NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-2022

MA2011 – MECHATRONICS SYSTEM INTERFACING

April/May 2022 Time Allowed: 2 ½ hours

INSTRUCTIONS

1. This paper contains **SECTION A & SECTION B** and comprises **FOUR (4)** pages.
2. **COMPULSORY** to answer **ALL** questions in both sections.
3. All questions carry equal marks.
4. This is a **RESTRICTED OPEN-BOOK** examination. One double-sided, A4-size reference sheet is allowed.

SECTION A

1. Given a signal *F*(*t*) below,

$$F(t) = \sum_{n=1}^{\infty} \frac{4\sqrt{2}}{(2n-1)} e^{-\frac{(2n-1)}{2}} \{Cos\ ((2n-1)\pi t) + Sin\ \ ((2n-1)\pi t)\}$$

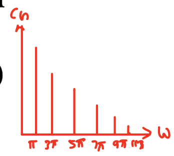
(a) Show the frequency-domain analysis of the signal for the first 6 terms (n= 1 to 6) with the aid of a spike drawing,

$n=1:\ 4\sqrt{2}\, e^{-\frac{1}{2}} (\cos \pi t + \sin \pi t)$ $\quad 2\pi f = \pi \Rightarrow f = \frac{1}{2}$

$n=2:\ \frac{4\sqrt{2}}{3} e^{-\frac{1}{2}} (\cos 3\pi t + \sin 3\pi t)$ $\quad 2\pi f = 3\pi \Rightarrow f = \frac{3}{2}$

$n=3:\ \frac{4\sqrt{2}}{5} e^{-\frac{1}{2}} (\cos 5\pi t + \sin 5\pi t)$ $\quad f = \frac{5}{2}$

(8 marks)



(b) Convert *F*(*t*) into the standard Fourier Series using a fundamental frequency ω_0,

$\sum_{n=1}^{\infty} 4\sqrt{2} e^{-\frac{1}{2}} \cos(\omega_0 t) + \sum_{n=1}^{\infty} 4\sqrt{2} e^{-\frac{1}{2}} \sin(\omega_0 t)$

(7 marks)

(c) Using Shannon-Nyquist Theory with a cut-off filter f_c=5 Hz, suggest a sampling rate with no aliasing effect.

$f_s > 2f_{max} = 2(5) = 10Hz$

$\therefore f_s > 10Hz$

(5 marks)

(d) Convert *F*(*t*) into Fourier Series in sine only representation.

(5 marks)

$$C_n = \sqrt{\left(\frac{4\sqrt{2}}{(2n-1)} e^{-\frac{2n-1}{2}}\right)^2 + \left(\frac{4\sqrt{2}}{(2n-1)} e^{-\frac{2n-1}{2}}\right)^2}$$

$$= \sqrt{2}\ \frac{4\sqrt{2}}{(2n-1)} e^{-\frac{2n-1}{2}}$$

$$= \frac{8}{2n-1} e^{\frac{1-2n}{2}}$$

$A_n = B_n \Rightarrow \phi = \tan^{-1}(1) = \frac{\pi}{4}$

$$\therefore F(t) = \sum_{n=1}^{\infty} \left(\frac{8}{2n-1} e^{\frac{1-2n}{2}}\right) \sin\left(n\omega_0 t + \frac{\pi}{4}\right)$$

2. Moon exploration is a highly comprehensive, interdisciplinary, and challenging task with lunar rover playing an utmost important and mission critical role.

(a) What knowledge of mathematics, natural science, and engineering in your opinion can be applied to the solution of complex problems of moon exploration?

(5 marks)

(b) Describe TWO most innovative ideas you have for design or develop a moon rover system.

(5 marks)

(c) In the moon exploration initiative, how do you function effectively as an individual, and as a member or leader in diverse teams and in multidisciplinary settings?

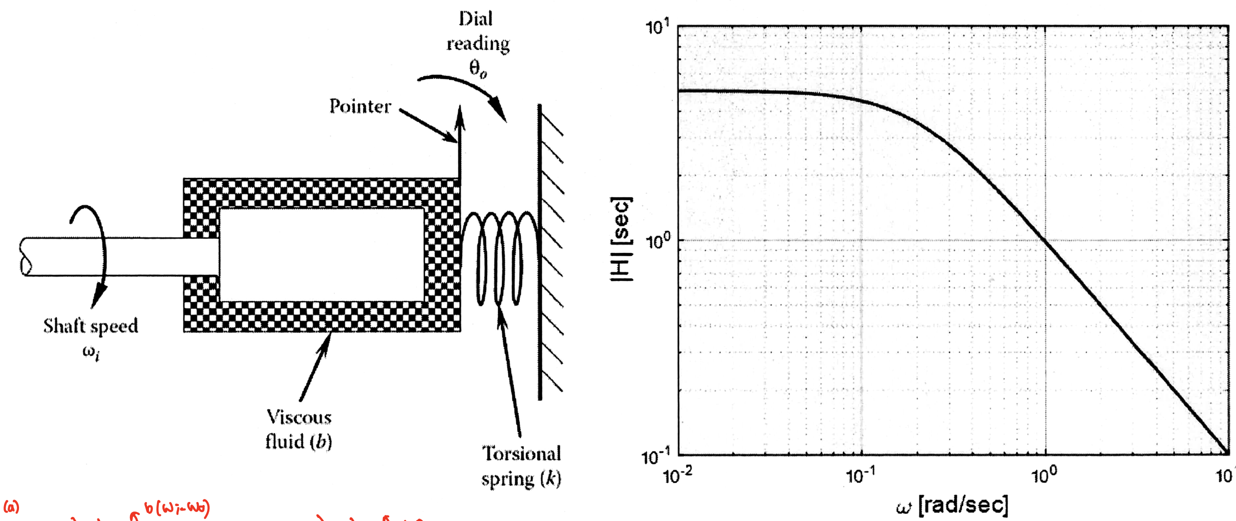
(5 marks)

