NANYANG TECHNOLOGICAL UNIVERSITY

MA2011 MECHATRONICS SYSTEMS INTERFACING

Tutorial 2

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RECAP OF L2

1. Fourier Theory is very important
2. The calculation of Fourier representation is not easy
3. There are several techniques to simplify the calculation
   - Uniqueness
   - Symmetry
   - Step-by-Step

(21 March 1768 – 16 May 1830)

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Q2 OF T1

What is the Fourier series and fundamental frequencies (in Hertz) and amplitudes of the following waveforms?

a) $f(t)$ = 5*sin($2\pi t$).
b) $f(t)$ = 5*cos($2\pi t$)
c) $f(t)$ = -5*sin($2\pi t$)

UNIQUE REPRESENTATION

$$F(t) = C_0 + \sum^{\infty} A_n \cos(n\omega_0 t) + \sum^{\infty} B_n \sin(n\omega_0 t)$$

UNIQUE REPRESENTATION

$\omega_0 = 2\pi$

$$5 * \sin(2\pi t) = \mathbf{0} + \sum_{n=1}^{\infty} \mathbf{0} * \boldsymbol{cos}(\mathbf{2}\boldsymbol{n}\pi t) \; + 5 * \sin(2\pi t) + \sum_{n=2}^{\infty} \mathbf{0} * \boldsymbol{sin}(\mathbf{2}\boldsymbol{n}\pi t)$$

$$F(t) = C_0 + \sum^{\infty} A_n \cos(n\omega_0 t) + \sum^{\infty} B_n \sin(n\omega_0 t)$$

UNIQUE FOURIER REPRESENTATION

For a periodic waveform with a given fundamental frequency, there will be only one Fourier series representation

$$F(t) = \boldsymbol{C}_0 + \sum_{n=1}^{\infty} (\boldsymbol{A}_n \cos(nw_0 \boldsymbol{t}) + \boldsymbol{B}_n \sin(nw_0 \boldsymbol{t}))$$

$$= \boldsymbol{C'}_0 + \textstyle\sum_{n=1}^{\infty} (\boldsymbol{A'}_n \cos(nw_0 \boldsymbol{t}) + \boldsymbol{B'}_n \sin(nw_0 \boldsymbol{t}))$$

Unique representation means

$For\, all\ n,\ \boldsymbol{C'}_0 = \boldsymbol{C}_0,\ \boldsymbol{A'}_n = \boldsymbol{A}_n,\ \boldsymbol{B'}_n = \boldsymbol{B}_n$

FOURIER SERIES REPRESENTATION OF SIGNALS

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t) \qquad (2)$$

where C_0 is the DC component of the signal, A_n and B_n are coefficients. They are given by

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt,\ B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$$
$$C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{A_0}{2}. \qquad (3)$$

Note: C_0 is the average value of the waveform over its period.

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UNIQUE REPRESENTATION

Additional Question: Uniqueness?

$$5*\sin(2\pi t) = \mathbf{0} + 5*\sin(2\pi t) + \sum_{n=2}^{\infty} 0 * sin(2n\pi t)$$

$$5*\sin(2\pi t) = \mathbf{0} + 0*\sin(\pi t)+5*\sin(2\pi t) + \sum_{n=3}^{\infty} 0 * sin(n\pi t)$$

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UNIQUE FOURIER SERIES REPRESENTATION OF PERIODIC WAVEFORMS

$$F(t) = C_0 + \sum^{\infty} A_n \cos(n\omega_0 t) + \sum^{\infty} B_n \sin(n\omega_0 t)$$

FOURIER SERIES REPRESENTATION

FOR

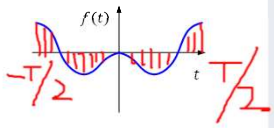
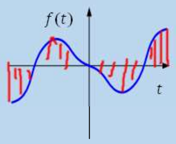
NON-PERIODIC WAVEFORMS?

SYMMETRICAL PERIODIC WAVEFORMS?

SYMMETRIC PERIODIC FUNCTION

Properties of symmetric functions

f(t) has period of T

f(t)	$\int_{-T/2}^{T/2} f(t)dt$	
even f(-t)=f(t)	$\int_{-T/2}^{T/2} f(t)dt = 2\int_{0}^{T/2} f(t)dt$	
odd f(-t)=-f(t)	$\int_{-T/2}^{T/2} f(t)dt = 0$	

Q1

***f(t)* and *g(t)* are two symmetric and periodic functions with period *T*. Answer the followings**

Quiz 2
At
NTULearn

	f(t)	*g(t)*	*-f(t)*	*f(t)+g(t)*	*f(t) - g(t)*	*f(t)* g(t)*	*f(t)/ g(t)*	$\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$	$\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)d$
Function symmetry	even	even							
	odd	odd							
	even	odd							
	odd	even							

