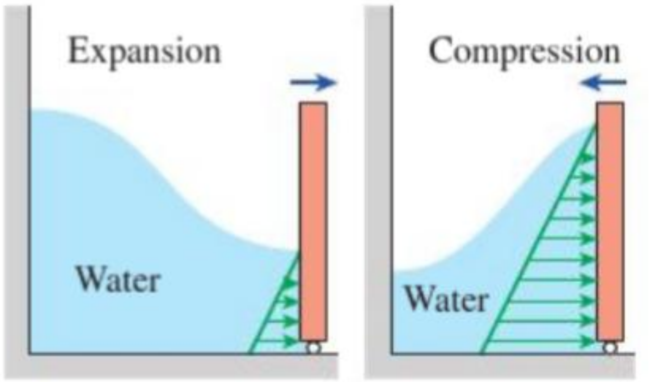
- An ideal pendulum would be a *frictionless* pendulum
  - ‘Perfect’ conversion between kinetic and potential energy
- A reversible process is defined as one which can be reversed without leaving any trace on the surroundings
  - The state of the *system & surroundings* can be reverted to initial states at the end of the reverse process
  - Theoretical/ideal process
- An irreversible process is the opposite of a reversible process
  - Characteristic of ***all*** processes in nature
- Reversible processes deliver the most and consume the least work
  - Serve as the theoretical limit for its corresponding irreversible process
  - Easy to analyse

Reversible & Irreversible Processes

- Reversible processes are *idealizations* of actual processes
- Actual devices/systems can be approximated as reversible processes at best
- Actual processes are compared against their corresponding idealized/reversible processes to determine its efficiency



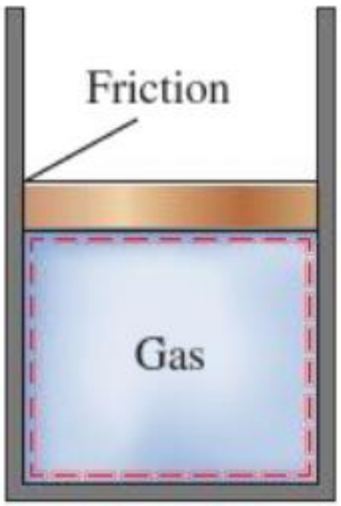
(*a*) Slow (reversible) process



(*b*) Fast (irreversible) process

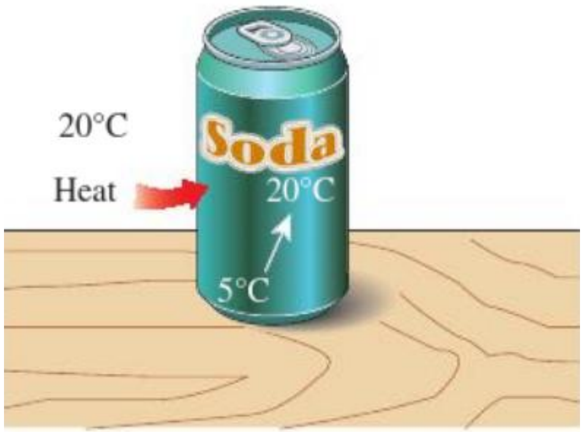
Irreversibilities

- Factors that cause a process to be irreversible
  - Friction
  - Heat transfer across finite temperature difference
  - Mixing of two fluids
  - Unrestrained expansion
  - Electrical resistance
  - Inelastic deformation of solids
  - Chemical reaction

