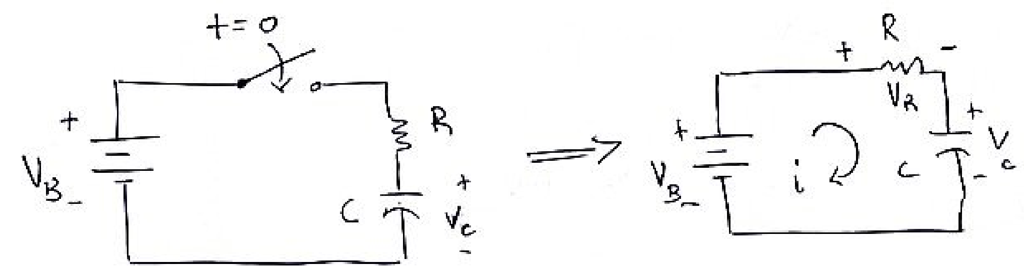
MA 2011 - Tutorial 1

1. (i)



KVL (at $t \geq 0$)

$$V_B = V_R + V_c \quad — (1)$$

$$V_R = i \cdot R \quad — (2)$$

$$i = C\frac{d}{dt}V_c \quad — (3)$$

Sub. (3) in (2),

$$V_R = C\frac{d}{dt}V_c \cdot R \quad — (4)$$

Sub. (4) in (1),

$$V_B = C\frac{d}{dt}V_c \cdot R + V_c$$

$$\frac{d}{dt}V_c = -\frac{1}{RC}V_c + \frac{V_B}{RC} \quad — (5) \qquad \frac{V}{t} = \frac{V}{sec}$$

Initial condition,

$$V_c(t=0) = 0V$$

Final condition,

$$V_c(t=\infty) = V_\infty = ?$$

$$\cancel{\frac{d}{dt} V_\infty} = -\frac{1}{RC} V_\infty + \frac{V_B}{RC}$$

$$-\frac{1}{RC} \cdot V_\infty + \frac{V_B}{RC} \quad — (6)$$

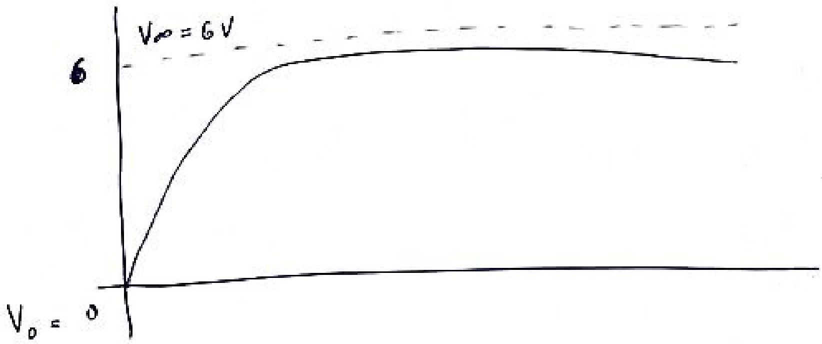
$$V_c(t=\infty) \Rightarrow V_\infty = V_B = 6V$$

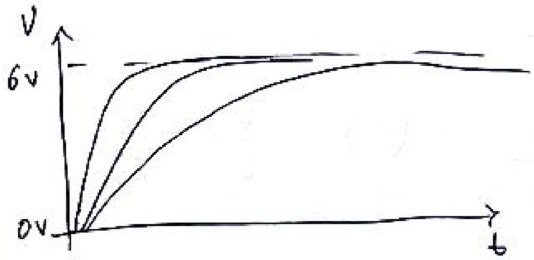
(ii) $\tau = RC = 1k\Omega \cdot 1mF = 1s$

(iii)+(iv) $V_c(t)$ for $t > 0$

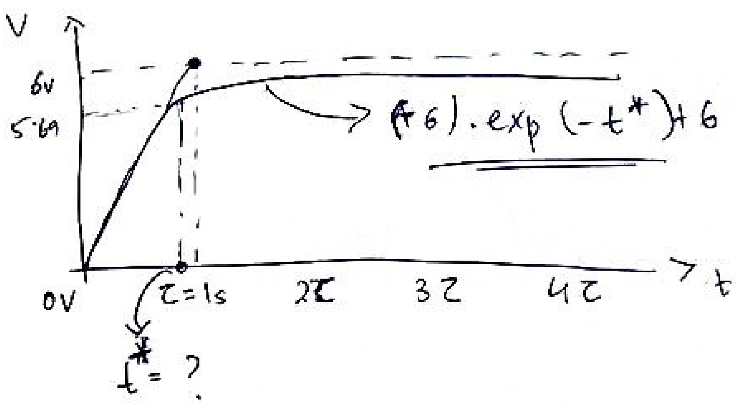
$$\boxed{V_c(t) = (V_0 - V_\infty) \cdot e^{-\frac{t}{\tau}} + V_\infty}$$

$V_0 = 0V$
$V_\infty = 6V$
$\tau = 1s.$





loot at $\tau = 1s$.