(d) Discuss one possible application of Fourier Series in signal processing and communication, robotics navigation, and path planning, etc., for rover based lunar exploration.

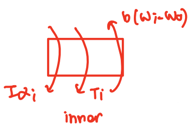
(10 marks)

SECTION B

3. The angular *tachometer* shown in Figure 1 (left) is a device meant to measure angular velocity. It is mainly based on a viscous damper (with damping constant b) consisting of two cylinders. The inner cylinder is connected to a shaft assumed to be revolving at the angular speed ω_i to be measured. The outer cylinder is compliantly held in place by a torsional spring of stiffness k. Without the torsional spring, the outer cylinder would simply spin, dragged along by the rotating shaft.



(a)



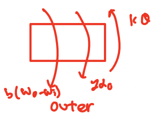


Figure: 1

$k\ddot{\theta}$ = rotational force − damper force

$I\alpha$ = rotational force + damper force

$T_i = b(\omega_i - \omega_o)$

$b(\omega_o - \omega_i) = k\theta_o$

Note: Question 3 continues on page 3.

(b) Let $\theta_o = Ae^{j\omega t}$ $\Rightarrow$ $\omega_o = j\omega A e^{j\omega t}$

$\omega_i = Be^{j\omega t}$

$b(j\omega A e^{j\omega t} - Be^{j\omega t}) = k(Ae^{j\omega t})$

$j\omega b A e^{j\omega t} - kAe^{j\omega t} = bBe^{j\omega t}$ $\Rightarrow$ $\frac{A}{B} = \frac{be^{j\omega t}}{j\omega b e^{j\omega t} - ke^{j\omega t}} = \frac{b}{j\omega b - k}$ #

(a) Draw the free-body diagram of the system and write the dynamic equations (Newton's law for rotating systems) for the two rigid bodies.

(7 marks)

(b) Assuming negligible inertia of all moving parts, write the frequency response $H = \frac{\theta_o}{\omega_i}$.

(8 marks)

(c) Based on the experimental frequency plot of the amplitude $|H|$ as given in Figure 1 (right), determine the numerical values and units of the following:

(i) Ratio $\frac{b}{k}$

(ii) Cutoff frequency ω^*

$H = \frac{b}{j\omega b - k}$

$|H| = \frac{b}{\sqrt{(\omega b)^2 + k^2}}$

$\frac{1}{|H|} b = \sqrt{\omega^2 b^2 + k^2}$

$\left[\left(\frac{1}{|H|}\right)^2 - \omega^2\right] b^2 = k^2 \Rightarrow \frac{b}{k} = \sqrt{\frac{1}{\frac{1}{|H|^2} - \omega^2}}$

At $|H| = 5$ and $\omega = 0.01$,

$b/k = 5.006$

$H = \frac{b}{j\omega b - k}$

$\lim_{\omega \to 0} \frac{b}{j\omega b - k} = \frac{b}{k}$

$\lim_{\omega \to \infty} \frac{b}{\cancel{j\omega b - k}} = 0$

(6 marks)

$\frac{b}{\sqrt{(\omega b)^2 + k^2}} = \frac{b/k}{\sqrt{2}} \Rightarrow \frac{b/k}{\sqrt{(\omega \frac{b}{k})^2 + 1}} = \frac{b/k}{\sqrt{2}}$

$(\omega \frac{b}{k})^2 + 1 = 2$

$\omega = \frac{k}{b}$

$= 0.1997$

(d) We neglected the system inertia up to this point, but had we accounted for it, we would have expected a resonance. Assuming a stiffness $k = 10$ Nm/rad, determine the value for the inertia for which the resonance frequency of the system would coincide with the cutoff frequency ω^* computed in (c).

$W_{res} = W_{cutoff} = 0.199749$ rad/s

$\omega = \sqrt{\frac{k}{I}}$ $\quad I = \frac{k}{\omega^2} = \frac{10}{0.199749^2} = 250.63$

(4 marks)

4. The loudspeaker converts electrical energy into acoustical energy; in an electromagnetic loudspeaker, this is achieved with a voice coil actuating a membrane which, in turn, displaces the surrounding air. As sketched in Figure 2, the loudspeaker can be modelled via an electrical system, a mechanical system and a set of coupling equations relating electrical and mechanical quantities via a constant 'T'.