Q1

***f(t)* and *g(t)* are two symmetric and periodic functions with period *T*. Answer the followings**

even	**$f_n(-t)=f_n(t)$**
odd	**$f_n(-t)=-f_n(t)$**

1) $f(t)$ even,
$h(t)\equiv -f(t)$: $h(-t)=-f(-t)= -f(t)=h(t)$ ➔ $-f(t)$ even

2) $f(t)$ & $g(t)$ even :
$h(t)\equiv f(t)+g(t)$: $h(-t)\equiv f(-t)+g(-t)=f(t)+g(t)= h(t)$ ➔ $f(t)+g(t)$ even
$h(t)\equiv f(t)-g(t)$: $h(-t)\equiv f(-t)-g(-t)=f(t)-g(t)= h(t)$ ➔ $f(t)-g(t)$ even

3) $f(t)$ & $g(t)$ even :
$h(t)\equiv f(t)*g(t)$: $h(-t)\equiv f(-t)*g(-t)= f(t)*g(t)=h(t)$ ➔ $f(t)*g(t)$ even
$h(t)\equiv f(t)/g(t)$: $h(-t)\equiv f(-t)/g(-t)= f(t)/g(t)=h(t)$ ➔ $f(t)/g(t)$ even

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Q1

***f(t)* and *g(t)* are two symmetric and periodic functions with period *T*. Answer the followings**

even	**$f_n(-t)=f_n(t)$**
odd	**$f_n(-t)=-f_n(t)$**

$f(t)$ even: $f(-t)= f(t)$
$g(t)\equiv \cos\left(\frac{2\pi nt}{T}\right)$ even: $\cos\left(\frac{2\pi n(-t)}{T}\right)=\cos\left(\frac{2\pi nt}{T}\right)$
$h(t)\equiv f(t)*g(t)$: ➔ even

So $\int_{-T/2}^{T/2} h(t)dt = 2 * \int_{0}^{T/2} h(t)dt$

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Q1

f(t) **and** ***g(t)*** **are two symmetric and periodic functions with period** ***T*****.**
Answer the followings

even $f_n(-t)=f_n(t)$
odd $f_n(-t)=-f_n(t)$

$f(t)$ even: $f(-t)= f(t)$

$g(t)\equiv \boldsymbol{sin}\left(\frac{2\pi nt}{T}\right)$ odd: $\boldsymbol{sin}\left(\frac{2\pi n(-t)}{T}\right) = -\boldsymbol{sin}\left(\frac{2\pi nt}{T}\right)$

$h(t)\equiv f(t)*g(t)$: ➔ odd

so $\int_{-T/2}^{T/2} \boldsymbol{h}(\boldsymbol{t})dt = 0$

Q1

f(t) **and** ***g(t)*** **are two symmetric and periodic functions with period** ***T*****.**
Answer the followings

	f(t)	g(t)	-f(t)	f(t)+ g(t)	f(t) - g(t)	f(t)* g(t)	f(t)/ g(t)	$\int_{-T/2}^{T/2} f(t) sin\left(\frac{2\pi nt}{T}\right)dt$	$\int_{-T/2}^{T/2} f(t) cos\left(\frac{2\pi nt}{T}\right)dt$
Function symmetry	even	even							
	odd	odd							
	even	odd							
	odd	even							

Q1

***f(t)* and *g(t)* are two symmetric and periodic functions with period *T*. Answer the followings**

?	$f(t)$	$g(t)$	$-f(t)$	$f(t)+g(t)$	$f(t)-g(t)$	$f(t)*g(t)$	$f(t)/g(t)$	$\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$	$\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$
Function symmetry	even	even	even	even	even	even	even	0	$2\int_0^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$
	odd	odd	odd	odd	odd	even	even	$2\int_0^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$	0
	even	odd	even	?	?	odd	odd	0	$2\int_0^{T/2} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$
	odd	even	odd	?	?	odd	odd	$2\int_0^{T/2} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$	0

Q1

even
$f_n(-t)=f_n(t)$

odd
$f_n(-t)=-f_n(t)$

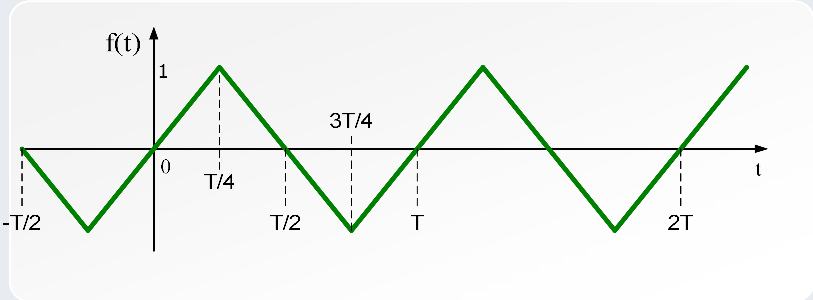
$f(-t)= f(t)$ even
$g(-t)= - g(t)$ odd
$h(t)\equiv f(t)+g(t)$: *even or odd?*

Proof by contradiction!
Assume even result
$h(-t)= h(t)$;
$f(-t)+g(-t)= (f(t)+g(t))$;
$f(t)-g(t)= f(t)+g(t)$
$2*g(t)=0 \rightarrow$ *only if* $g(t)=0$

Assume odd result
$h(-t)= -h(t)$;
$f(-t)+g(-t)= -(f(t)+g(t))$;
$f(t)-g(t)= -f(t)-g(t)$
$2*f(t)=0 \rightarrow$ *only if* $f(t)=0$

Q2

f(t) is a function defined as follows.