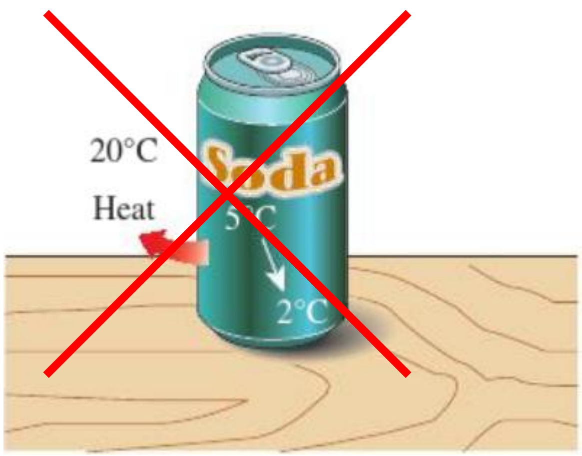


- Presence of any one factor would cause the process to be irreversible

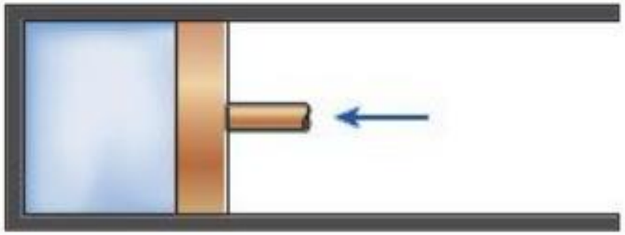
Irreversibilities



Heat transfer across
finite temp. difference



Impossible reverse process



(*a*) Fast compression

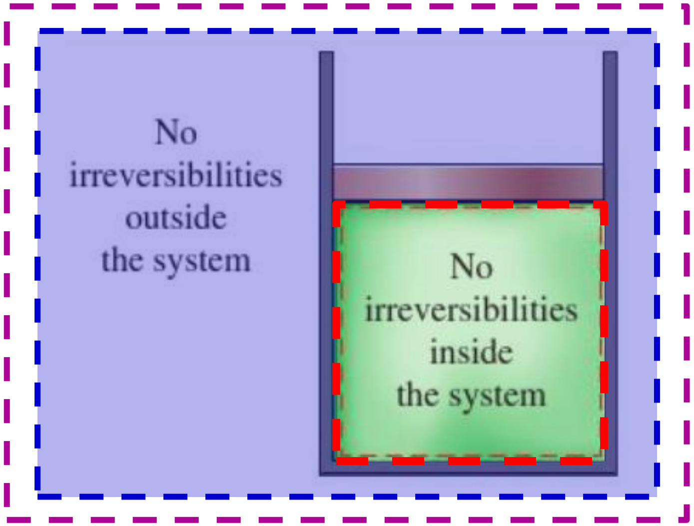
(*b*) Fast expansion

(*c*) Unrestrained expansion

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

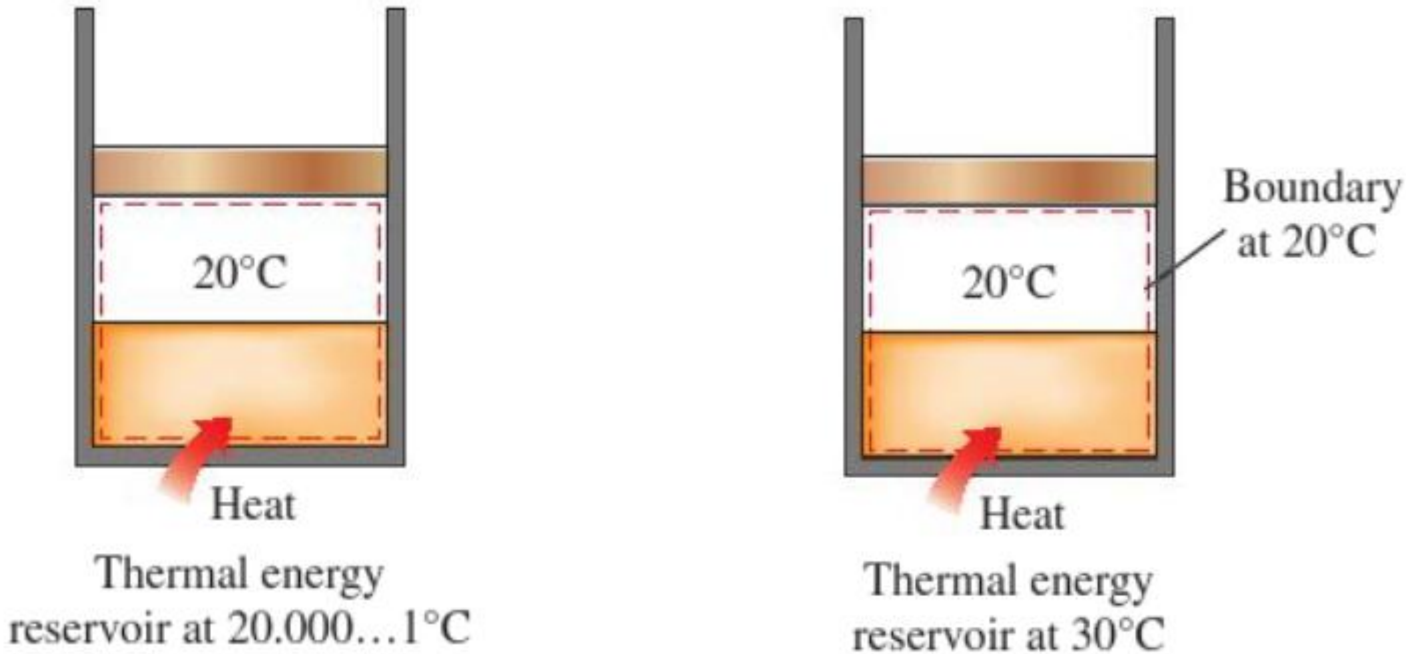
Internally & Externally Reversible Processes

- **Internally reversible**: if no irreversibilities occur within the system boundaries (red box)
- **Externally reversible**: if no irreversibilities occur outside the system boundaries (blue box)
- **Totally reversible**: internally + externally reversible; i.e. no irreversibilities within the system or surroundings (magenta box)



Internally & Externally Reversible Processes

- Example of internally reversible process: boiling of a fluid (constant temperature & pressure process)



(*a*) Totally reversible

(*b*) Internally reversible

$$\Delta T = 20.00\ldots1 - 20$$
$$= 0.000\ldots1°\mathrm{C}$$
$$\approx 0°\mathrm{C}$$

$$\Delta T = 10°\mathrm{C}$$

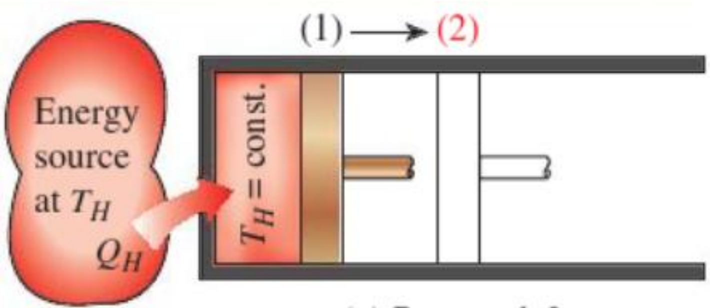
Carnot Cycle

- Proposed by French engineer Sadi Carnot in 1824
- Theoretical cycle
- Consists of 4 reversible processes:
  - 2 isothermal processes
  - 2 adiabatic processes
- Applicable to closed systems or steady flow systems
- Sets the theoretical limits for heat engines, refrigerators and heat pumps

Carnot Cycle

a) Reversible Isothermal Expansion (1 – 2, T_H = const.)

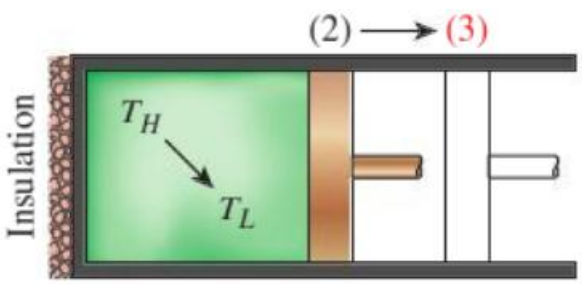
– Gas expands at constant temp. while absorbing heat from energy source



(a) Process 1-2

b) Reversible Adiabatic Expansion (2 – 3, T_H drops to T_L)

– Gas does work on surroundings and expands while its temp. drops



(b) Process 2-3

Carnot Cycle

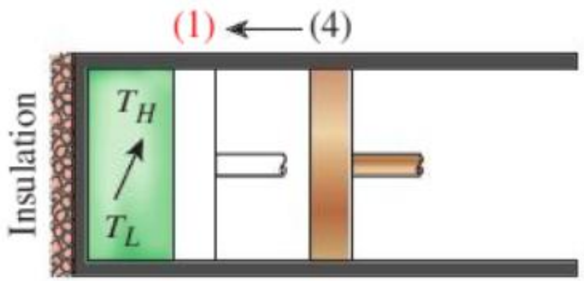
c) Reversible Isothermal Compression (3 – 4, T_L = const.)

– Gas compression at constant temp. while losing heat to energy sink

(c) Process 3-4

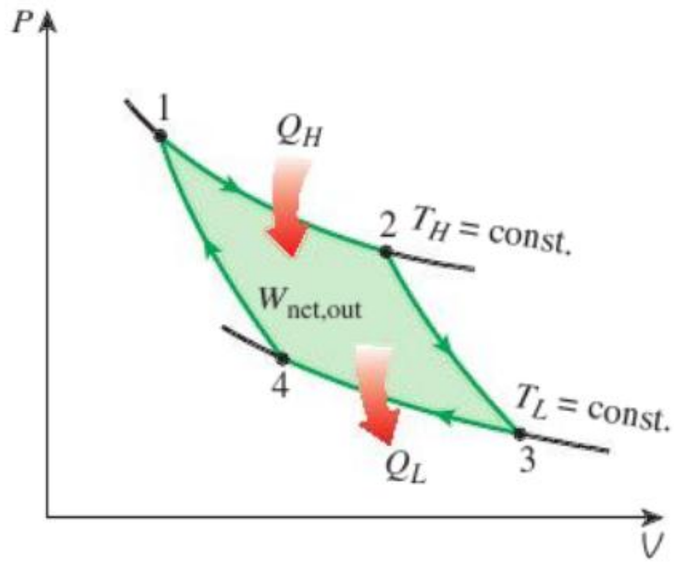
d) Reversible Adiabatic Compression (4 – 1, T_L rises to T_H)

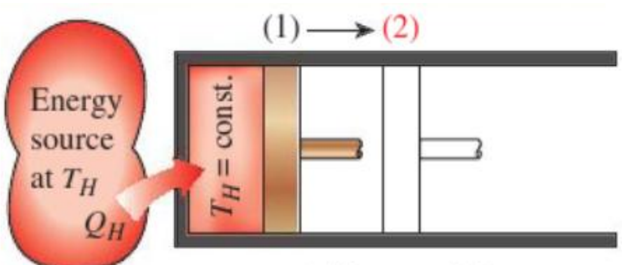
– Work done on gas to compress it and its temp. rises



