NANYANG TECHNOLOGICAL UNIVERSITY

MA2011 MECHATRONICS SYSTEMS INTERFACING

Tutorial 1

Prof. Cai Yiyu

College of Engineering
School of Mechanical and Aerospace Engineering

1

HIGHLIGHT OF LECTURE #1

 2

2

LEARNING GOALS

1. 3 main characteristics of a good measurement system of Amplitude **Linearity**, Phase **Linearity**, and Adequate Bandwidth.
2. Fourier series representation of a signal and its applications.
3. Relationship between an instrument's bandwidth and spectra of its input and output signals.
4. Dynamic response of zero-, first-, and second-order systems.
5. System response to step and sinusoidal inputs.
6. Analogies among mechanical, electrical and hydraulic systems.

 3

3

MEASUREMENT SYSTEMS - COMPONENTS

INPUT

Physical quantity to be measured is called input to a measurement system.

OUTPUT

Transducer transforms the input into a form compatible with the processor to be processed then becomes the output of a measurement system.

DIFFERENCE

Usually, the input is different from the output of a measurement system.

CHARACTERISATION

A good measurement system is characterised by phase linearity, amplitude linearity, and adequate bandwidth.

 4

4

FOURIER SERIES REPRESENTATION OF SIGNALS

1. To study **phase linearity and bandwidth** (applied to frequency components of an input signal), it is necessary to review Fourier series representation of a signal.
2. Any periodical waveform can be represented as an infinite series of sine and cosine waveforms of different amplitudes and frequencies.
3. Summing up this infinite series gives the original periodical waveform.
4. Practically, a finite number of the sine and cosine waveforms can adequately represent a periodical waveform.

TUTORIAL #1 – Q1

Q1: A QUICK HANDS-ON USING A QR-CODE

If the following input (V_{in}) and output (V_{out}) relationships exist for different measurement systems, indicate whether each is linear or nonlinear and explain why:

a) $V_{out}(t) = 5V_{in}(t);$
b) $V_{out}(t)/V_{in}(t) = 5t;$
c) $V_{out}(t) = V_{in}(t) + 5;$
d) $V_{out}(t) = V_{in}(t) + V_{in}(t);$
e) $V_{out}(t) = V_{in}(t)*V_{in}(t);$
f) $V_{out}(t) = V_{in}(t) + 10t;$
g) $V_{out}(t) = V_{in}(t) + \sin(5)$
h) If $(V_{out}(t) - V_{out}(0)) = \alpha(V_{in}(t) - V_{in}(0))$, what will be the relation between $W_{out}(t) = \beta V_{out}(t) + C$ and $V_{in}(t)$?

Quiz at NTULearn

 7

7

ANSWER TO Q1

a) $V_{out}(t) = 5V_{in}(t)$: Linear or Non-linear?

Answer: Existing Magic $\alpha = 5$, ➔ Linear

 8

8

AMPLITUDE LINEARITY

Magic α

The output always changes with the same factor times by the change in the input, i.e.,

$$V_{out}(t) - V_{out}(0) = \alpha(V_{in}(t) - V_{in}(0))$$

Where α is the proportional constant (gain).

ANSWER TO Q1

Uniqueness

If $(V_{out}(t) - V_{out}(0)) = \alpha(V_{in}(t) - V_{in}(0))$, and $(V_{out}(t) - V_{out}(0)) = \beta(V_{in}(t) - V_{in}(0)) \Rightarrow \alpha = \beta$

Since $(V_{out}(t) - V_{out}(0)) = \alpha(V_{in}(t) - V_{in}(0))$, and $(V_{out}(t) - V_{out}(0)) = \beta(V_{in}(t) - V_{in}(0))$

We have $\alpha(V_{in}(t) - V_{in}(0)) = \beta(V_{in}(t) - V_{in}(0))$

Thus $(\alpha - \beta)V_{in}(t) = (\alpha - \beta)V_{in}(0)$

If $\alpha \neq \beta$, $V_{in}(t) = V_{in}(0)$

Therefore $\alpha = \beta$

AMPLITUDE LINEARITY

Ratio

The output always changes with the same factor times by the change in the input, i.e.,

$$(V_{out}(t) - V_{out}(0)) / (V_{in}(t) - V_{in}(0)) = \alpha$$

Where α is the proportional constant (gain).