(a) Is it a periodic function? If yes, what is the period, and write the function of the waveform defined at [0, T]
(b) Is it a symmetric function?
(c) Can the waveform be represented by Fourier Series? If yes, what are the DC C_0, A_n and B_n of the waveform and the Fourier Series?
(d) What are the peak amplitude, and peak-to-peak amplitude?
(e) If the peak amplitude is changed to A, what will be the function of f(t) and its Fourier Series Representation?

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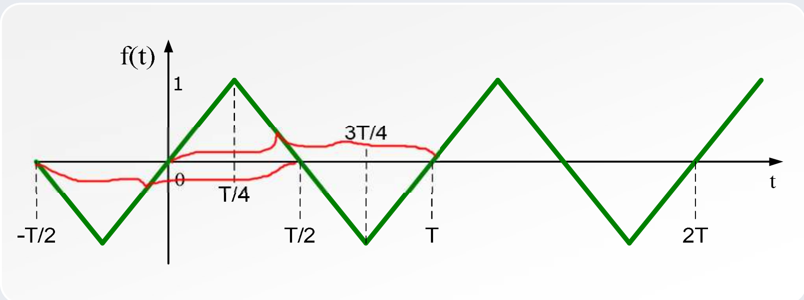
ANSWER TO Q2

(a) Is it a periodic function? If yes, what is the period, and write the function of the waveform defined at [0, T]



Answer:
(a) It is a periodic function with Period=T: $f(t+T)=f(t)$

And the function can be defined as

$$f(t)=\begin{cases}\frac{4}{T}t, & 0\le t\le\frac{T}{4}\\ \frac{4}{T}(-t+\frac{T}{2}), & \frac{T}{4}\le t\le\frac{3T}{4}\\ \frac{4}{T}(t-T), & \frac{3T}{4}\le t\le T\end{cases}$$

$$or\ f(t)=\begin{cases}\frac{4}{T}\left(-t-\frac{T}{2}\right), & -\frac{T}{2}\le t\le-\frac{T}{4}\\ \frac{4}{T}t, & -\frac{T}{4}\le t\le\frac{T}{4}\\ \frac{4}{T}\left(-t+\frac{T}{2}\right), & \frac{T}{4}\le t\le\frac{T}{2}\end{cases}$$

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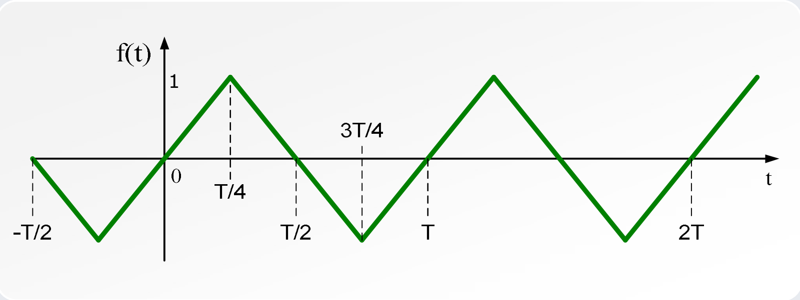
ANSWER TO Q2

(b) Is it a symmetric function?



Answer:

(b) This is an odd periodic function: $f(-t) = -f(t)$

As a periodic odd function, it can be represented by Fourier Series.

For the odd periodic function, the DC C_0=0 and $A_n = 0$

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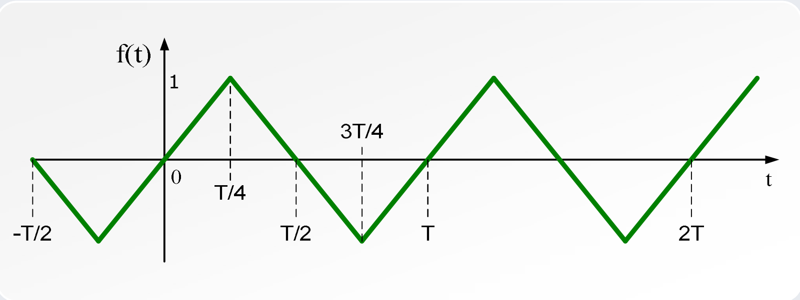
ANSWER TO Q2

(c) Can the waveform be represented by Fourier Series? If yes, what are the DC C_0, A_n and B_n of the waveform and the Fourier Series?