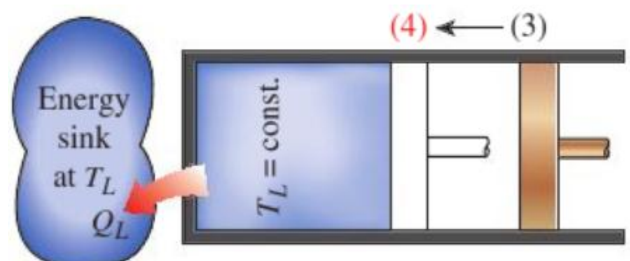
(d) Process 4-1

Carnot Cycle – PV Diagram

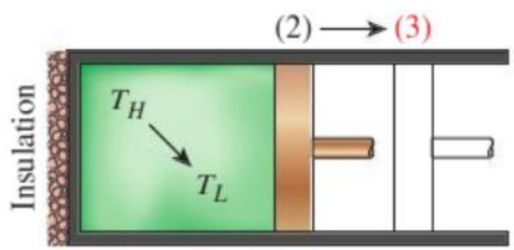




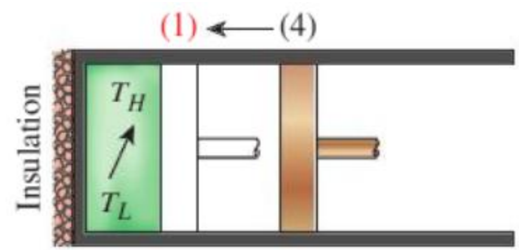
(a) Process 1-2



(c) Process 3-4



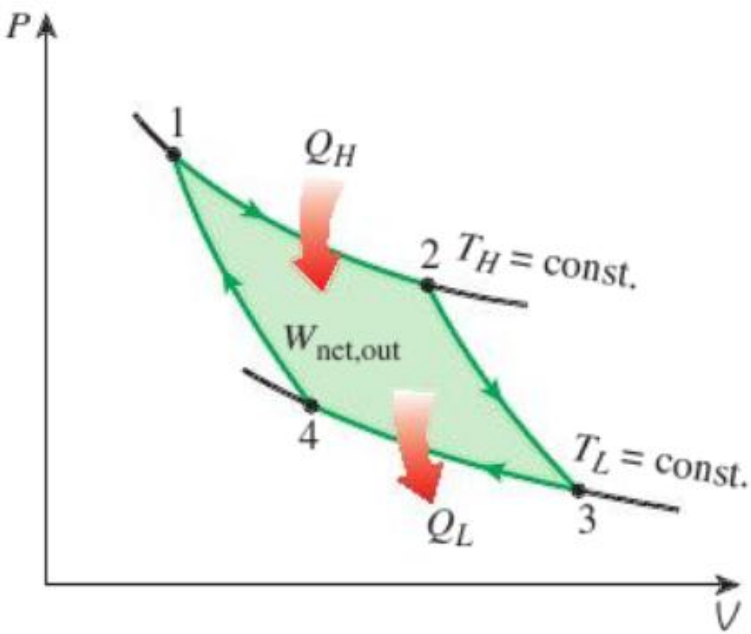
(b) Process 2-3



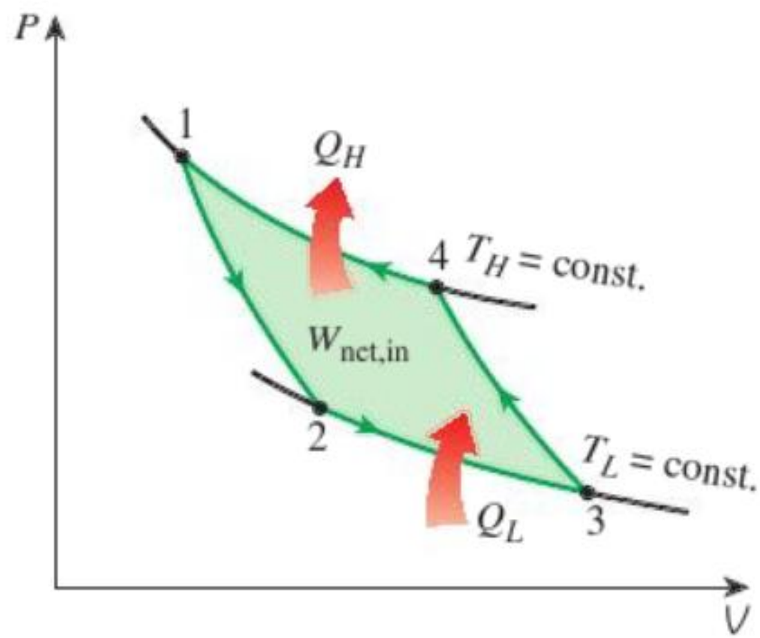
(d) Process 4-1

Reversed Carnot Cycle

- The Carnot cycle is a totally reversible cycle
- A reversed Carnot cycle becomes the Carnot refrigeration cycle



Carnot heat-engine cycle



Reversed Carnot heat-engine cycle

Carnot refrigeration cycle

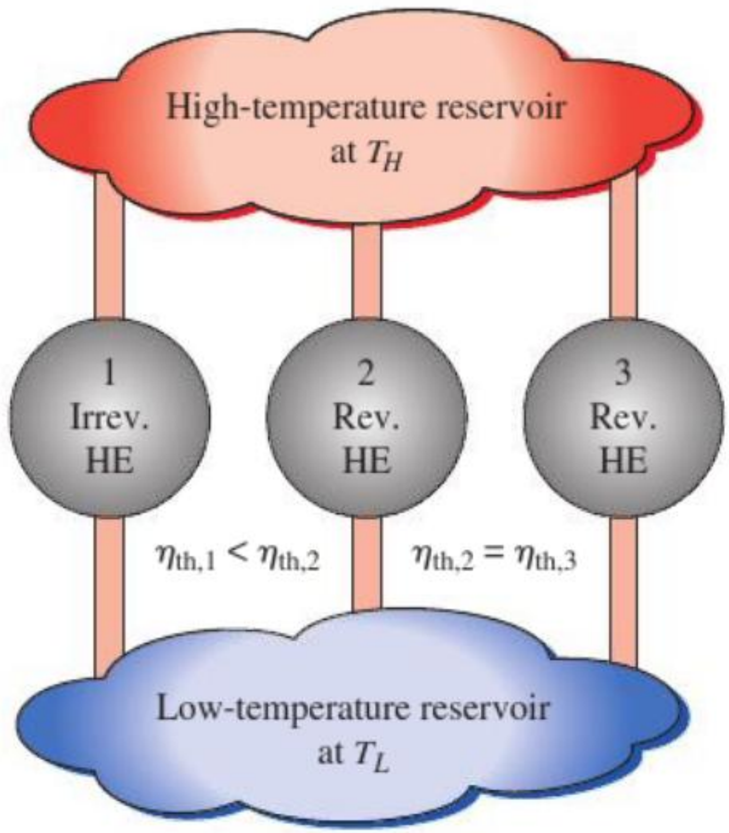
Carnot Principles

1. The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs.

$$\eta_{th,1,\text{irrev}} < \eta_{th,2,\text{rev}}$$

2. The efficiencies of all reversible heat engines operating between the same two reservoirs are the same.

$$\eta_{th,2,\text{rev}} = \eta_{th,3,\text{rev}}$$



Proof of Carnot Principles – Part 1

- Assume a violation of the principles such that:

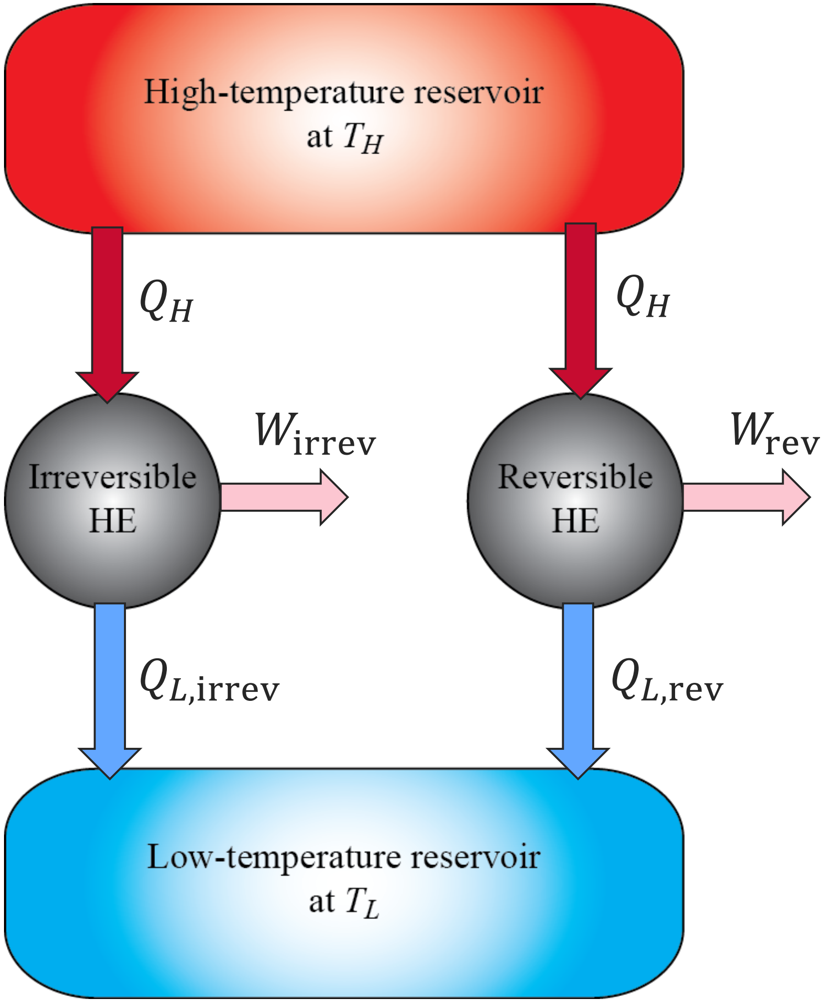
$$\eta_{th,\text{irrev}} > \eta_{th,\text{rev}}$$

- The output work from an irreversible HE would be higher than a reversible HE in this case:

$$W_{\text{irrev}} > W_{\text{rev}}$$

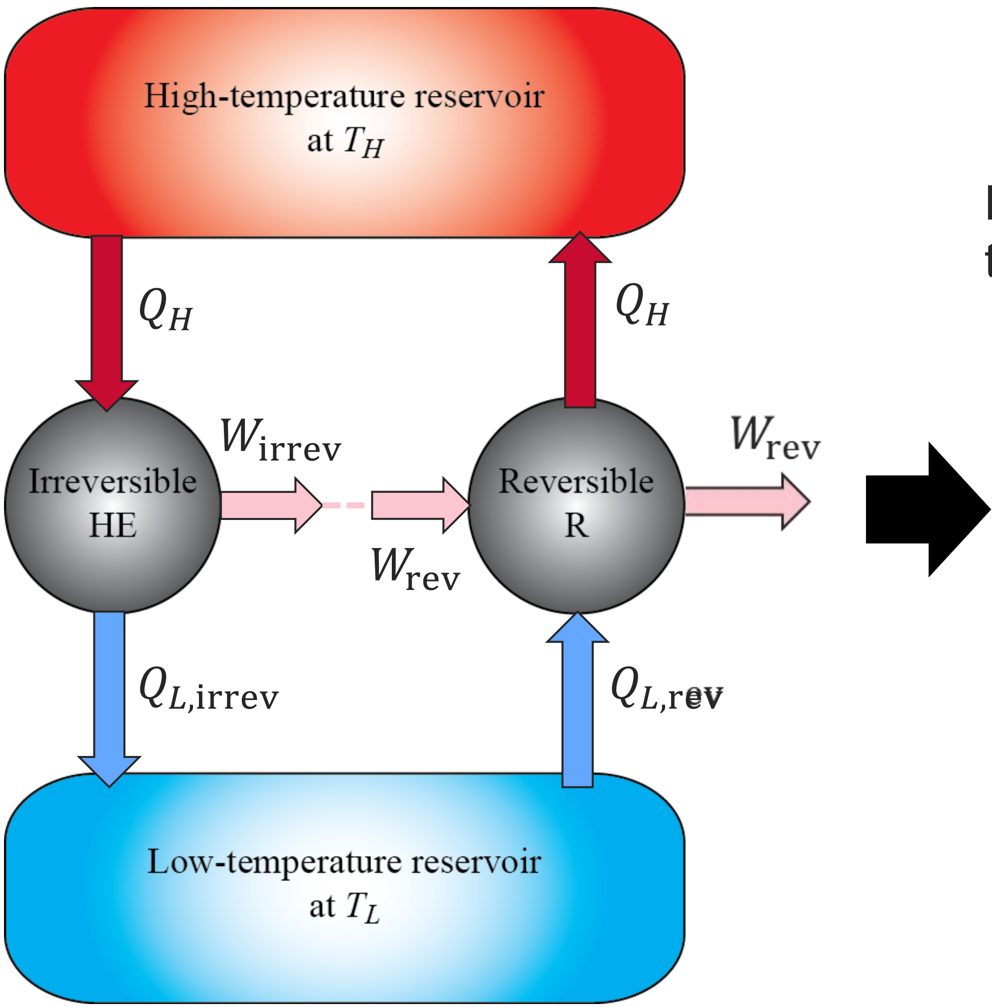
- The heat rejected for an irreversible HE would consequently lower than a reversible HE:

$$Q_{L,\text{irrev}} < Q_{L,\text{rev}}$$

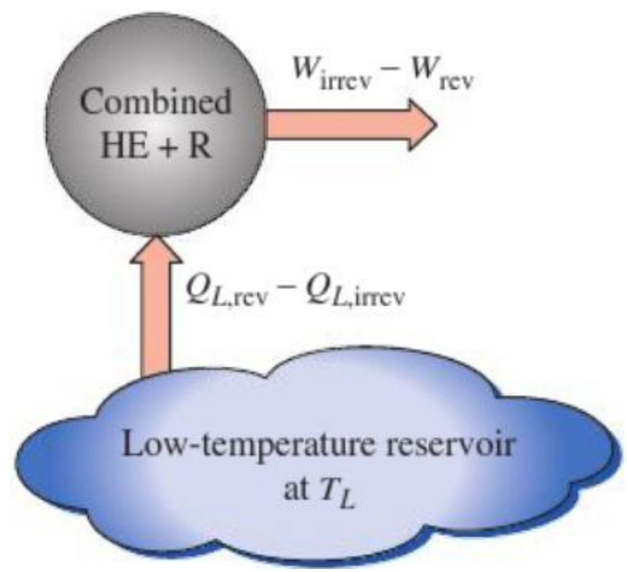


Proof of Carnot Principles – Part 1

- Reverse the reversible HE, power the reversible R with the HE:



Equivalent machine that violates the Kelvin-Planck statement!



