NANYANG TECHNOLOGICAL UNIVERSITY

MA2011 MECHATRONICS SYSTEMS INTERFACING

Tutorial 6

Prof. Cai Yiyu

College of Engineering
School of Mechanical and Aerospace Engineering

A/D CONVERSION

&

D/A CONVERSION

Q1

Specify an appropriate ± 5V n-bit A/D converter (8- or 12-bit), sample rate (up to 100 Hz) and signal conditioning to convert the following analog signal into digital series. Estimate the relative quantization error in quantizing the specified input voltage:

$$E(t) = 1.5 \sin \pi t + 20 \sin 32\pi t - 3 \sin (60\pi t + \pi/4) \text{ V}$$

(Many possible solutions.)

ANSWER TO Q1

$$\textit{The relative quantization error} = \frac{\textit{Quantization error}}{\textit{Gain} * \textit{Amptitude}}$$

ANSWER TO Q1

Question Analysis

KNOWN: A/D converter: $n = 8$ or 12; $V_{max} = 5$ V; $V_{min} = -5$ V

$0 < f_s \leq 100$ Hz

Estimate the maximum Quantifying Error for your choice

ANSWER TO Q1

$$E(t) = 1.5 \sin \pi t + 20 \sin 32\pi t - 3 \sin (60\pi t + \pi/4) \text{ V}$$

This input signal contains amplitudes C_1, C_2 and C_3 with frequencies of $f_1 = 0.5$, $f_2 = 16$ and $f_3 = 30$ Hz, respectively.

ANSWER TO Q1

to be able to reconstruct the signal correctly, sampling theorem must be met:

$$f_s > 2f_{max}$$

The maximum frequency in the signal, f_{max}, is 30 Hz. So we must select $f_s > 60$ Hz.

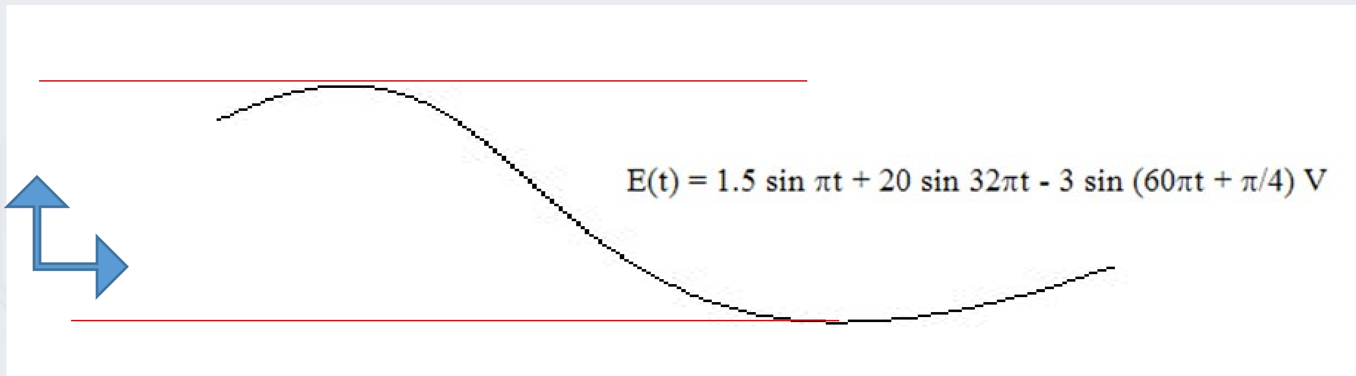
ANSWER TO Q1

The choice of number of bits for the A/D converter depends on the required accuracy as well as the accuracy of the transducer. For 8-bit A/D converter,

$$Q = \frac{10\ \text{V}}{2^8} = 39.1\ \text{mV}$$

For 12-bit A/D converter, $Q = \dfrac{10}{2^{12}} = 2.44\ \text{mV}$

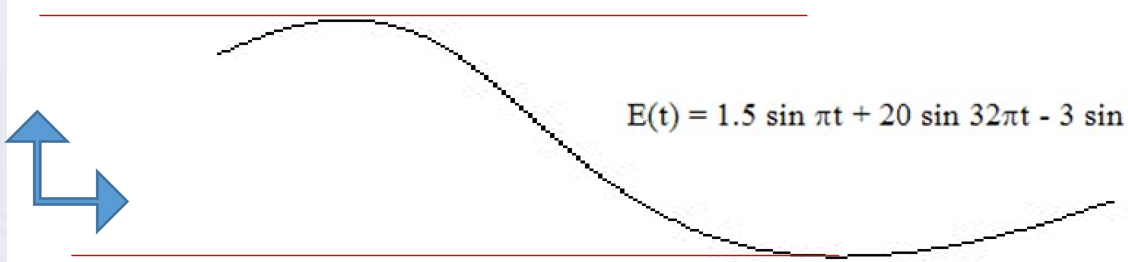
ANSWER TO Q1



Finding the Amplitude Range

Amplitude range of the signal E(t) can only be found from the time domain plot or by differentiating the equation.