11

ANSWER TO Q1

b) $V_{out}(t)/V_{in}(t) = 5t$*: Linear or Non-linear?*

Do we have a magic α *existing?*

$\alpha = 5t$ is a function of t, not a constant ➔ *Non-linear*

Proof by contra-diction

Assume: There is an α such that $V_{out}(t)-V_{out}(0) = \alpha\,(V_{in}(t) - V_{in}(0))$

This means $V_{out}(t)-V_{out}(0) = 5tV_{in}(t)-0= \alpha\,(V_{in}(t) - V_{in}(0))$

Or $V_{in}(t)(5t- \alpha)= -\alpha V_{in}(0)$

In other words, $V_{in}(t) = \alpha V_{in}(0)/(\alpha - 5t)$

which is generally not correct

So no *magic* α *existing* !

12

ANSWER TO Q1

c) $V_{out}(t) = V_{in}(t) + 5;$

Answer: $V_{out}(t)\text{-}V_{out}(0) = V_{in}(t) + 5 - (V_{in}(0)+5) = 1*(V_{in}(t) - V_{in}(0))$

Existing Magic α =1 is a constant ➔ Linear

***Generalisation**: Any constant C,* $V_{out}(t) = V_{in}(t) + C$ *➔ Linear*

ANSWER TO Q1

d) $V_{out}(t) = V_{in}(t) + V_{in}(t)$;

Answer: $V_{out}(t)\text{-}V_{out}(0) = V_{in}(t) + V_{in}(t) - (V_{in}(0)+V_{in}(0)) = 2*(V_{in}(t) - V_{in}(0))$

Magic α =2 is a constant ➔ Linear

***Generalization**: Any constant C,* $V_{out}(t) = CV_{in}(t)$ *➔ Linear*

ANSWER TO Q1

e) $V_{out}(t) = V_{in}(t) * V_{in}(t)$;

Answer: $V_{out}(t)-V_{out}(0) = V_{in}(t) * V_{in}(t) - V_{in}(0)*V_{in}(0)$

Proof by contradiction

Assume: we have a constant α so that ➔ *Mostly Non-linear*

$V_{out}(t)-V_{out}(0) = \alpha *(V_{in}(t) - V_{in}(0))$

$V_{in}(t) * V_{in}(t)- V_{in}(0) * V_{in}(0) = \alpha *(V_{in}(t) - V_{in}(0))$

$V_{in}(t) * V_{in}(t)- a\ V_{in}(t) + (aV_{in}(0)-V_{in}(0) * V_{in}(0))=0$

$D= \alpha * \alpha -4*(\alpha V_{in}(0)-V_{in}(0) * V_{in}(0))$

$V_{in}(t)=(\alpha \pm sqr(D))/2$

So, no magic α existing

ANSWER TO Q1

f) $V_{out}(t) = V_{in}(t) + 10t$;

Answer: $V_{out}(t)-V_{out}(0) = V_{in}(t) +10t -(V_{in}(0) +10*0)= (V_{in}(t) -V_{in}(0)) +10t$

Usually, we do not have a constant α ➔ *Non-linear*

Note: *Any constant C,* $V_{out}(t) =V_{in}(t)+Ct$ ➔ *Non-linear*

Proof by contradiction

Assume, we have a constant α so that

$V_{out}(t)-V_{out}(0) = \alpha *(V_{in}(t) - V_{in}(0))$

$V_{in}(t) + Ct - V_{in}(0)- C0 = \alpha *(V_{in}(t) - V_{in}(0))$

$V_{in}(t) *(1- \alpha) =V_{in}(0) + C0- Ct - \alpha V_{in}(0)$

$V_{in}(t)=(V_{in}(0) + C0- Ct - \alpha V_{in}(0))/(1- \alpha)$

No magic α existing

ANSWER TO Q1

g) $V_{out}(t) = V_{in}(t) + C;$

Answer: C=Sin(5), magic α=1 existing ➔ *Linear*

ANSWER TO Q1

Additional Question h):

If $(V_{out}(t) - V_{out}(0)) = \alpha(V_{in}(t) - V_{in}(0))$,

Will $W_{out}(t) = \beta V_{out}(t)+C$ and $W_{in}(t) = \beta V_{in}(t)+C$ have

$\boldsymbol{W_{out}(t) - W_{out}(0) = \alpha(W_{in}(t) - W_{in}(0))}$?

