NANYANG TECHNOLOGICAL UNIVERSITY

MA2011 MECHATRONICS SYSTEMS INTERFACING

Tutorial 5

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Recap

1. It has infinite impedance at both inputs, so no current drawn from the input circuit: $I_+ = I_- = 0$.

2. It has infinite gain, so difference between input voltages is zero: $V_+ = V_-$.

3. It has zero output impedance, so output voltage does not depend on output current.

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Recap

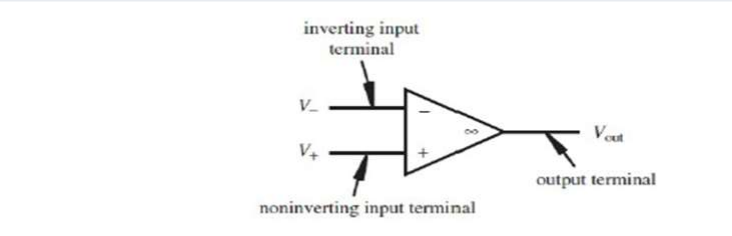
INFINITE or ZERO

The Open-Loop gain A is VERY VERY LARGE = can be considered as INFINITE

The input impedances of the two terminals are VERY VERY LARGE = can be considered as INFINITE

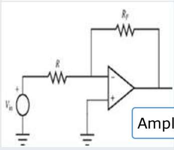
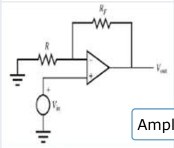
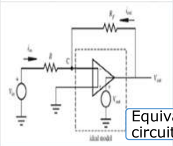
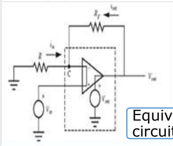
The output impedance is VERY VERY SMALL = can be considered as ZERO

Schematic and Nomemchature



A differential input: The inverting input (-) vs The non-inverting input (+)

COMPARISON OF INVERTING AND NONINVERTING AMPLIFIER

Inverting	Non-inverting
 Amplifier	 Amplifier
 Equivalent circuit	 Equivalent circuit
$V_{out} = -\frac{R_F}{R} V_{in}$ Equation & negative gain	$V_{out} = (1 + \frac{R_F}{R}) V_{in}$ Equation & positive gain

Q1

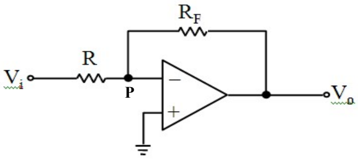
The inverting amplifier shown below uses an op-amp with infinite input resistance, zero output resistance, and finite loop gain A. Show that the closed loop gain G is given by

$$G = \frac{-AR_F}{AR + R + R_F}$$

And for large value of A, G can be approximated by

$$G' = -\frac{R_F}{R}$$

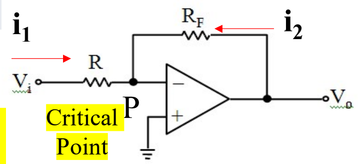
Inverting amplifier



For the above amplifier, compare the results of the two equations for A = 10, 100, 10^4, and $\boxed{10^6}$ with $R/(R + R_F)$ varying from 0 to 1.

ANSWER TO Q1

SOLUTION:



$$i_1 = \frac{(V_P - Vi)}{R}, \; i_o = \frac{(VP - Vo)}{R_F}$$

For an op-amp with infinite input resistance and zero output resistance, $i_o = - i_1$

So, $\frac{(V_P - Vi)}{R} = -\frac{(VP - Vo)}{R_F}$

And $R_F\,(V_P - Vi) = -R(VP - Vo)$

$$V_P\,(R_F + R) = VoR + V_iR_F$$

ANSWER TO Q1

Finite loop gain $A = - V_o/V_P$

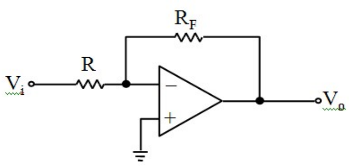
But $V_P = - V_o/ A$ $\qquad V_P(R_F + R) = V_oR + V_iR_F$

So $\frac{-V_o(R+R_F)}{A} = V_o R + V_i R_F$

$-V_o\,(R + R_F + AR) = V_i\,A\,R_F$

Op-amp gain
$G = V_o/V_i$

By definition $G = \frac{V_o}{V_i}$



Giving $\quad G = \frac{-AR_F}{AR+R+R_F}$

ANSWER TO Q1

For large open-loop gain A, AR >> (R + R_F), and so the equation becomes

$$G' = \frac{-AR_F}{AR} = -\frac{R_F}{R}$$

$$\frac{G'}{G} = \frac{R_F}{R} \times \frac{AR + R + R_F}{AR_F} = 1 + \frac{R + R_F}{AR}$$