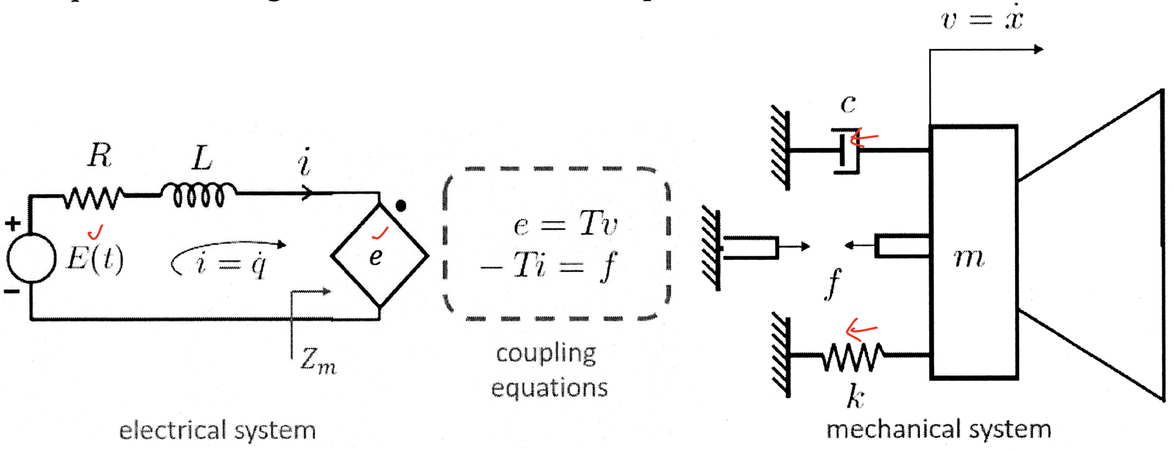


Figure:2

(a) For the electrical system, using Kirchhoff's voltage law, write the (first order) differential equation for the unknown current $i(t)$ and its time derivative assuming the input voltage $E(t)$, electrical resistance R, electrical inductance L, and electromotive voltage e as given.

(5 marks)

(a) $E(t) - iR - L\frac{di}{dt} - e = 0$

$iR + L\frac{di}{dt} = E(t) - e$

$i + \frac{L}{R}\frac{di}{dt} = \frac{1}{R}(E(t) - e)$ *

Note: Question 4 continues on page 4.

$m\ddot{x} = -c\dot{x} - kx + f$

$m\ddot{x} + cv + kx = f$

(b) For the mechanical system, using Newton's law, write the (second order) differential equations relative to dynamics of the membrane, assuming its position x and its velocity v as unknown and with given force f, mass m, stiffness k and damping c.

(5 marks)

(c) Using the coupling equations in Figure 2 and assuming T as a given constant, rewrite the electromechanical equations above in state-space form, by defining the state $[x\ v\ i]^T$. *[Hint: use the coupling equations to express the electromotive voltage as a function of membrane velocity and the force as a function of the electrical current.*

(10 marks)

(d) Operating in frequency domain (i.e. formally replacing derivatives and integrals with '$j\omega$' and '$1/j\omega$', respectively), determine the mechanical impedance Z_m seen at the electrical port (see Figure 2). [*Hint: focus on the mechanical equation and express 'x' in terms of 'v', in frequency domain*]

(5 marks]

(c) $iR + L\frac{di}{dt} = E(t) - e$

$\frac{di}{dt} = \frac{E(t) - iR - e}{L}$

$\dot{i} = \frac{E(t) - iR - Tv}{L}$

$m\ddot{x} + cv + kx = f$

$$e = Tv$$
$$-Ti = f$$

Let $v = \dot{x}$

$\dot{v} = \ddot{x}$

END OF PAPER

$m\dot{v} + cv + kx = f \Rightarrow \dot{v} = -\frac{c}{m}v - \frac{k}{m}x - \frac{1}{m}Ti$

$\dot{x} = v$

$\dot{i} = \frac{E(t) - iR - Tv}{L} = -\frac{T}{L}v - \frac{R}{L}i + E(t)$

$$\frac{d}{dt}\begin{bmatrix} x \\ v \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -k/m & -c/m & -1/m\,T \\ 0 & -T/L & -R/L \end{bmatrix}\begin{bmatrix} x \\ v \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ c \\ 1 \end{bmatrix} E(t)$$

(d) $m\ddot{x} + cv + kx = f$

$v = \dot{x}e^{jw} \Rightarrow$

$\dot{v} = jw\ddot{x}e^{jw} \Rightarrow \ddot{x} = \frac{\dot{v}}{jwe^{jw}}$

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1. Figure 1 shows an inverting amplifier.

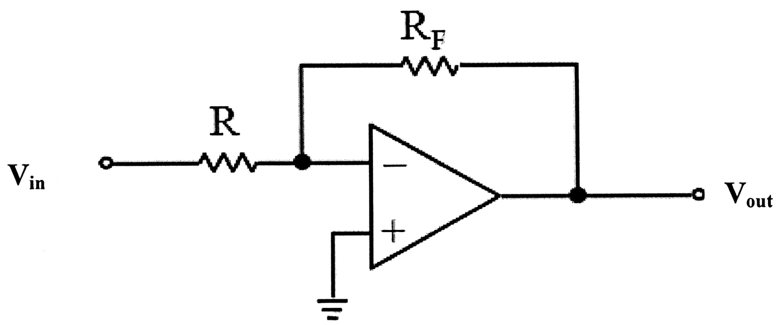


Figure 1: An inverting amplifier.