Answer:

DC C_0=0 and A_n =0

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)sin\left(\frac{2\pi nt}{T}\right)dt$$

Since f(t) and $sin\left(\frac{2\pi nt}{T}\right)$ are two odd function, we have f(t)*$sin\left(\frac{2\pi nt}{T}\right)$ is an even function

$$B_n = \frac{4}{T}\int_{0}^{T/2} f(t)sin\left(\frac{2\pi nt}{T}\right)dt$$

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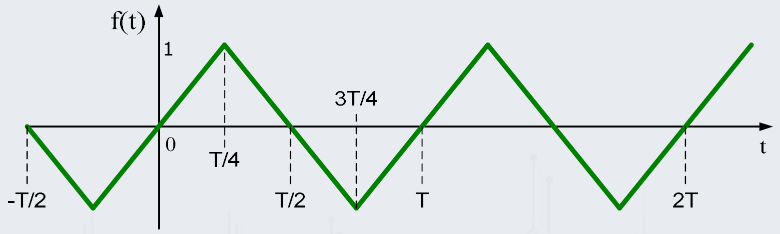
ANSWER TO Q2



$$f(t) = \begin{cases} \frac{4}{T}\left(-t - \frac{T}{2}\right), & -\frac{T}{2} \le t \le -\frac{T}{4} \\ \frac{4}{T}t, & -\frac{T}{4} \le t \le \frac{T}{4} \\ \frac{4}{T}\left(-t + \frac{T}{2}\right), & \frac{T}{4} \le t \le \frac{T}{2} \end{cases}$$

$$B_n == \frac{4}{T}\int_0^{T/2} f(t) \sin\left(\frac{2\pi nt}{T}\right) dt$$

$$= \frac{4}{T}\int_0^{T/4} \frac{4t}{T} \sin\left(\frac{2\pi nt}{T}\right) dt + \frac{4}{T}\int_{T/4}^{T/2} \frac{4}{T}\left(\frac{T}{2} - t\right) \sin\left(\frac{2\pi nt}{T}\right) dt$$

ANSWER TO Q2

$$B_n = \underbrace{\frac{4}{T}\int_0^{T/4} \frac{4t}{T} \sin\left(\frac{2\pi nt}{T}\right) dt}_{X_n} + \underbrace{\frac{4}{T}\int_{T/4}^{T/2} \frac{4}{T}\left(\frac{T}{2} - t\right) \sin\left(\frac{2\pi nt}{T}\right) dt}_{Y_n}$$

$$X_n = \frac{4}{T}\int_0^{T/4} \frac{4t}{T} \sin\left(\frac{2\pi nt}{T}\right) dt = \left(\frac{4}{T}\right)^2 \int_0^{T/4} t \sin\left(\frac{2\pi nt}{T}\right) dt$$

$$= -\left(\frac{4}{T}\right)^2 \frac{T}{2\pi n} \int_0^{T/4} t \, d\cos\left(\frac{2\pi nt}{T}\right)$$

$$= -\frac{8}{\pi nT}\left(t * \cos\left(\frac{2\pi nt}{T}\right)\right)\Bigg|_0^{T/4} + \frac{8}{\pi nT}\int_0^{T/4} \cos\left(\frac{2\pi nt}{T}\right) dt$$

Because **Integration by Parts**: $\int_a^b f(t)dg(t) = f(t)g(t)\Big|_a^b - \int_a^b g(t)df(t)$

ANSWER TO Q2

Because of **Integration by Parts**: $\int_a^b f(t)dg(t) = f(t)g(t)\Big|_a^b - \int_a^b g(t)df(t)$

$$X_n = -\frac{8}{\pi nT}\left(t * \cos\left(\frac{2\pi nt}{T}\right)\right)\Bigg|_0^{T/4} + \frac{8}{\pi nT}\int_0^{T/4}\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$= -\frac{8}{\pi nT}\left(T/4 * \cos\left(\frac{2\pi nT/4}{T}\right)\right) + \frac{8}{\pi nT} * \frac{T}{2\pi n}\int_0^{T/4} d\sin\left(\frac{2\pi nt}{T}\right)$$

$$= -\frac{2}{\pi n}\left(\cos\left(\frac{\pi n}{2}\right)\right) + \frac{4}{(\pi n)^2} * \left(\sin\left(\frac{2\pi nt}{T}\right)\right)\Bigg|_0^{T/4}$$

$$= -\frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right)$$

ANSWER TO Q2

Because of **Integration by Parts**: $\int_a^b f(t)dg(t) = f(t)g(t)\Big|_a^b - \int_a^b g(t)df(t)$

$$Y_n = \frac{4}{T}\int_{T/4}^{T/2}\frac{4}{T}\left(\frac{T}{2}-t\right)\sin\left(\frac{2\pi nt}{T}\right)dt = \left(\frac{4}{T}\right)^2\int_{T/4}^{T/2}\left(\frac{T}{2}-t\right)\sin\left(\frac{2\pi nt}{T}\right)dt$$

$$= -\left(\frac{4}{T}\right)^2\frac{T}{2\pi n}\int_{T/4}^{T/2}\left(\frac{T}{2}-t\right)d\cos\left(\frac{2\pi nt}{T}\right)$$

$$= -\frac{8}{\pi nT}\left(\left(\frac{T}{2}-t\right) * \cos\left(\frac{2\pi nt}{T}\right)\right)\Bigg|_{T/4}^{T/2} + \frac{8}{\pi nT}\int_{T/4}^{T/2}\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$= -\frac{8}{\pi nT}\left(0 - T/4 * \cos\left(\frac{2\pi nT/4}{T}\right)\right) - \frac{8}{\pi nT} * \frac{T}{2\pi n}\int_{T/4}^{T/2} d\sin\left(\frac{2\pi nt}{T}\right)$$

$$= \frac{2}{\pi n}\left(\cos\left(\frac{\pi n}{2}\right)\right) - \frac{4}{(\pi n)^2} * \left(\sin\left(\frac{2\pi nt}{T}\right)\right)\Bigg|_{T/4}^{T/2}$$

$$z = \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) - \frac{4}{(\pi n)^2} * \left(0 - \sin\left(\frac{2\pi nT/4}{T}\right)\right) = \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right)$$