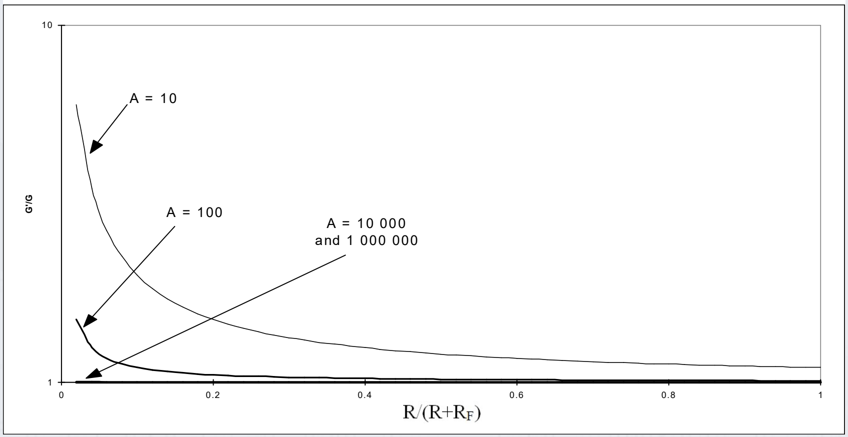
A: Finite loop gain $A = -V_o/V_P$

G: Closed loop (op-amp) gain $G = \frac{-AR_F}{AR + R + RF}$

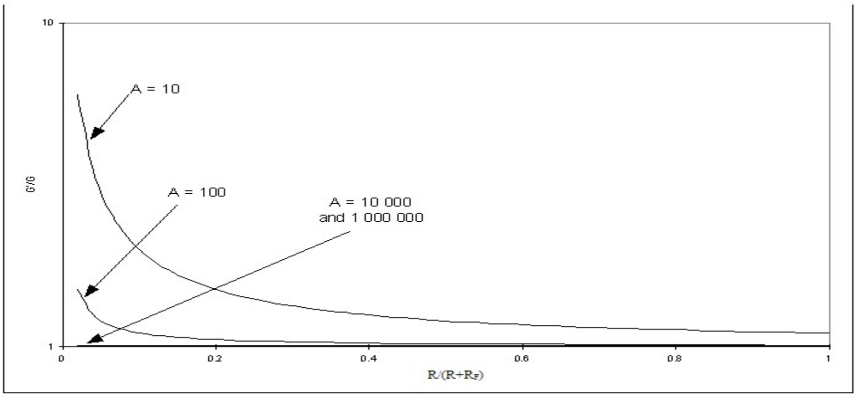
G': Closed-loop inverting amp gain $G' = \frac{-R_F}{R}$

ANSWER TO Q1

Plots of G'/G versus R/(R+R_F) for different values of A are shown below. For A which are very large in values, G'/G approaches 1 or G' → G.



ANSWER TO Q1

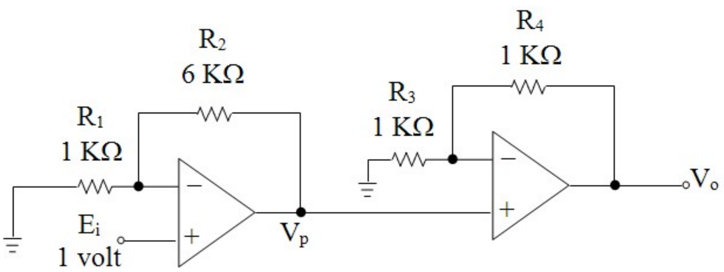


COMMENT: A practical op-amp has very high input impedance ($Z_i > 10^7\ \Omega$), low output impedance ($Z_o < 100\ \Omega$), and high open-loop gain (A about 10^5 to 10^6), and the values of R and R_F are typically in the kΩ to MΩ range. So, it is valid to use the second equation.

Q2

Find V_o for the following op-amp circuits:

a) Two non-inverting amplifiers in series.



ANSWER TO Q2

SOLUTION:

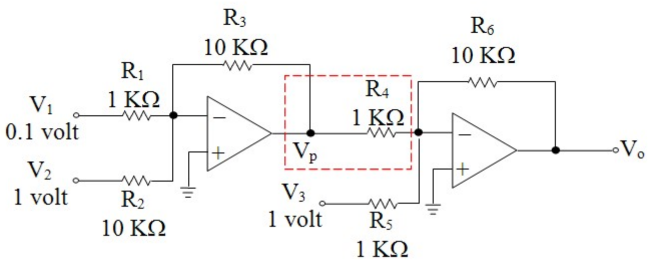
$$G = \frac{R + R_F}{R} = 1 + \frac{R_F}{R} \text{ (General equation)}$$

$$V_p = \left(1 + \frac{R_2}{R_1}\right)V_i = \left(1 + \frac{6}{1}\right) \times 1 = 7\ V$$

$$V_o = \left(1 + \frac{R_4}{R_3}\right)V_p = \left(1 + \frac{1}{1}\right) \times 7 = 14\ V$$

ANSWER TO Q2

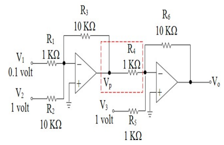
b) Two summing amplifiers in series.