(a) Choose the right answer(s) below and write on the answer sheet for the concept of open-loop gain.

(i) explicitly involved in feedback path
(ii) explicitly involved in feedforward path
(iii) related to the ratio R_F and R
(iv) related to the ratio V_{out} and V_{in} ✓

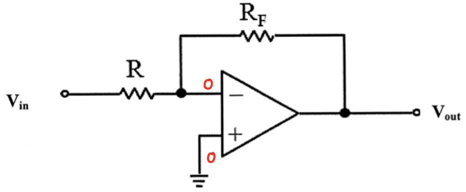
(2 marks)

(b) Choose the right answer(s) below and write on the answer sheet for the concept of closed-loop gain.

(i) it is for feedback amplifier✓
(ii) it is for operational amplifier
(iii) it is due to the loop broken
(iv) it is nothing to do with the loop broken✓

(2 marks)

Note: Question 1 continues on page 2.



(c) infinite input resistance => $I^+ = I^- = 0$

finite loop gain A

$V_o = A V_{in}$

$V_o = A(V_+ - V_-)$, $V_+ = 0$ => $V_o = A(-V^-)$ => $V^- = -\frac{V_o}{A}$

(c) If the inverting amplifier in Figure 1 has a finite loop gain A, an infinite input resistance and a zero output resistance, what is the operational amplifier gain G?

$\frac{V_{in} - V(-)}{R} = \frac{V(-) - V_{out}}{R_F}$ => $V_i R_F - V(-) R_F = V(-) R - V_o R$

$V_i R_F = V(-)[R_F + R] - V_o R$ => $V_i R_F = -\frac{V_o}{A}(R_F + R) - V_o R$ => $V_i = -\frac{V_o}{A}(1 + \frac{R}{R_F}) - V_o \frac{R}{R_F}$ => $\frac{V_o}{V_i} = -\frac{1}{\frac{1}{A}(1+\frac{R}{R_F}) - \frac{R}{R_F}}$ #

(12 marks)

(d) Given the same conditions above, what are the difference and relations between the operational amplifier gain G and the closed-loop gain? Discuss and elaborate in detail.

(9 marks)

2. Shazam is an app that recognises music around you for discovery, exploration and sharing. It is available on Apple, Android and Windows devices.

(a) Describe the music term "Pitch" in relation to Fourier Series Representation.

Pitch is determined by the frequency w_0 in the fourier series terms $\sin(nw_0 t)$ and $\cos(nw_0 t)$, where n is the n th harmonic frequency in the signal.

(5 marks)

(b) Shazam has a music database which are represented in frequency domain acting as a "fingerprint" or signature of the time domain signal of the music. Design your approach to recognising a music from the database for a sound wave you recorded.

(10 marks)

(c) Discuss the sampling technique to convert analogue sound to digital sound. Elaborate the potential problems and your solution involved in this ADC process.

(10 marks)

3. A mass (m) is suspended through a linear spring (k) and a linear damper (b) against gravity (g) and subjected to a vertical force $f(t)$, where t denotes time. We are interested in determining the system response in terms of motions $z(t)$ along the vertical z-axis.

$m\ddot{z} + c\dot{z} + kz = 0$

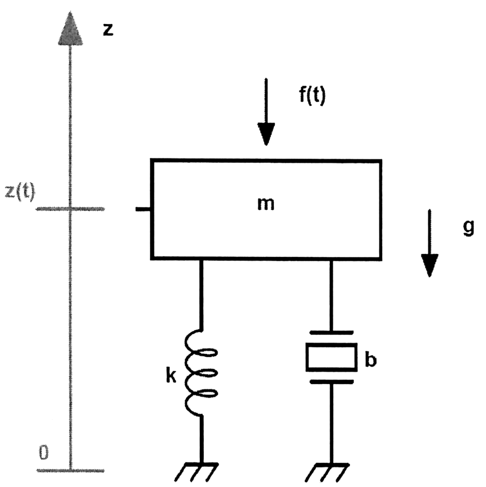
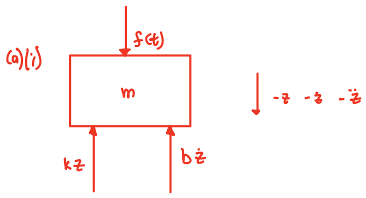


Figure 2

(a)(i)



(ii) $m(-\ddot{z}) = -f(t) + kz + b\dot{z}$

$m\ddot{z} + b\dot{z} + kz = f(t)$ #

(iii) $\ddot{z} = \frac{f(t) - b\dot{z} - kz}{m}$

$v = \dot{z}$

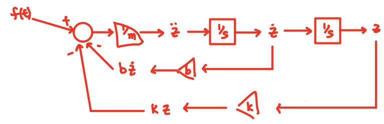
$\dot{v} = \ddot{z}$

$\dot{v} = \frac{-bv - kz + ft}{m}$

$\dot{z} = v$

$\frac{d}{dt}\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix}\begin{bmatrix} z \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} f(t)$

(iv) $\ddot{z} = \frac{f(t) - b\dot{z} - kz}{m}$



Note: Question 3 continues on page 3.