ANSWER TO Q2

$$B_n = X_n + Y_n$$
$$= -\frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right) + \frac{2}{\pi n}\cos\left(\frac{\pi n}{2}\right) + \frac{4}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right)$$
$$= \frac{8}{(\pi n)^2} * \sin\left(\frac{\pi n}{2}\right)$$

$$\sin\left(\frac{\pi n}{2}\right) = \begin{cases} 0, & n = 2k \\ 1, & n = 4k+1 \\ -1, & n = 4k-1 \end{cases}$$

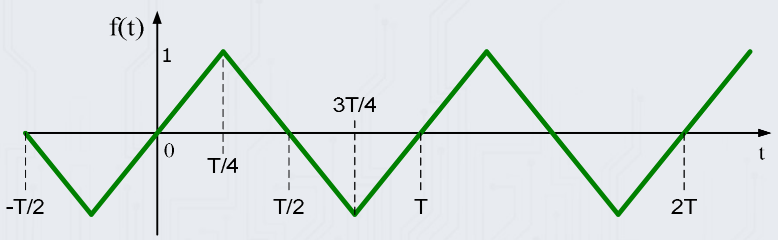
Therefore, the Fourier series is

$$F(t) = \frac{8}{\pi^2}\left[\sin\left(\frac{2\pi}{T}t\right) - \frac{1}{3^2}\sin\left(\frac{6\pi}{T}t\right) + \frac{1}{5^2}\sin\left(\frac{10\pi}{T}t\right) - \cdots\right]$$

ANSWER TO Q2

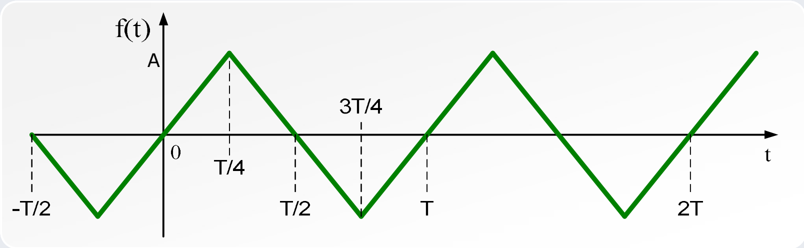
(d) The peak amplitude=1

and peak-to-peak amplitude =2



ANSWER TO Q2

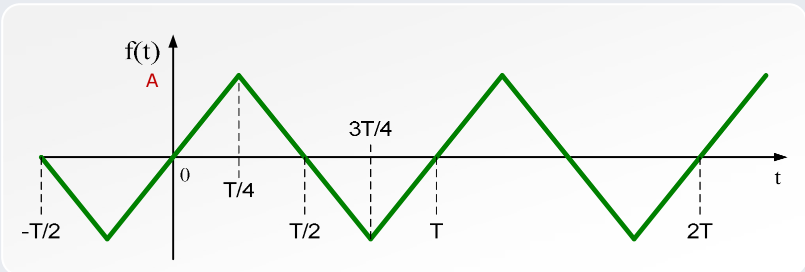
(e) Find the Fourier series of the following periodic function.



It is a periodic function with Period $=T$. $f(t+T)=f(t)$

ANSWER TO Q2

(e) Find the Fourier series of the following periodic function.

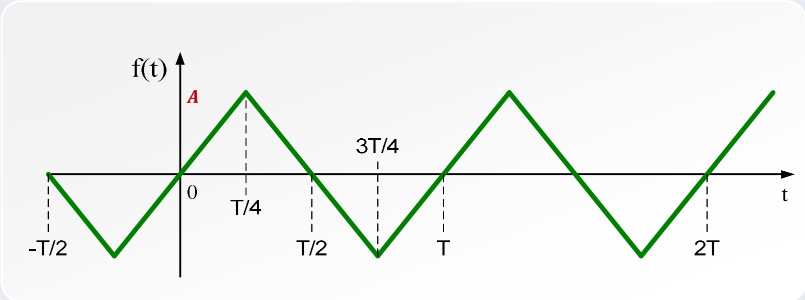


It is a periodic function with Period$=T$:

And the function can be defined as

$$f(t)=\begin{cases} A\dfrac{4}{T}t, & 0\le t\le \dfrac{T}{4} \\ A\dfrac{4}{T}(-t+\dfrac{T}{2}), & \dfrac{T}{4}\le t\le \dfrac{3T}{4} \\ A\dfrac{4}{T}(t-T), & \dfrac{3T}{4}\le t\le T \end{cases}$$

ANSWER TO Q2



This is an odd periodic function: $f(-t) = -f(t)$

As a periodic old function, it can be represented by Fourier Series.