Proof of Carnot Principles – Part 2

- Assume a violation of the principles such that:

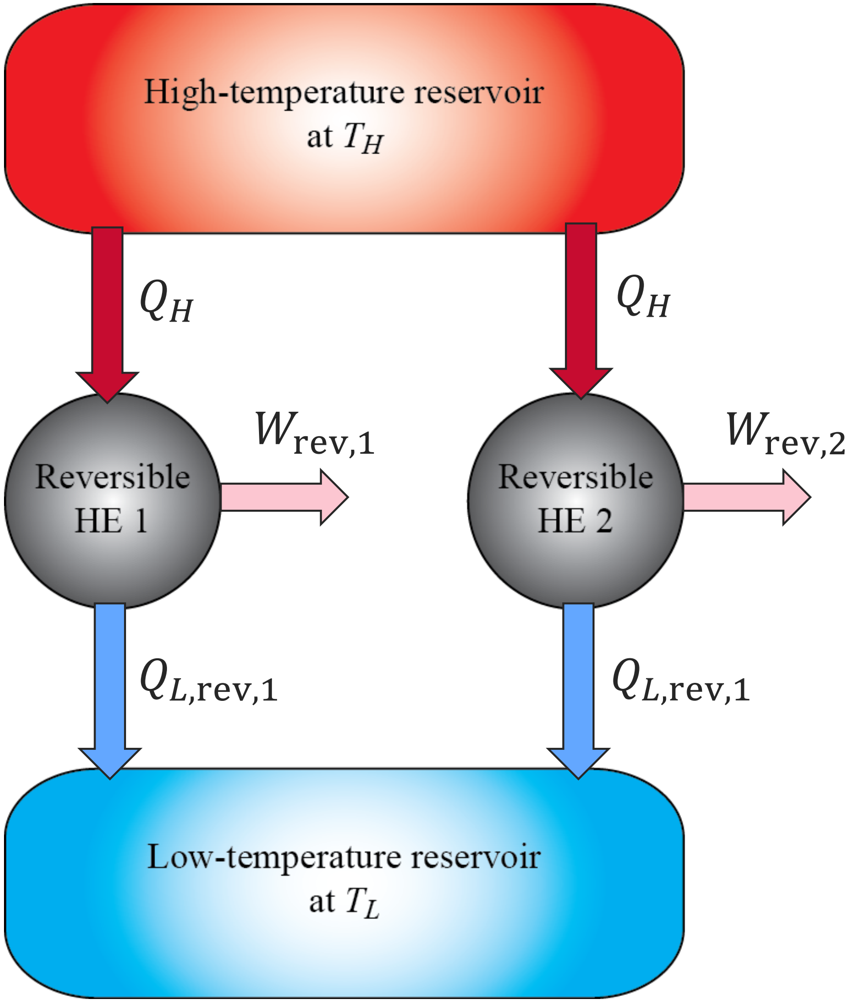
$$\eta_{th,\mathrm{rev},1} > \eta_{th,\mathrm{rev},2}$$

- The output work from reversible HE 1 would be higher than reversible HE 2:

$$W_{\mathrm{rev},1} > W_{\mathrm{rev},2}$$

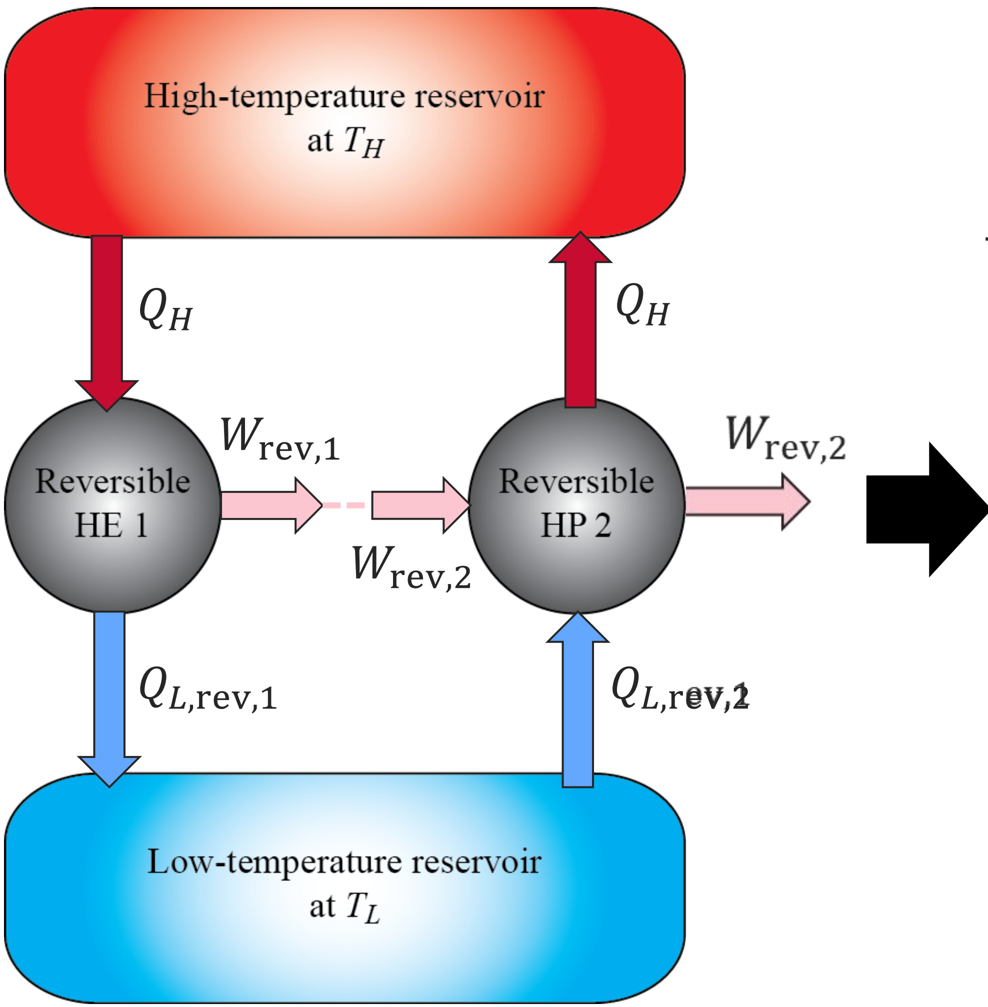
- The heat rejected for reversible HE 1 would then be lower than reversible HE 2:

$$Q_{L,\mathrm{rev},1} < Q_{L,\mathrm{rev},2}$$

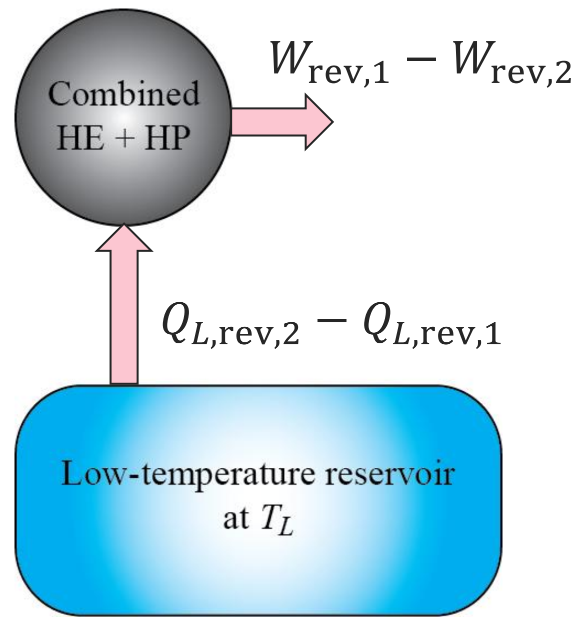


Proof of Carnot Principles – Part 2

- Reverse HE 2, power it with HE 1:



Equivalent machine that violates the Kelvin-Planck statement!