A useful solution approach: not perfect but practical

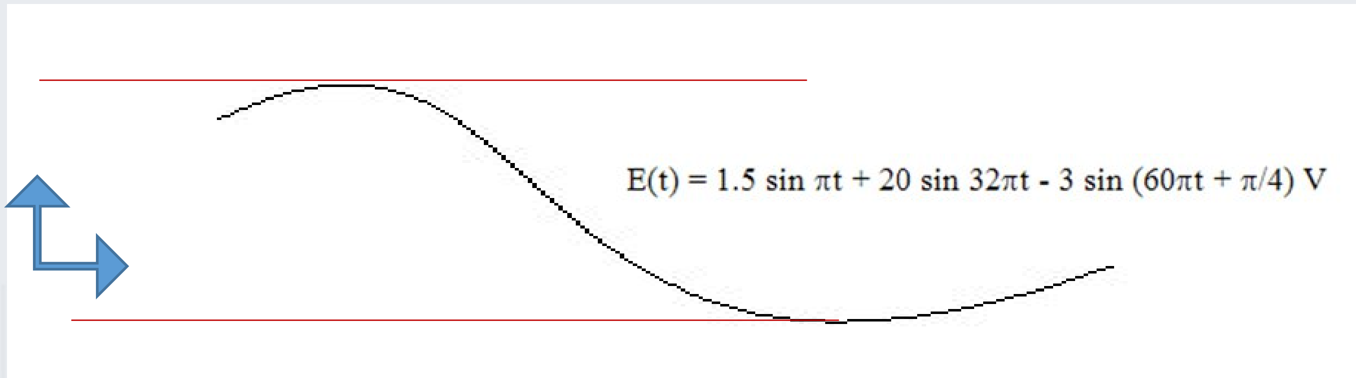


$E(t) = 1.5 \sin \pi t + 20 \sin 32\pi t - 3 \sin (60\pi t + \pi/4)$ V

Estimation of Amplitude Range

However, we know that it will never exceed

$$\pm (1.5 + 20 + 3) = \pm 24.5 \text{ V}.$$



Estimated Gain

$$A = Vout/Vin \approx \pm 24.5V/\pm 5V \approx 0.2$$

ANSWER TO Q1

$$\textit{The relative quantization error} = \frac{\textit{Quantization error}}{\textit{Gain} * \textit{Amptitude}} = \frac{Q}{G * A}$$

Gain G=0.2
Quantization errors: Q(8-bits)=39.1mV or Q(12-bits)=2.44mV
Amplitudes A= 1.5, 20, 3 and $24.5V$

ANSWER TO Q1

For each frequency component and the signal

Component	Frequency	Max. Amplitude	Relative Quantization Error $Q/(G*A)*100\%$	
			8-bits	12-bits
$1.5\sin\pi t$	0.5	1.5	39.1mV/0.2/1.5*100%=13%	2.44mV/0.2/1.5*100%=0.81%
$20\sin 32\pi t$	16	20	39.1mV/0.2/20*100%=9.8%	2.44mV/0.2/20*100%=0.06%
$3\sin(60\pi t+\pi/4)$	30	3	39.1mV/0.2/3*100%=6.5%	2.44mV/0.2/3*100%=0.41%
E(t)		<+24.5	39.1mV/0.2/24.5*100%=0.8%	2.44mV/0.2/24.5*100%=0.05%

As the relative quantization error for both 8-bit and 12-bit A/D converters are less than 1% for E(t), either is suitable for most applications. If the accuracies for the low-level components are important, then the 12-bit A/D converter is required.