For the odd periodic function, the DC $C_0=0$ and $A_n = 0$

ANSWER TO Q2

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) sin\left(\frac{2\pi nt}{T}\right) dt$$

$$= \frac{4}{T}\int_0^{T/2} f(t) sin\left(\frac{2\pi nt}{T}\right) dt$$

$$= \frac{4}{T}\int_0^{T/4} A\frac{4t}{T}\, sin\left(\frac{2\pi n}{T}\right) dt + \frac{4}{T}\int_{T/4}^{T/2} A\frac{4}{T}\left(\frac{T}{2}-t\right) sin\left(\frac{2\pi n}{T}\right) dt$$

$$= \frac{4}{T} * \frac{4}{T} * A\left[2\left(\frac{T}{2\pi n}\right)^2 sin\left(\frac{\pi n}{2}\right)\right]$$

$$= 8A\left(\frac{1}{\pi n}\right)^2 sin\left(\frac{\pi n}{2}\right)$$

$B_n = 0$ when n is even.

ANSWER TO Q2

Given wave form *f*(t)

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t) \quad (2)$$

where C_0 is the DC component of the signal, A_n and B_n are coefficients. They are given by

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt, \; B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$$
$$C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{A_0}{2}. \quad (3)$$

Note: C_0 is the average value of the waveform over its period.

Now with new wave form K•*f*(t)

$$F'(t) = KF(t)$$

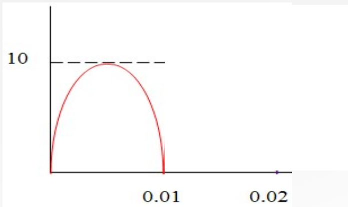
FOURIER SERIES REPRESENTATION

FOR

WAVEFORMS IN A GIVEN INTERVAL?

Q3

A signal $y(t)=|A\sin(100\pi t)|$ below is defined in the time interval [0, 0.01] of unit second.



(c) If a periodic function *g(t)* of odd symmetry is created also based on the given signal y(t) with a period *T=0.02* seconds. What will be *g(t)* and its Fourier Series

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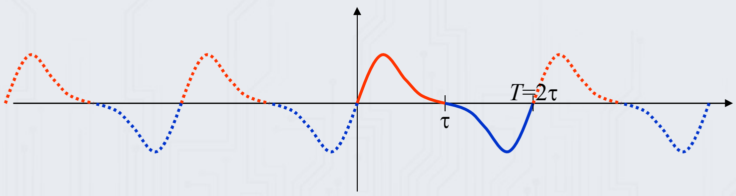
CONVERSION FROM NON-PERIODIC TO PERIODIC

Expansion Into Odd-function Symmetry

A second pattern can be created by mirroring the original one against the time axis and then the axis t= τ.

An odd-function periodic waveform can be generated by offsetting the two patterns merged along the time axis by a distance *nT* (*T*=2τ), *n*=±1, ±2, ±3, …



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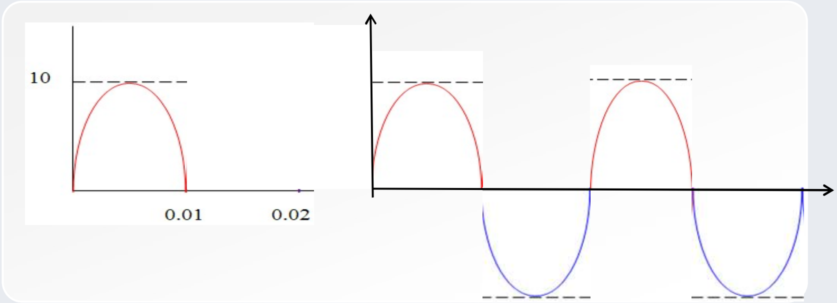
Q3

A signal $y(t)=|A\sin(100\pi t)|$ below is defined in the time interval [0, 0.01] of unit second.

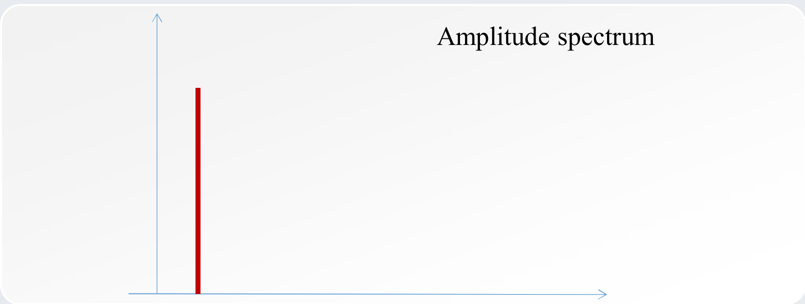


(c) If a periodic function *g(t)* of odd symmetry is created also based on the given signal y(t) with a period *T=0.02* seconds. What will be *g(t)* and its Fourier Series

$\boldsymbol{g}(\boldsymbol{t}) = \mathrm{A}\sin(100\pi t), 0 \le t \le T, or\ g(t) = \mathrm{A}\sin(100\pi t), 0 \le t \le T,$

$\boldsymbol{G}(\boldsymbol{t}) = \mathrm{A}\sin(100\pi t)$: Uniqueness of the Fourier Representation

ANSWER TO Q3(C)



C_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}
0	10	0	0	0	0	0	0	0	0	0	0	0

$B_n=0$

Because of the uniqueness

Q3

A signal $y(t)$ is shown below for 2 complete cycles. It has a period of 0.02 s.