ANSWER TO Q1

Generalization & Inheritance

h) If we have amplitude linearity wave $V_{out}(t)$ and $V_{in}(t)$: $(V_{out}(t)- V_{out}(0)) = \alpha(V_{in}(t)- V_{in}(0))$,

What will be the relation with constant C and β between $W_{out}(t)$ and $W_{in}(t)$:

$W_{out}(t) = \beta V_{out}(t)+C$ **and** $W_{in}(t) = \beta V_{in}(t)+C$ **?**

$(V_{out}(t)- V_{out}(0)) = \alpha(V_{in}(t)- V_{in}(0))$,

$W_{out}(t) = \beta V_{out}(t)+C$

$W_{out}(t) - W_{out}(0) = \beta V_{out}(t)+C - \beta V_{out}(0)-C = \beta(V_{out}(t)- V_{out}(0)) = \alpha\,\beta\,(V_{in}(t)- V_{in}(0))$

$W_{out}(t) - W_{out}(0) = \alpha(\beta V_{in}(t)+C- \beta V_{in}(0)-C)=\alpha(W_{in}(t) - W_{in}(0))$

$W_{out}(t)$ and $W_{in}(t)$ *are Linearly related*

TUTORIAL #1 – Q2

Q2

What is the Fourier series and fundamental frequencies (in Hertz) and amplitudes of the following waveforms?

a) $F(t) = 5*\sin(2\pi t)$.
b) $F(t) = 5*\cos(2\pi t)$
c) $F(t) = -5*\sin(2\pi t)$

FOURIER SERIES REPRESENTATION OF SIGNALS

The Fourier series representation of a periodical waveform *f(t)* is

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

where

C_0 is the DC component of the signal, and the average value of the waveform over its period

ω_0 is the fundamental or first (lowest) harmonic frequency defined as

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

f_0 is fundamental frequency in Hertz (Hz).

T is period

ANSWER TO Q2

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

	Term	Fundamental frequency	Amplitude	Remark
F(t)=5sin(2πt)	n=1	$f_0=\omega_0/2\pi=2\pi/2\pi=1$Hz	5	F(t)=5cos(2πt-π/2)
F(t) = 5cos(2πt)	n=1	$f_0=\omega_0/2\pi=2\pi/2\pi=1$Hz	5	
F(t) = -5cos(2πt)	n=1	$f_0=\omega_0/2\pi=2\pi/2\pi=1$Hz	5	Amplitude is the absolute value

 23

23

TUTORIAL #1 – Q3

 24

24

Q3

The power we use at home has a frequency of 60 Hz. What is the period of this sine wave?

Answer: 0.0166s=16.6ms

25

ANSWER TO Q3

The power we use at home has a frequency of 60 Hz. The period of this sine wave can be determined as follows:

$$\boxed{T = \frac{1}{f} = \frac{1}{60} = 0.0166\ \text{s} = 0.0166 \times 10^3\ \text{ms} = 16.6\ \text{ms}}$$

26

TUTORIAL #1 – Q4

TRIGONOMETRY FORMULA

Common trigonometry formulas	
variations on the Pythagorean theorem:	$\sin^2 A + \cos^2 A = 1$ $\tan^2 A + 1 = \sec^2 A$ $1 + \cot^2 A = \csc^2 A$
half-angle formulas:	$\sin^2\left(\frac{A}{2}\right) = \frac{1-\cos A}{2}$ $\cos^2\left(\frac{A}{2}\right) = \frac{1+\cos A}{2}$
double-angle formulas:	$\sin(2A) = 2\sin A \cos A$ $\cos(2A) = \cos^2 A - \sin^2 A$
addition formulas:	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
law of sines:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
law of cosines:	$c^2 = a^2 + b^2 - 2ab\cos C$ $b^2 = a^2 + c^2 - 2ac\cos B$ $a^2 = b^2 + c^2 - 2bc\cos A$

Q4

For the Fourier series given by

$$y(t) = 4 + \sum_{n=1}^{\infty} \frac{2n\pi}{10} \cos \frac{n\pi}{4} t + \frac{120n\pi}{30} \sin \frac{n\pi}{4} t$$

where t is the time in seconds,

c. Express this Fourier series as an infinite series containing sine terms only.