Thermodynamic Temperature Scale

- A temperature scale that is independent of the properties of substances that are used to measure temperature is called a thermodynamic temperature scale
- Offers great convenience for thermodynamic calculations

- Recall the Carnot principle: all reversible heat engines operating between the same two reservoirs have the same efficiency.
- Thermal reservoirs are characterized only by their temperatures
- Thermal efficiencies of reversible heat engines can be expressed as a function of reservoir temperatures

Thermodynamic Temperature Scale – Derivation

- Thermal efficiencies of rev. HE is a function of temperature only:

$$\eta_{th,\text{rev}} = g(T_H, T_L) = 1 - \frac{Q_L}{Q_H}$$

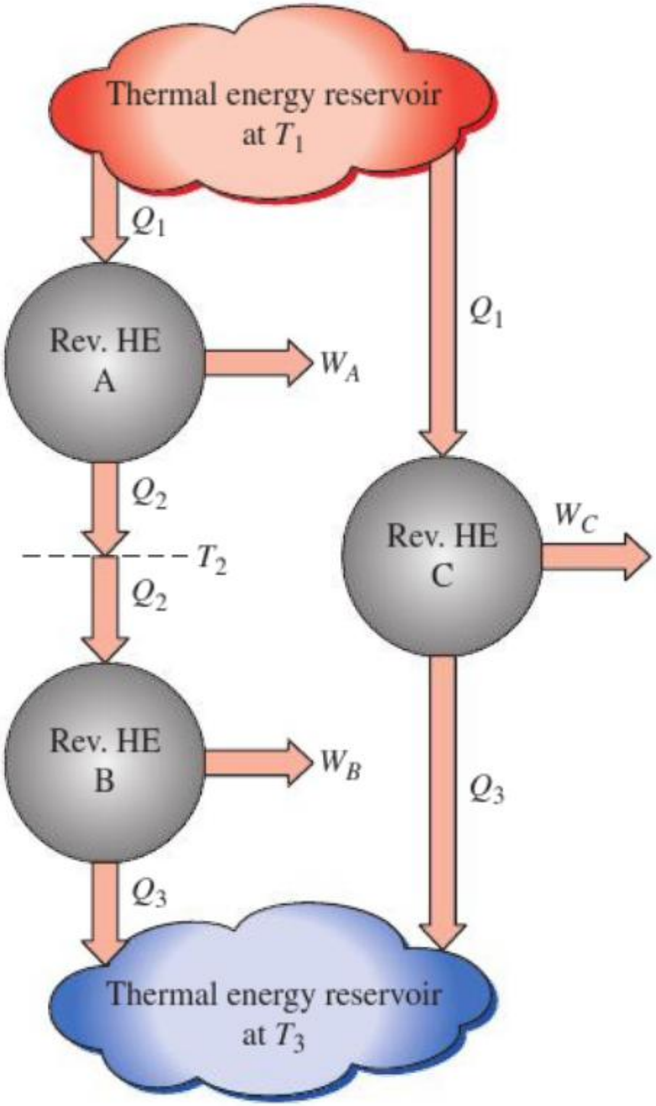
$$\rightarrow \frac{Q_L}{Q_H} \text{ or } \frac{Q_H}{Q_L} = f(T_H, T_L)$$

- For the three heat engines:

$$\frac{Q_1}{Q_2} = f(T_1, T_2)$$

$$\frac{Q_2}{Q_3} = f(T_2, T_3)$$

$$\frac{Q_1}{Q_3} = f(T_1, T_3)$$



Thermodynamic Temperature Scale – Derivation

- Let's consider:

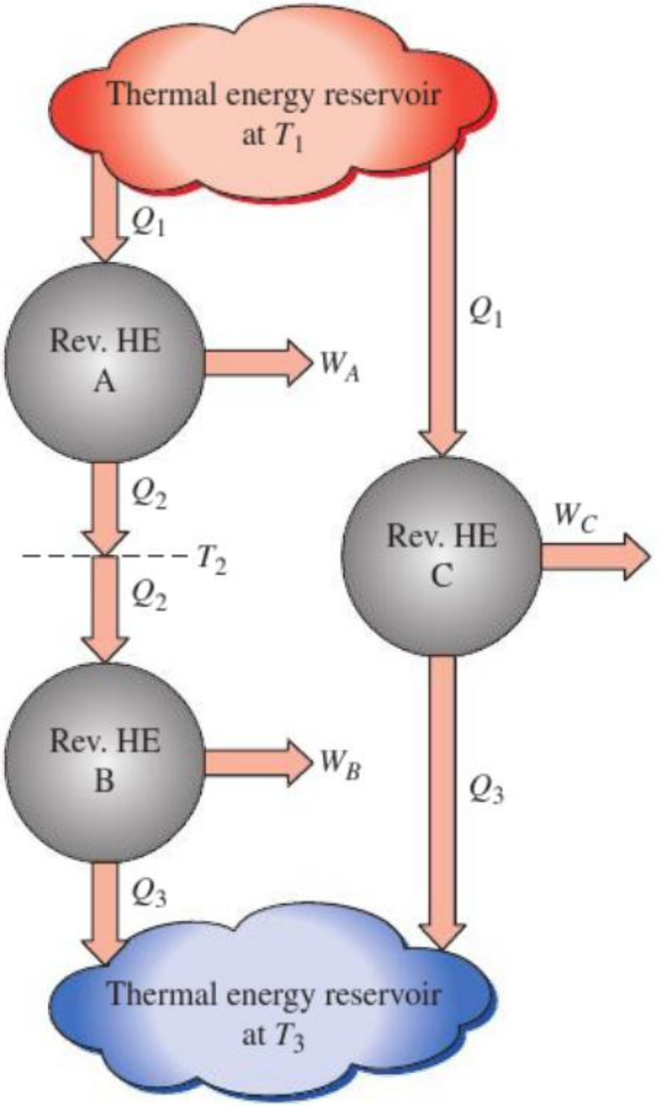
$$\because \frac{Q_1}{Q_3} = \frac{Q_1}{Q_2} \cdot \frac{Q_2}{Q_3}$$

$$\therefore f(T_1, T_3) = f(T_1, T_2) \cdot f(T_2, T_3)$$

- Product of functions on RHS must cause the T_2 term to disappear
- Only possible if function f is of the form:

$$f(T_1, T_2) = \frac{\phi(T_1)}{\phi(T_2)} \qquad f(T_2, T_3) = \frac{\phi(T_2)}{\phi(T_3)}$$

$$\rightarrow f(T_1, T_3) = \frac{\phi(T_1)}{\phi(T_3)} = \frac{Q_1}{Q_3}$$



Thermodynamic Temperature Scale – Derivation

- Therefore, for a reversible HE:

$$\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{\phi(T_H)}{\phi(T_L)}$$

- Lord Kelvin thus propose and define:

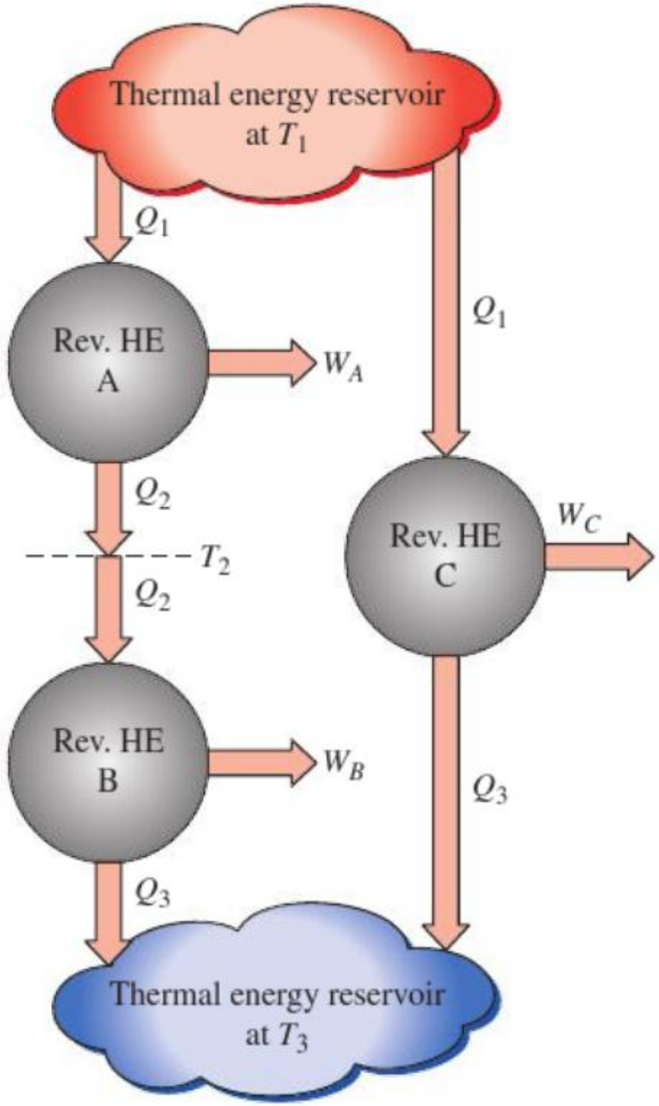
$$\phi(T) = T$$

- The thermodynamic temperature scale is therefore defined as:

$$\boxed{\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{T_H}{T_L}}$$

Note: $Q_H \neq T_H$

$Q_L \neq T_L$



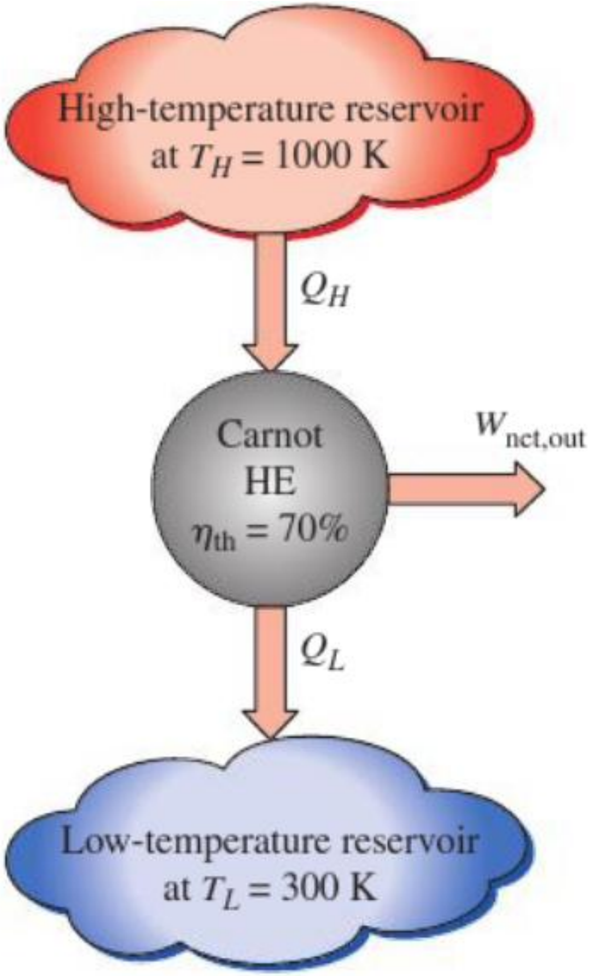
Thermodynamic Temperature Scale

- This temperature scale is called the **Kelvin scale**
- Temperatures on this scale are called absolute temperatures

$$T(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15$$

- Magnitudes of temperature units on the Kelvin and Celsius scales are the same:

$$1\ \mathrm{K} \equiv 1^\circ\mathrm{C}$$



NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Carnot Heat Engines

- Hypothetical heat engine operating on the Carnot cycle
- Most efficient (ideal) heat engine

- Recall that the thermal efficiency of ***any*** heat engine is:

$$\eta_{th} = 1 - \frac{Q_L}{Q_H}$$

- From the thermodynamic temperature scale,

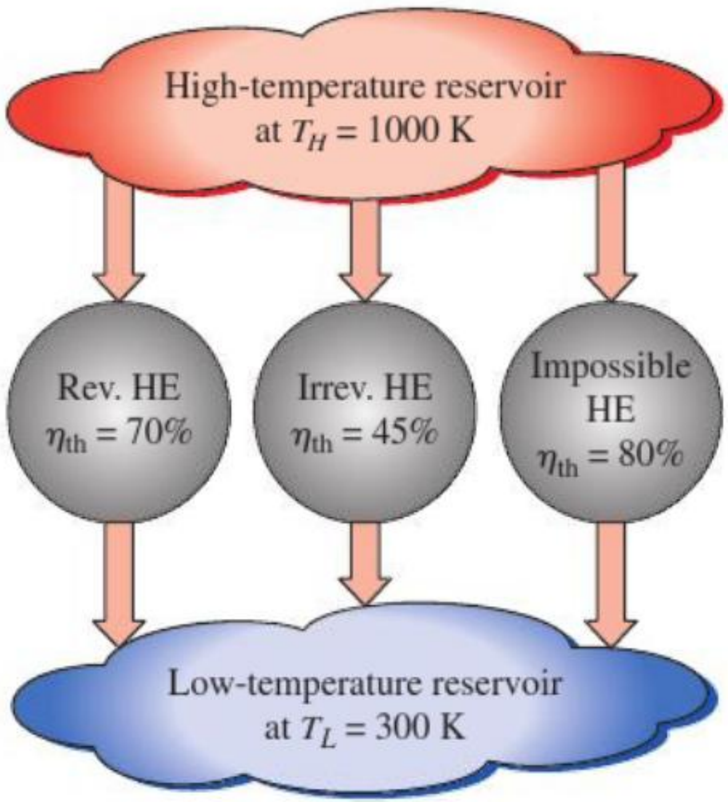
$$\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{T_H}{T_L} \quad \text{or} \quad \left(\frac{Q_L}{Q_H}\right)_{\text{rev}} = \frac{T_L}{T_H}$$

- Thermal efficiency of a Carnot heat engine:

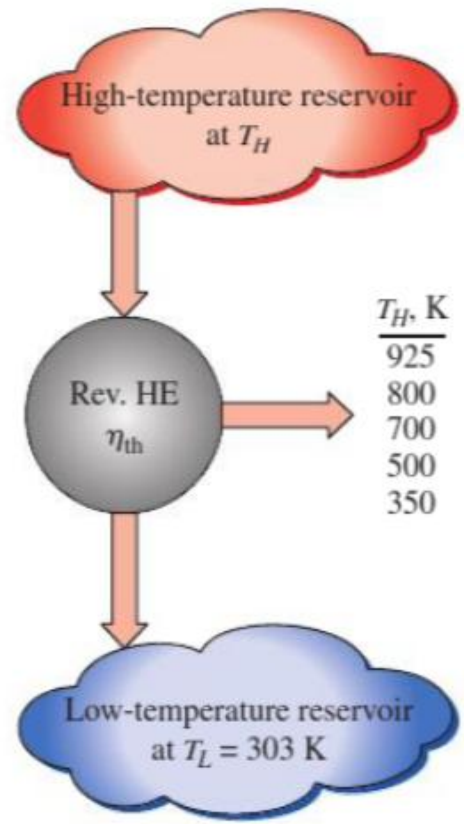
$$\boxed{\eta_{th,\text{rev}} = 1 - \frac{T_L}{T_H}} \qquad \text{(Carnot efficiency)}$$

Absolute temperatures only!