(v) Ultrasound

(a) With reference to the mechanical configuration sketched in Figure 2 and assuming zero-length for the spring at rest:

(i) Draw the free body diagram for the mass m, indicating all the forces and their expressions in terms of vertical position $z(t)$ of the mass and its higher derivatives.

(ii) Apply Newton's law and write the second order differential equation relative to system.

(iii) Derive the state-space representation of the same equations in (ii).

(iv) Sketch the block-diagram representing the same system, using only integrator, gain and adder blocks.

(v) Describe a possible sensor to measure $z(t)$ without disturbing the dynamics of the system itself.

(8 marks)

$Z(j\omega) = Ze^{j\omega t}$

$\dot{z}(j\omega) = j\omega Ze^{j\omega t}$

$\ddot{z}(j\omega) = (j\omega)^2 Ze^{j\omega t}$

$= -\omega^2 Ze^{j\omega t}$

$f(t) = m\ddot{z} + b\dot{z} + kz$

$F(j\omega) = \cancel{A}e^{j\omega t} = (m\ddot{z} + b\dot{z} + kz)e^{j\omega t}$

$= (-m\omega^2 + j\omega b + k)Ze^{j\omega t}$

$H(j\omega) = \frac{Z(j\omega)}{F(j\omega)}$

$= \frac{1}{-m\omega^2 + j\omega b + k}$

(b) Let $F(j\omega)$ and $Z(j\omega)$ be the Fourier transforms for $f(t)$ and $z(t)$, respectively.

(i) Determine the frequency response $H(j\omega) = Z(j\omega)/ F(j\omega)$ of the system. (Note: you will have to remove any offset term first).

(ii) While $H(j\omega)$ is in general a complex function of the frequency ω, determine the frequency Ω for which the real part of $H(j\Omega)$ is zero.

$-m\omega^2 + k = 0$

$\omega = \sqrt{\frac{k}{m}}$

(7 marks)

(c) Assuming that the external force is given as $f(t) = A*exp(i\omega t) + B$, where A is the amplitude, ω is the radian frequency, i is the imaginary constant, and B is an offset.

(i) What is the (DC) steady-state solution when A=0?

$f(t) = Ae^{j\omega t} + B$

If $A = 0$, $f(t) = B$

$B = m\ddot{z} + b\dot{z} + kz$

$\ddot{z}$ and $\dot{z} = 0$ (stdy state)

$B = kz \Rightarrow z = \frac{B}{k}$

(ii) Assuming a generic solution $z(t) = C*exp(i\omega t) + D$, determine the (complex) constants C and D which satisfy the second order differential equation derived above.

(10 marks)

$z(t) = Ce^{j\omega t} + D$

$\dot{z}(t) = j\omega Ce^{j\omega t}$

$\ddot{z}(t) = -\omega^2 Ce^{j\omega t}$

$f(t) = m\ddot{z} + b\dot{z} + kz$

$Ae^{j\omega t} + B = [-\omega^2 m + j\omega b + k]Ce^{j\omega t} + Dk$

$C = \frac{A}{-\omega^2 m + j\omega b + k}$ $\qquad D = \frac{B}{k}$

4. A simplified electromechanical model of a DC-motor (DCM) is shown in Figure 3. It mainly consists of an electrical port (with armature current i flowing through a resistance R) and a mechanical port at the shaft (with angular velocity w, mechanical damping b and inertia J) connected through an *ideal gyrator*.

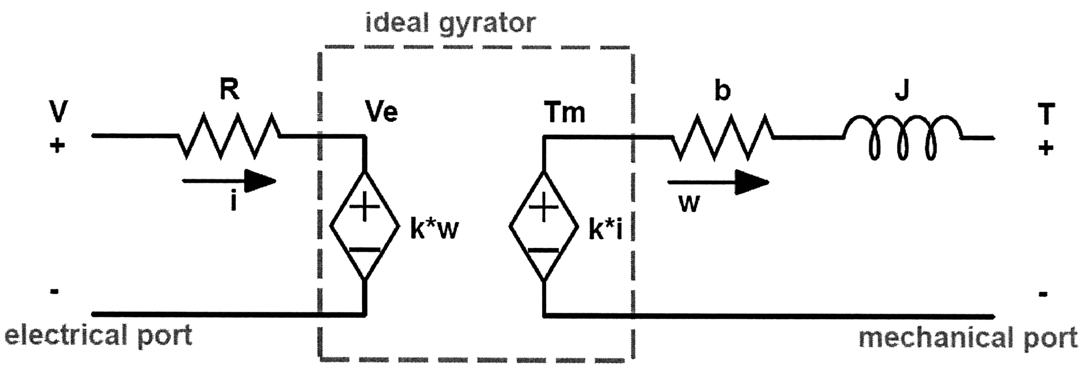


Figure 3

$Ve\, i = Tm\, \omega$ $\Sigma P: 2k\omega i$

$k\omega i = k i \omega$

Down ele   power mech

(a) As highlighted by the dashed box in Figure 3, the gyrator is itself a 2-port system, with one electrical port (current i, voltage Ve) and a mechanical port (angular speed w and torque Tm). Determine the power at each of the two ports of the gyrator and their sum (i.e. total power).