$$y(t) = 4 + 4n\pi \sum_{n=1}^{\infty} sin(^{n\pi t}/_{4} + 0.05)$$

$$y(t) = 4 + \sum_{n=1}^{\infty} 4\pi sin(^{n\pi t}/_{4} + 0.05)$$

$$y(t) = 4 + \sum_{n=1}^{\infty} 4n\pi sin(^{n\pi t}/_{4} + 0.05)$$

$$y(t) = 4 + \sum_{n=1}^{\infty} 4n\pi cos(^{n\pi t}/_{4} + 0.05)$$

Q4

For the Fourier series given by

$$y(t) = 4 + \sum_{n=1}^{\infty} \frac{2n\pi}{10} \cos \frac{n\pi}{4} t + \frac{120n\pi}{30} \sin \frac{n\pi}{4} t$$

where t is the time in seconds,

a. What is the fundamental frequency in hertz and radians per second, rad s^{-1}?

b. What is the period T associated with the fundamental frequency?

c. Express this Fourier series as an infinite series containing sine terms only.

$$Ans: fo = {}^{1}/_{8}\,Hz, T = 8s; y(t) = 4 + \sum_{n=1}^{\infty} 4n\pi sin(^{n\pi t}/_{4} + 0.05)$$

ANSWER TO Q4

SOLUTION:

A_n B_n

Given $$y(t) = 4 + \sum_{n=1}^{\infty} \frac{2n\pi}{10} \cos\left(\frac{n\pi t}{4}\right) + \sum_{n=1}^{\infty} \frac{120n\pi}{30} \sin\left(\frac{n\pi t}{4}\right)$$

a. At $n = 1$, we get $\omega_0 = \frac{\pi}{4}$ rad s^{-1} or $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{8}$ Hz.

Note that frequency may be in rad s^{-1} or Hz. When the unit is rad s^{-1}, we use the symbol ω. When the unit is Hz, we use the symbol f.

b. Hence the fundamental period $T = 8$ sec.

 31

31

FOURIER SERIES REPRESENTATION OF SIGNALS

Define

$$C_n = \sqrt{{A_n}^2 + {B_n}^2}$$

$$\boldsymbol{\phi_n}^{*} = arctan\left(\frac{A_n}{B_n}\right)$$

Then

$$\boxed{F(t) = C_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_0 t + \boldsymbol{\phi_n}^{*})}$$

That is, a period waveform can be represented by an infinite series of sine of single amplitude and phase

 32

32

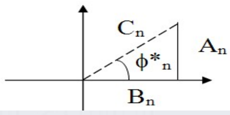
ANSWER TO Q4

c. To convert to the form $y(t) = C_0 + \sum_{n=1}^{\infty} C_n \, sin\left(\frac{2n\pi t}{T} + \phi_n^*\right)$

Where $C_n = \sqrt{A_n^2 + B_n^2} = \sqrt{\left(\frac{2n\pi}{10}\right)^2 + \left(\frac{120n\pi}{30}\right)^2}$

$$C_n = n\pi\sqrt{\left(\frac{2}{10}\right)^2 + \left(\frac{120}{30}\right)^2} \approx 4n\pi$$

$$tan(\phi_n^*) = \frac{A_n}{B_n} = \frac{1}{20} = 0.05$$



 33

33

ANSWER TO Q4

$$F(t) = C_0 + \sum_{n=1}^{\infty} \left(A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t)\right)$$

$$= C_0 + \sum_{n=1}^{\infty} \sqrt{A_n^2 + B_n^2}\left(\frac{A_n}{\sqrt{A_n^2 + B_n^2}}\cos(n\omega_0 t) + \frac{B_n}{\sqrt{A_n^2 + B_n^2}}\sin(n\omega_0 t)\right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \left(\sin(\phi_n^*)\cos(n\omega_0 t) + \cos(\phi_n^*)\sin(n\omega_0 t)\right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \boxed{\sin(n\omega_0 t + \phi_n^*)}$$

$$\phi_n^* = arctan\left(\frac{A_n}{B_n}\right), \ \sin(\phi_n^*) = \frac{A_n}{\sqrt{A_n^2 + B_n^2}}, \ \cos(\phi_n^*) = \frac{B_n}{\sqrt{A_n^2 + B_n^2}}$$