Question Analysis

Static pressures are to be measured at eight locations (at 8-cylinder points) under the hood of a NASCAR race car (with a V-8 engine). The pressure transducers to be used have an output span of ±1 V for an input span of ±25 cm H_2O. The signals are measured and recorded on a portable DAS/computer, which uses a 10-bit, ±5V A/D converter. Pressure needs to be resolved to within 0.25 cm H_2O. The dynamic content of the signal is important and has a fundamental period of about 0.5 s. Suggest an appropriate sample rate and signal conditioning (i.e. amplifier gain G and anti-aliasing filter cut off frequency f_c) for this application.

NASCAR: National Association of Stock Car Auto Racing
DAS: Direct-attached storage

(Many possible solutions.)

ANSWER TO Q2

KNOWN: Transducer: ±1 V output; ±25 cm H_2O input

DAS (ADC): $n = 10$; $V_{max} = 5$ V; $V_{min} = -5$ V

FIND: f_s, f_c, G.

The Static Sensitivity

$$K = \Delta V_{out}/\Delta V_{in}$$

ΔV_{in}: The change in input
ΔV_{out}: The change in output

Sensitivity may have different units depending on the instrument being considered.

The transducer sensitivity

$$K = \Delta V_{out}/\Delta V_{in} = (1-(-1))\text{V}/(25-(-25))\ \text{cmH}_2\text{O}$$

$$= 2\text{V}/50\text{cmH}_2\text{O}$$

$$= 0.04\text{V}/\text{cmH}_2\text{O}$$

ANSWER TO Q2

The A/D resolution is:

$$\text{Qres} = \frac{10V}{2^{10}} = 0.00976V$$

Pressure needs within 0.25 cm H_2O?

$$K = \Delta V_{out}/\Delta V_{in} = \text{Qres}/\Delta V_{in}$$

$$\Delta V_{in} = \frac{\text{Qres}}{K} = \frac{0.00976V}{0.04V/cmH2O} = 0.244cmH_2O < 0.25cmH_2O$$

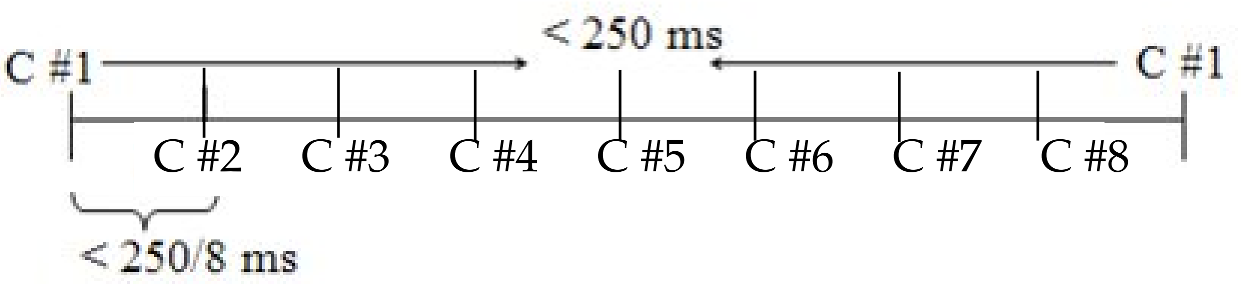
The sensitivity meets problem constraints. However, an analog amplifier between the transducer and A/D converter with a gain of G = 5 will take full advantage of the A/D range and improve resolution:

$$\Delta V_{in} = \frac{0.00976V}{5*0.04V/cmH2O} = 0.0488cmH_2O < 0.25cmH_2O$$

ANSWER TO Q2

For one pressure sensor, the fundamental period of the signal T=0.5 s (500 ms), or f_0=2 Hz.

According to Shannon-Nyquist theorem, the minimum sampling frequency $f_s > 2^* f_0 = 4$ Hz, or maximum sampling period 250 ms.



For 8 pressure sensors (8 cylinders for the V-8 engine), the DAS has to sample at least up to 8 times faster than the maximum sampling period for each sensor 250/8 ms or a minimum frequency (cutoff frequency) of 4*8=32 Hz.

Q3

The range of a signal is between ±5 V and it is required to make measurements with a quantization size no more than 5 mV. What is the minimum resolution of a ADC needed?

ANSWER TO Q3

Since

$$\frac{5-(-5)}{2^N} = Q \le 0.005$$

$$\Rightarrow \quad 2^N \ge \frac{10}{0.005} = 2000$$

If $N = 11$, $2^N = 2048$ and if $N = 12$, $2^N = 4096$. Although a 11-bit ADC will suffice, it is not commercially available.

Hence a <u>12-bit ADC</u> will be needed.