(5 marks)

(b) The electromechanical model in Figure 3 is general, in the sense that it does not specify any particular electrical or mechanical connection. In the specific case of a DCM driven by a voltage source $v(t)$, where t is time, and mechanically free to rotate (i.e. mechanically unloaded),

(i) Redraw the corresponding electromechanical model on your answer book.

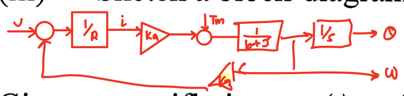
(ii) For the same specific case, write the dynamical equations for $w(t)$, i.e. the angular velocity as function of time t, and its derivatives.

(iii) Sketch a block-diagram representing the system drawn in (i).

(ii) $Tm - b\omega - J\dot{\omega} = 0$

$b\omega = Tm - J\dot{\omega}$

$\omega = \frac{Tm - J\dot{\omega}}{b}$



(7 marks)

(c) Given a specific input $v(t) = A\ exp(i\omega\, t)$, and assuming a response w(t) = $B\ exp(i\omega\, t)$, determine the constant B as function of input amplitude A and frequency ω using the dynamical equations derived in (b).

(8 marks)

(d) Sketch on your answer book and describe the mechanical structure of a simple brushed DC-motor.

(5 marks)

(c) $V(t) = A e^{i\omega t}$ $\omega(t) = B e^{i\omega t}$ $Tm = Te$ $= i\, V(t)$

$\omega = \frac{K - (R) Tm}{(R)(J+b) + K^2}$

$= \frac{K - R\, i\, v(t)}{RJ + Rb + K^2} = \frac{K - A^2 e^{2i\omega t}}{RJ + Rb + K^2}$

End of Paper

$B e^{i\omega t} = \frac{K - A^2 e^{2i\omega t}}{RJ + Rb + K^2}$

$B = \frac{K - A^2 e^{i\omega t}}{RJ + Rb + K^2}$

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1(a) Give five different definitions of "Mechatronics".

(10 marks)

(b) Based on Figure 1, describe in detail what is a typical mechatronics system.

(15 marks)

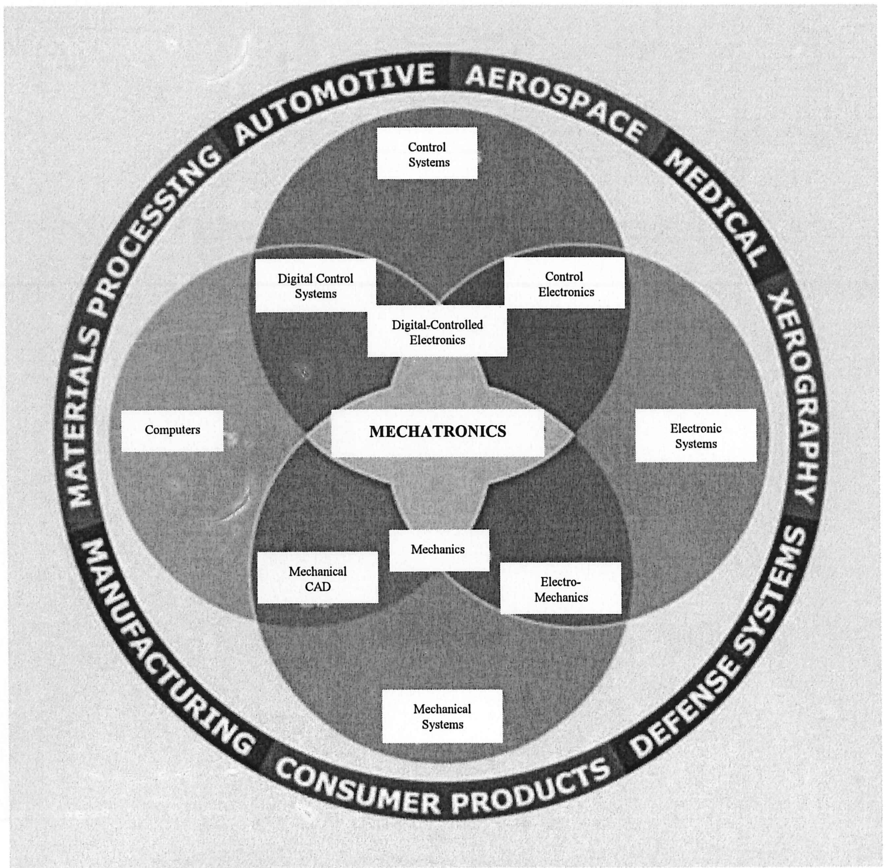


Figure 1: A Mechatronics System.

Mechatronics system is when control systems, computers, electronic systems and mechanical systems are all integrated as one. If one is missing, it will not be a machatronics system and will become another genre of system.

A machatronic system can be used in many fields ranging from aerospace, xerography, defense system, consumer products, manufacturing, material processing and automotive.