$y(t) = |A\sin(100\pi t)|$, where A is 10 volt

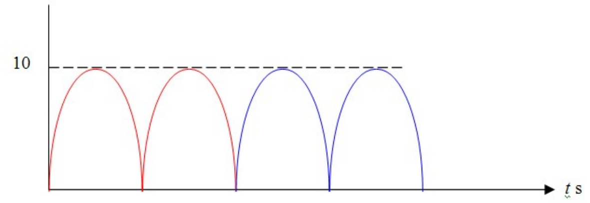


Figure 1: The signal $y(t)$

Find the Fourier Series for $y(t)$ and sketch the amplitude spectrum from DC to the 12th harmonics.

Ans: $y(t) = \frac{20}{\pi} + \sum_{n=2,4,\ldots}^{\ldots} A_n \cos(n\omega_o t)$, $A_n = \frac{10(2)}{\pi(n+1)} - \frac{10(2)}{\pi(n-1)}$,

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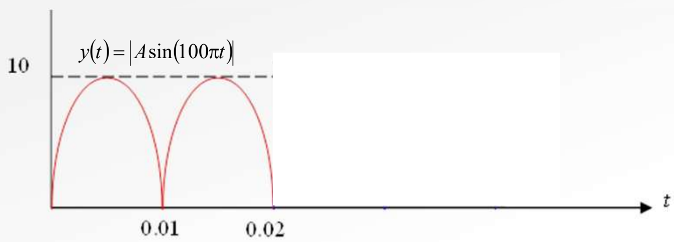
Q4

A signal is defined in the time interval [0, 0.02] of unit second.



(a) Create a periodic function f(t) of even symmetry based on the given signal y(t) with a period T=0.02 seconds.

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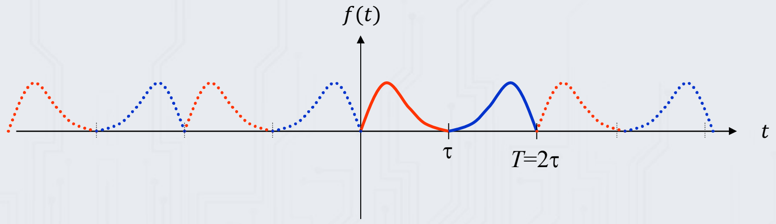
CONVERSION FROM NON-PERIODIC TO PERIODIC

Expansion Into Even-function Symmetry

A second pattern can be created by mirroring the original one against an axis $t = \tau$.

An even-function symmetric periodic waveform can be generated by offsetting the two patterns merged along the time axis by a distance nT ($T=2\tau$), $n=\pm1, \pm2, \pm3, \ldots$



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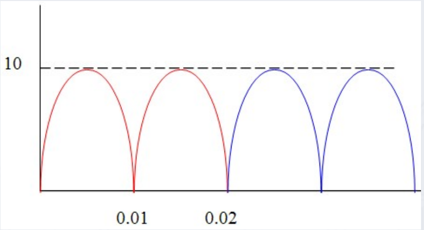
Answer to Q3(A)

(a) Create a periodic function f(t) of even symmetry based on the given signal y(t) with a period T=0.02 seconds.



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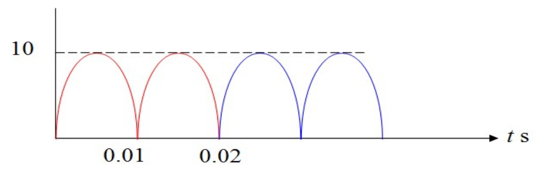
Answer to Q3(B)

(b) Find the Fourier Series for $f(t)$ and sketch the amplitude spectrum from DC to the 12th harmonics.

$y(t) = |A\sin(100\pi t)|$, where A is 10 volt

A=10



The period T = 0.02 s. The fundamental frequency is 50 Hz.

$$f(t) = \begin{cases} A\sin(\mathbf{100\pi t}), \mathbf{0} \le \boldsymbol{t} \le \frac{\boldsymbol{T}}{\mathbf{2}} \\ -A\sin(\mathbf{100\pi t}), \frac{\boldsymbol{T}}{\mathbf{2}} \le \boldsymbol{t} \le \boldsymbol{T} \end{cases} \qquad f(t) = \begin{cases} A\sin(\mathbf{100\pi t}), \mathbf{0} \le \boldsymbol{t} \le \frac{\boldsymbol{T}}{\mathbf{2}} \\ -A\sin(\mathbf{100\pi t}), -\frac{\boldsymbol{T}}{\mathbf{2}} \le \boldsymbol{t} \le \mathbf{0} \end{cases}$$