(v) $E = \frac{1}{2} C V^2$

(vi) $E_{max} = \frac{1}{2} C V_\infty^2 = \frac{1}{2} \cdot 1mF \cdot (6V)^2 = 18 mJ$

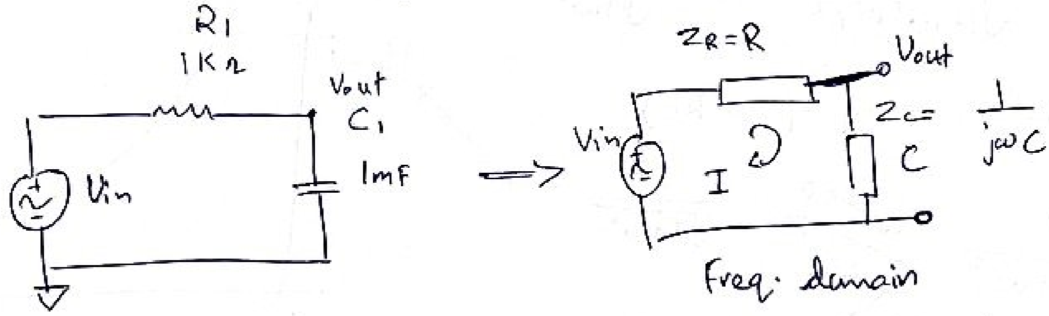
$E_{90\%} = 90\% \cdot E_{max} = \frac{1}{2} C (V_{90\%})^2$

$V_{90\%} = \sqrt{\frac{2 \cdot 16.2}{1mF}} = 5.69V$

$V_{90\%} = 5.69 = -6 \cdot \exp(-t^*) + 6$

$t^* = -\ln\left(\frac{6 - V_{90\%}}{6}\right) = \underline{\underline{2.96s}}$

2.(i)



$$I = \frac{V_{in}}{Z_R + Z_c} = \frac{V_{in}}{R + \frac{1}{j\omega C}} \quad -(1)$$

$$V_{out} = I \cdot Z_c = I \cdot \frac{1}{j\omega C} \quad -(2)$$

Sub. (1) in (2),

$$V_{out} = \frac{V_{in}}{R + \frac{1}{j\omega C}} \cdot \frac{1}{j\omega C} = \frac{V_{in}}{1 + j\omega RC} \quad -(3) \rightarrow \tau = RC = 1s$$

(ii) $H = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega \underbrace{RC}_{1sec}} = \frac{1}{1 + j\omega}$

(iii) $|H| \rightarrow$ Amplitude

$\measuredangle H \rightarrow$ Phase

$$z = a + jb$$

$$|z| = \sqrt{a^2+b^2} \rightarrow \text{Amp.}$$

$$\frac{1}{z} = \frac{1}{\sqrt{a^2+b^2}}$$

$$\measuredangle z = tg^{-1}\left(\frac{b}{a}\right) \rightarrow \text{Phase}$$

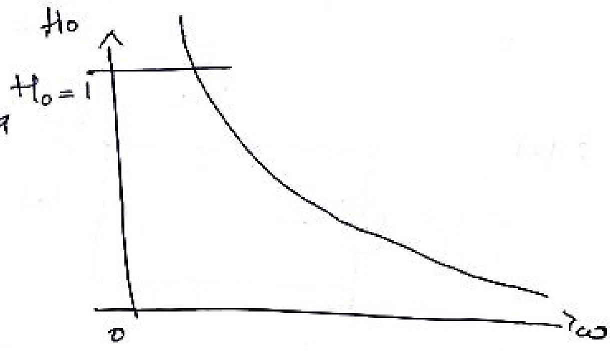
$$\measuredangle \frac{1}{z} = -tg^{-1}\left(\frac{b}{a}\right)$$

$$|H| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}} = \frac{1}{\sqrt{1+\omega^2}}$$

$$\measuredangle H = -tg^{-1}(\omega RC)$$

$$\frac{1}{\sqrt{1+\omega^2}} \xrightarrow{\omega \to 0} \frac{1}{\sqrt{1+0^2}} = 1 \rightarrow H_0 = 1$$

$$\frac{1}{\sqrt{1+\omega^2}} \xrightarrow{\omega \to \infty} \frac{1}{\sqrt{\cancel{1}+\omega^2}} \approx \frac{1}{\omega}$$



$$|H(j\omega_0)| = \frac{H_0}{\sqrt{2}}$$

$$\frac{1}{\sqrt{1+\omega_0^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

$$\sqrt{1+\omega_0^2 R^2 C^2} = \sqrt{2}$$

$$1 + \omega_0^2 R^2 C^2 = 2$$

$$\omega_0^2 R^2 C^2 = 1$$

$$\omega_0 = \frac{1}{RC} = 1 \text{ rad/s}$$

(iv) $\omega_0 = 2\pi f_0$

$$f_0 = \frac{1}{2\pi} = 0.159 \text{ Hz}$$