2. Figure 2 shows a periodic waveform.

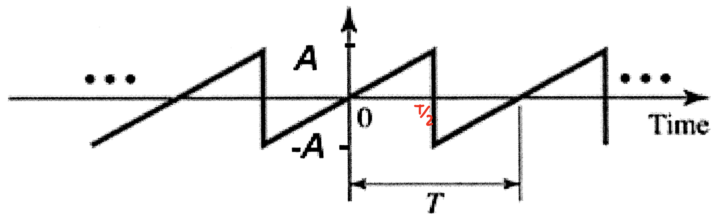


Figure 2: A waveform.

(a) Gradient for $0 \le t \le T/2 = \frac{A-0}{T/2 - 0} = \frac{2A}{T}$

$f(t) = \frac{2A}{T} t$

Gradient for $\frac{T}{2} \le t \le T = \frac{0-(-A)}{T - T/2} = \frac{2A}{T}$

$f(t) = \frac{2A}{T} t + C$

$0 = \frac{2A}{T}(T) + C \Rightarrow C = -2A$

$f(t) = \frac{2A}{T} t - 2A$

$$\therefore f(t) = \begin{cases} \frac{2A}{T} t & \text{for } 0 \le t \le T/2 \\ \frac{2A}{T} t - 2A & \text{for } T/2 \le t \le T \end{cases}$$

(a) Define the function $f(t)$ over the period T.

(5 marks)

(b) Calculate the coefficients A_n and B_n in the Fourier series of the waveform.

(10 marks)

(c) Using the Frequency-Domain representation of the Fourier Series, show the spike diagram of the frequency spectrum.

(10 marks)

(b)

$$f(t) = \begin{cases} \frac{2A}{T} t & \text{for } 0 \le t \le T/2 \\ \frac{2A}{T} t - 2A & \text{for } T/2 \le t \le T \end{cases}$$

$A_n = 0$ (Odd function)

$B_n = \frac{4}{T} \int_0^{T/2} \frac{2A}{T} t \sin(n\omega_0 t)\, dt$

$u = t \qquad \frac{dv}{dt} = \sin(n\omega_0 t)$

$\frac{du}{dt} = 1 \qquad v = -\frac{1}{n\omega_0} \cos(n\omega_0 t)$

$= \frac{8A}{T^2} \int_0^{T/2} t \sin(n\omega_0 t)\, dt$

$= \frac{8A}{T^2} \left[-\frac{t}{n\omega_0} \cos(n\omega_0 t) + \int_0^{T/2} \frac{1}{n\omega_0} \cos(n\omega_0 t)\, dt \right]$

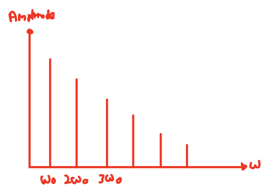
$= \frac{8A}{T^2} \left[-\frac{t}{n\omega_0} \cos(n\omega_0 t) + \frac{1}{(n\omega_0)^2} \sin(n\omega_0 t) \right]_0^{T/2}$