Answer to Q3(B)

the function is an even function, there are only the dc and the cosine terms. The standard formulas are:

$$A_o = \frac{1}{T}\int_{-T/2}^{T/2} y(t)dt \text{ and } A_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\cos(100n\pi t)dt$$

The terms can be obtained from 0 to T/2 and then multiplied by 2:

$$A_o = \frac{2}{T}\int_0^{T/2} 10\sin(100\pi t)dt = \frac{1000}{100\pi}\cos(100\pi t)\Big|_0^{100}\Big|$$

$$A_0 = \frac{2}{T}\int_0^{T/2} y(t)dt = \frac{2}{100\pi T}\int_0^{T/2} A\sin(100\pi t)dt$$

$$= \frac{-2}{100\pi T}\cos(100\pi t)\Big|_0^{\frac{T}{2}} = \frac{-2A}{100\pi T}\left(\cos\left(\frac{100\pi T}{2}\right) - \cos(100\pi * 0)\right)$$

$$= \frac{2A}{100\pi T}\left(1 - \cos\left(\frac{100\pi T}{2}\right)\right) = \frac{2*10}{100*\pi*0.02}\left(1 - \cos\left(\frac{100\pi*0.02}{2}\right)\right) = \frac{10}{\pi}*(1 - \cos(\pi)) = \frac{20}{\pi}$$

Answer to Q3(B)

$$A_n = 2 \times \frac{2}{T} \int_0^{T/2} 10\sin(100\pi t)\cos(100n\pi t)\,dt$$

Use the identity:

2sin(X)cos(Y) = sin(X+Y) + sin(X–Y):

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Answer to Q3(B)

$$A_n = 1000 \int_0^{T/2} 2\sin(100\pi t)\cos(100n\pi t)\,dt$$

$$= 1000 \int_0^{T/2} \sin(100\pi[1+n]t) + \sin(100\pi\underline{[1-n]}t)\,dt$$

$$= -1000\frac{\cos(100\pi[n+1]t)}{100\pi(n+1)}\Bigg|_0^{1/100} \underline{+}1000\frac{\cos(100\pi[n-1]t)}{100\pi\underline{(n-1)}}\Bigg|_0^{1/100}$$

$$= -\frac{10}{(n+1)\pi}[\cos(n+1)\pi - 1] + \frac{10}{(n-1)\pi}[\cos(n-1)\pi - 1]$$

For n = 1, or odd: cos(2π) = cos(2kπ) = 1; for k = 2, 3, 4 …A_n = 0

For n = 2, or even: cos(π) = cos((2k+1)π) = –1, for k = 1, 2, 3, 4

$$A_n = \frac{10(2)}{\pi(n+1)} - \frac{10(2)}{\pi(n-1)}$$

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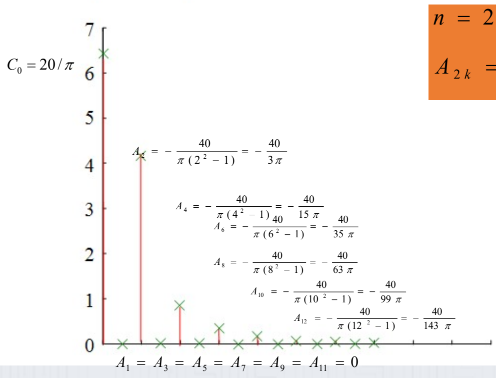
Answer to Q3(B)

$$y(t)=\frac{20}{\pi}+\sum_{n=2,4,\ldots}^{\ldots}A_n\cos(n\omega_o t)\text{ where }A_n=\frac{10(2)}{\pi(n+1)}-\frac{10(2)}{\pi(n-1)}$$

The amplitude spectrum is as shown below:

$$n=2k$$
$$A_{2k}=-\frac{40}{\pi(4k^2-1)}$$



PROJECT - OBJECTIVE

This project is designed for MA2011 students to better understand Mechatronics through investigation in a BIG and GLOBAL picture for UN's Sustainability Development Goal and Net Zero.

PROJECT - APPLICATIONS

Specific applications (e.g., EV Manufacturing, Petrochemical Plants, etc.) should be identified with the project. The investigation is to address industrial pain points in the given field.

PROJECT - PROTOTYPE

Due to resource and time constraints, the project does not require students to develop prototypes (software or hardware). Thus, it will be a bonus if a valid prototype is included as part of the delivery with the project.

PROJECT - DEMONSTRATION

Use of existing solutions and/or data (commercially or non-commercial) are allowed to demonstrate the viability of the project ideas.

PROJECT – AI TOOLS AND DECLARATION

ChatGPT and other AI tools can be used in the project with proper declaration wherever applicable.

PROJECT – ELEVATOR PITCH PRESENTATION

The 5 min presentation is an Elevator Pitch. Video, audio, and other multimedia can be included in the presentation PPT. Role pay can be also used in the presentation.

The PPT slides must be submitted immediately after the presentation to the File Exchange Folder with Group under MA2011 of NTULearn.

PROJECT - RUBRIC

The rubric of the project assessment will be seen mainly as follows:

•Team efforts (not one man show)
•Novelty and Innovation
•Elevator Pitch Presentation
•Bonus