Carnot Heat Engines



$$\eta_{th} \begin{cases} < \eta_{th,rev} & \text{irreversible heat engine} \\ = \eta_{th,rev} & \text{reversible heat engine} \\ > \eta_{th,rev} & \text{impossible heat engine} \end{cases}$$



T_H, K	η_{th}, %
925	67.2
800	62.1
700	56.7
500	39.4
350	13.4

- Efficiency increases with source temperature
- Energy has higher quality at higher temperatures.

Carnot Refrigerators & Heat Pumps

- A device that operates on the reversed Carnot cycle

- Recall that the coefficient of performance for any refrigerator or heat pump are:

$$\mathrm{COP_R} = \frac{1}{Q_H/Q_L - 1} \quad \& \quad \mathrm{COP_{HP}} = \frac{1}{1 - Q_L/Q_H}$$

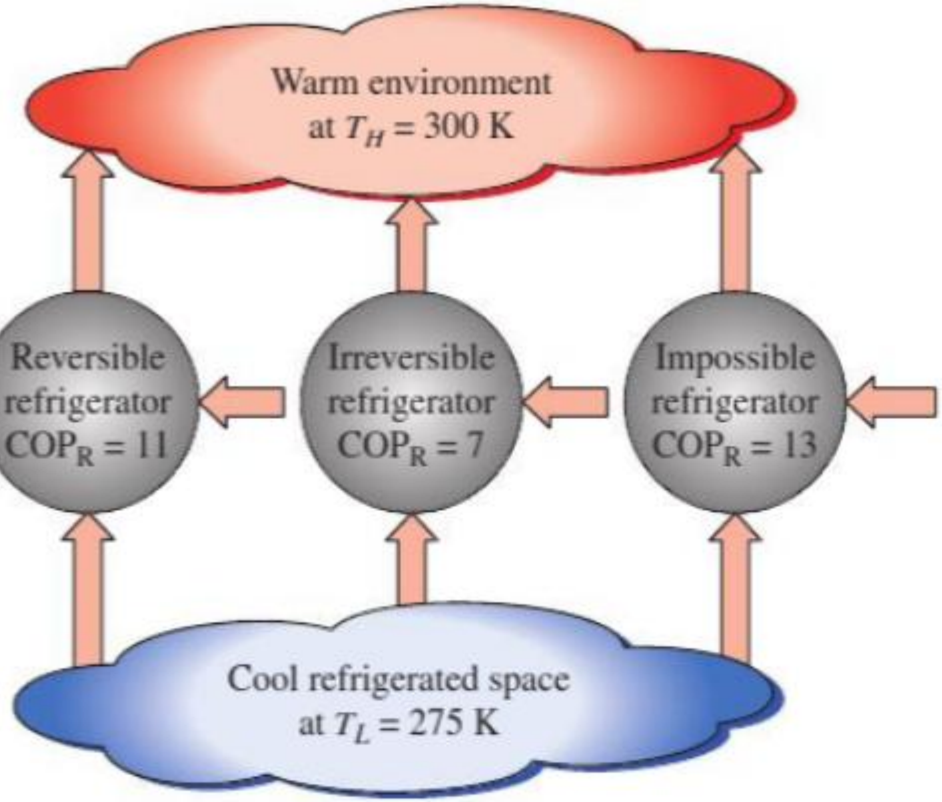
- Coefficient of performance for a Carnot refrigerator:

$$\boxed{\mathrm{COP_{R,rev}} = \frac{1}{T_H/T_L - 1}}$$

- Coefficient of performance for a Carnot heat pump:

$$\boxed{\mathrm{COP_{HP,rev}} = \frac{1}{1 - T_L/T_H}}$$

Carnot Refrigerators & Heat Pumps



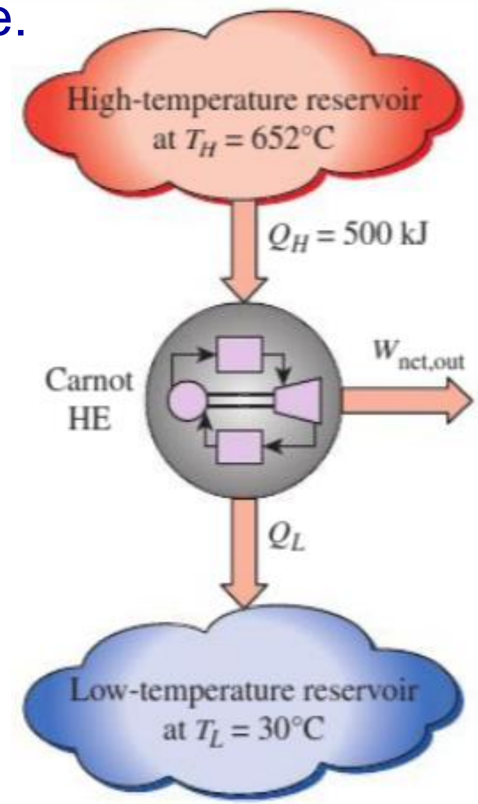
$$COP_R \begin{cases} < COP_{R,rev} & \text{irreversible refrigerator} \\ = COP_{R,rev} & \text{reversible refrigerator} \\ > COP_{R,rev} & \text{impossible refrigerator} \end{cases}$$

Same principles apply to heat pumps

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Example 3: Analysis of a Carnot Heat Engine

A Carnot heat engine receives 500 kJ of heat per cycle from a high-temperature source at 652°C and rejects heat to a low-temperature sink at 30°C. Determine the thermal efficiency of this Carnot engine and the amount of heat rejected to the sink per cycle.



$$\eta_{th,rev} = 1 - \left(\frac{Q_L}{Q_H}\right)_{rev} = 1 - \frac{T_L}{T_H}$$

$$= 1 - \frac{(30+273)\text{K}}{(652+273)\text{K}} = 0.672$$

$$\left(\frac{Q_L}{Q_H}\right)_{rev} = \frac{T_L}{T_H}$$

$$Q_{L,rev} = \frac{T_L}{T_H} \cdot Q_{H,rev}$$

$$= \frac{(30+273)\text{K}}{(652+273)\text{K}}(500\text{kJ}) = 164\text{ kJ}$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Summary

- Processes can only proceed in one direction and satisfy both the 1st and 2nd Laws of Thermodynamics
- Thermal energy reservoirs can absorb/supply finite amounts of heat without any change in temperature
- Two classes of cyclic devices that operate between T_H and T_L thermal energy reservoirs:
  - Heat engines that produce work output
  - Refrigerators absorb heat from T_L reservoirs and heat pumps supply heat to T_H reservoirs, both require work input
- Thermal efficiency of a heat engine: $$\eta_{\text{th}} = 1 - \frac{Q_L}{Q_H}$$
- Coefficients of performance (refrigerators & heat pumps):

$$\text{COP}_{\text{R}} = \frac{1}{Q_H/Q_L - 1} \qquad \text{COP}_{\text{HP}} = \frac{1}{1 - Q_L/Q_H}$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Summary

- Reversible processes are ideal processes that can be reversed and are the most efficient
- Actual processes are irreversible due to the presence of irreversibilities
- The Carnot cycle is a reversible cycle that possess the best possible efficiency which is dependent only on T_H and T_L
- The thermodynamic temperature scale is derived from the Carnot principles
- For a reversible Carnot cycle: $\left(\frac{Q_L}{Q_H}\right)_{\text{rev}} = \frac{T_L}{T_H}$
- Carnot efficiency for heat engines: $\eta_{th,\text{rev}} = 1 - \frac{T_L}{T_H}$
- Coefficients of performance for reversed Carnot cycle:

$$\text{COP}_{\text{R,rev}} = \frac{1}{T_H/T_L - 1} \qquad \text{COP}_{\text{HP,rev}} = \frac{1}{1 - T_L/T_H}$$