ANSWER TO Q2

SOLUTION:

General equation: $V_o = -\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots\right) R_F$



$$V_p = -\frac{R_3 V_1}{R_1} - \frac{R_3 V_2}{R_2} = -10 \times 0.1 - 1 \times 1$$

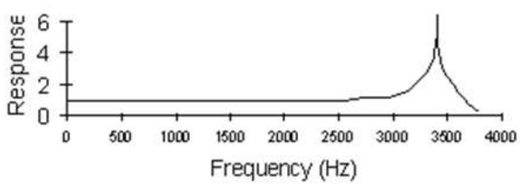
$$= -2\ \mathrm{V}$$

$$V_o = -\frac{R_6 V_p}{R_4} - \frac{R_6 V_3}{R_5} = -\{10 \times (-2)\} - (10 \times 1)$$

$$= 10\ \mathrm{V}$$

Q3

3. An accelerometer that is used to measure the vibration of a machine has a frequency response as shown in the figure below:



The vibration signal is filtered to ensure that the vibration signal beyond 2.5 kHz does not affect the recording of the measurements.

ANSWER TO Q3

Select an appropriate filter consisting of a resistor, with resistance R, and a capacitor, with capacitance C, that will remove the error due to the frequency response of the accelerometer. Suggest appropriate values of R and C. Sketch the frequency response of the filter, indicating all relevant parameters on the sketch.

KNOWN: To filter off signal of 2.5 kHz or more.

FIND: (i) Filter type, values of R and C, and frequency response of filter

ANSWER TO Q3

SOLUTION:

As frequencies below 2.5 kHz are required and those above have to be removed, a low pass filter is required.

Cut-off frequency must be at least 2.5 kHz (required) and less than about 3 kHz, (where the accelerometer response becomes > 1).

ANSWER TO Q3

Impedance-based relationship between frequency, resistance and capacity.

If the cut-off frequency is 2.5 kHz, $\boxed{f_c = \frac{1}{2\pi RC}}$

Or $RC = \frac{1}{2\pi f_c}$

So, $RC = \frac{1}{2\pi \times 2500} = 6.37 \times 10^{-5}$ s

ANSWER TO Q3

If the cut-off frequency is 2.5 kHz, $f_c = \frac{1}{2\pi RC}$

Or $RC = \frac{1}{2\pi f_c}$

So, $RC = \frac{1}{2\pi \times 2500} = 6.37 \times 10^{-5}$ s

Choose any value of R and C such that the combination results in 6.37×10^{-5} s.

For example, R = 63.7 kΩ, C = 1 000 pF (1 pF = 10^{-12} F).

ANSWER TO Q3

In many cases, $\frac{1}{\text{cutoff frequency}}$(time second) can be a rough estimate of the time constant

The frequency response of the filter, with the relevant parameters is:

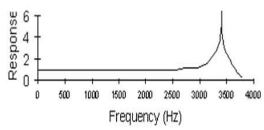
$$M(f) \approx \frac{1}{\sqrt{1+(f/fc)^2}} = \frac{1}{\sqrt{1+(f/2500)^2}}$$

$$M(3400) \approx \frac{1}{\sqrt{1+(3400/2500)^2}} = 0.592$$

For f=fc=2500, M(2500)= $\frac{1}{\sqrt{1+(2500/2500)^2}} = 1/\sqrt{2} =$ **0.707**

ANSWER TO Q3

3. An accelerometer that is used to measure the vibration of a machine has a frequency response as shown in the figure below:



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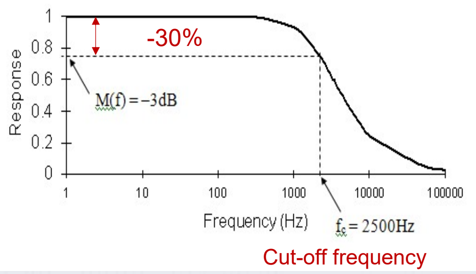

Bandwidth and frequency response

Frequency response curve or Bode plot plots A_{out}/A_{in} vs input frequency.