 34

34

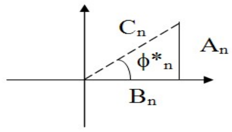
ANSWER TO Q4

c. To convert to the form $y(t) = C_0 + \sum_{n=1}^{\infty} C_n \sin\left(\frac{2n\pi t}{T} + \phi_n^*\right)$

Where $C_n = \sqrt{A_n^2 + B_n^2} = \sqrt{\left(\frac{2n\pi}{10}\right)^2 + \left(\frac{120n\pi}{30}\right)^2}$

$C_n = n\pi\sqrt{\left(\frac{2}{10}\right)^2 + \left(\frac{120}{30}\right)^2} \approx 4n\pi$

$\tan(\phi_n^*) = \frac{A_n}{B_n} = \frac{1}{20} = 0.05$



$\phi_n^* = \tan^{-1}(0.05) = 0.05$ radians

Hence, $y(t) = 4 + \sum_{n=1}^{\infty} 4n\pi \sin\left(\frac{n\pi t}{4} + 0.05\right)$

Phase Angle

$F(t) = C_0 + \sum_{n=1}^{\infty}(A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t))$

$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t - \boldsymbol{\phi_n})$

$\boldsymbol{\phi_n} = \arctan\frac{B_n}{A_n}, \cos(\boldsymbol{\phi_n}) = \frac{A_n}{\sqrt{A_n^2+B_n^2}}, \sin(\boldsymbol{\phi_n}) = \frac{B_n}{\sqrt{A_n^2+B_n^2}}, C_n = \sqrt{A_n^2 + B_n^2}$

$F(t) = C_0 + \sum_{n=1}^{\infty}(A_n \cos(n\omega_0 t) + B_n \sin(n\omega_0 t))$

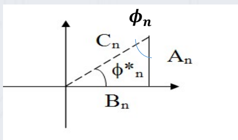
$= C_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_0 t + \boldsymbol{\phi_n}^*)$

$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \boldsymbol{\phi_n}^* - \frac{\pi}{2})$

$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t - \boldsymbol{\phi_n})$

$\boldsymbol{\phi_n}^* = \arctan\frac{A_n}{B_n}, \cos(\boldsymbol{\phi_n}^*) = \frac{A_n}{\sqrt{A_n^2+B_n^2}}, \sin(\boldsymbol{\phi_n}^*) = \frac{B_n}{\sqrt{A_n^2+B_n^2}}, C_n = \sqrt{A_n^2 + B_n^2}$

$\boldsymbol{\phi_n} = \frac{\pi}{2} - \boldsymbol{\phi_n}^*$



$Sin\ (\boldsymbol{\phi_n}) = sin(\frac{\pi}{2} - \boldsymbol{\phi_n}^*) = \cos\ (\boldsymbol{\phi_n}^*)$

FUNDAMENTAL FREQUENCY

Additional Question:

For example: $y(t) = 4 + \frac{2\pi}{10}\cos\frac{2\pi}{4}t + \frac{4\pi}{10}\cos\frac{3\pi}{4}t + \frac{8\pi}{10}\cos\frac{4\pi}{4}t + \cdots$

This can be written as $y(t) = 4 + \sum_{n=1}^{\infty} \boxed{\frac{2(n-1)\pi}{10}} \cos\frac{n\pi}{4}t$

What are the fundamentak frequency and amplitude?

n=1, A_1= 2*(1-1)π/10**=0,** fundamentak frequency **$\omega_0 = \pi/4$**

n=2, A_2= 2*(2-1)π/10**= π/5,** fundamentak frequency **$\omega_0 = \pi/4$**

$\frac{2\pi}{10}\cos\frac{2\pi}{4}t$

$\omega_0 \,!= \pi/2$

FUNDAMENTAL FREQUENCY

When n=1, the fundamental frequency is $\frac{\pi}{4}$ rad s^{-1} or $\frac{1}{8}$ Hz, although the amplitude is zero for this component, and not $\frac{\pi}{2}$ rad s^{-1} or $\frac{1}{4}$ Hz for n = 2.