$= \frac{8A}{T^2} \left[-\frac{T^2}{4n\pi} \cos(n\pi) + 0 \right]$

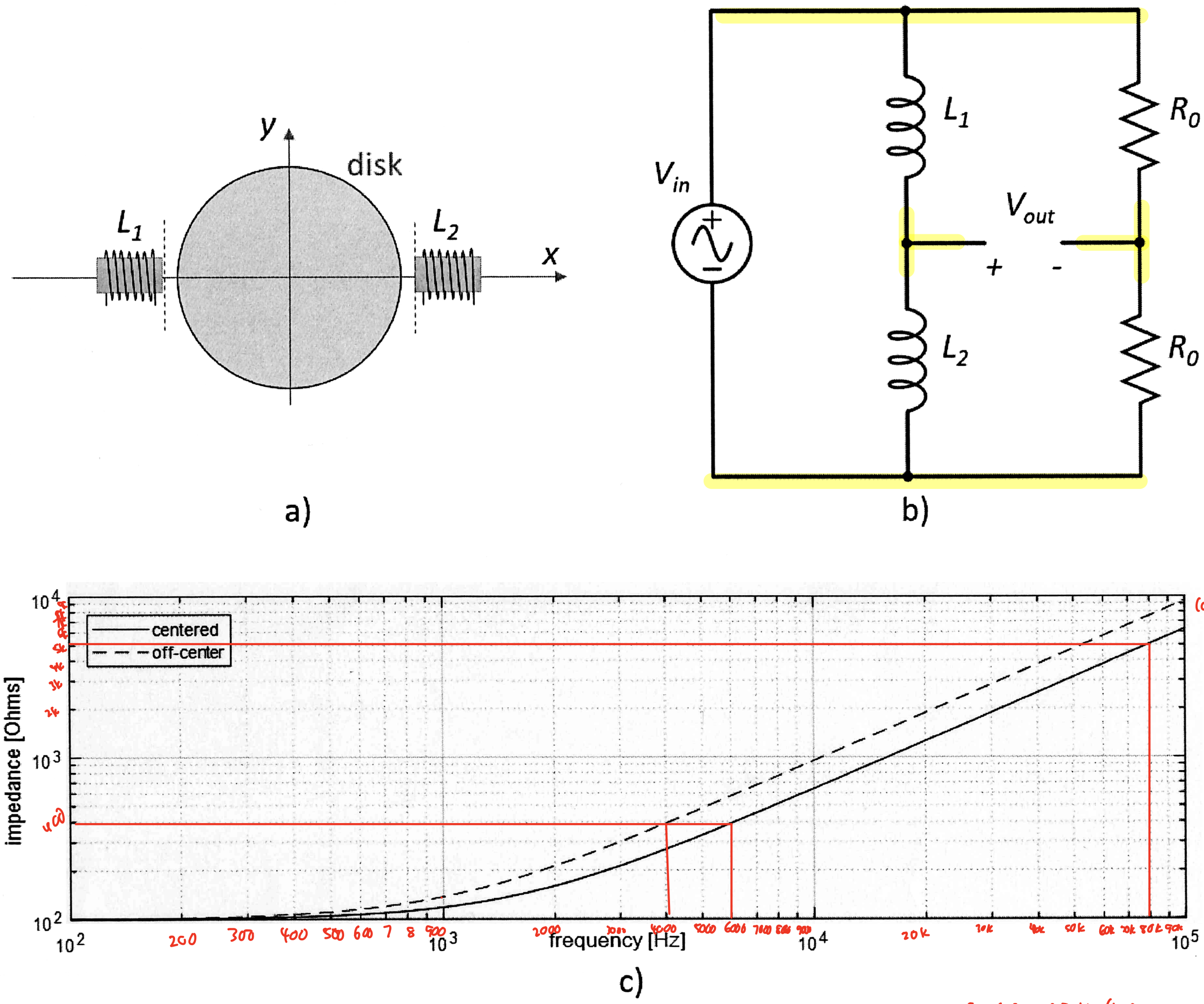
$= \frac{2A}{n\pi} (-1)^{n+1}$

(c) $F(t) = \sum_{n=1}^{\infty} \frac{2A}{n\pi} (-1)^{n+1} \sin(n\omega_0 t)$

$= \frac{2A}{\pi} \sin \omega_0 t - \frac{A}{\pi} (2\omega_0 t) + \frac{2A}{3\pi} \sin(3\omega_0 t)$



3. A solid disk, with its height along the z-direction, has to remain centered at the origin of the X-Y plane, as shown in the cross-sectional view in Figure 3(a). To monitor variations from the centered position, a pair of inductive sensors is used for each axis. Specifically for the x-axis, two inductive sensors L_1 and L_2, are arranged symmetrically, with respect to the disk, as shown in Figure 3(a), and electrically connected to a Wheatstone bridge, as shown in Figure 3(b).



(a) Z centred $= r_0 + L_0 j$

$|Z| = \sqrt{r_0^2 + (\omega L)^2}$

At $f = 6000$ $|Z| : 400$

$160 \times 10^3 = r_0^2 + 1421223034 L^2$

At $f = 80k$, $|Z| = 5k$

$25 \times 10^6 = r_0^2 + (2.52 \times 10^{11}) L^2$

$r_0^2 = 19484.606$

$L^2 = 0.0000988$

$r_0 = 139.587$

$L = 9.943 \times 10^{-3}$

(ii) At $f = 4k$, $|Z| : 400$

$L = \dfrac{\sqrt{|Z|^2 - r_0^2}}{\omega} = 0.149$

$\therefore \Delta L = 4.972 \times 10^{-3}$

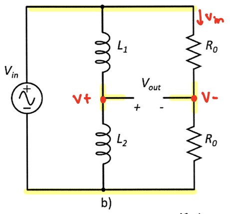
Figure 3

(a) When the disk is perfectly centered along the x-axis, each inductive sensor is characterized by intrinsic resistance r_0 and inductance L_0. However, an inductance change of $L_0 + \Delta L$ is observed when the distance from the disk is off-center. Based on the values of the amplitude response in Figure 3(c), determine

(i) an approximate value for r_0,
(ii) an approximate value for L_0
(iii) an approximate value for ΔL.

(10 marks)

Note: Question 3 continues on page 4.



$V- = \frac{R_o}{R_o+R_o} V_{in} = \frac{1}{2} V_{in}$

$V+ = \frac{r_o + j\omega L_2}{2r_o + j\omega(L_1+L_2)} V_{in} = \frac{r_o + j\omega(L_o+\Delta L)}{2r_o + 2j\omega(L_o+\Delta L+L_o-\Delta L)} V_{in} = \frac{r_o + j\omega(L_o+\Delta L)}{2(r_o+j\omega L_o)} V_{in}$

$V_{out} = V+ - V- = \left[\frac{r_o + j\omega(L_o+\Delta L)}{2(r_o+j\omega L_o)} - \frac{1}{2}\right] V_{in} = \frac{r_o + j\omega(L_o+\Delta L) - (r_o+j\omega L_o)}{2(r_o+j\omega L_o)} V_{in} = \frac{j\omega\Delta L}{2r_o+2j\omega L_o} V_{in}$

(b) Consider the Wheatstone bridge in Figure 3(b), with an AC driving input and with similar bridge resistances (R_0).

(i) Derive **a symbolic expression** of the bridge output V_{out} as a function of r_0, L_0, ΔL and input frequency, assuming that inductance varies linearly with the distance.

(Note: if the inductance of L_2 changes $L_0+\Delta L$, the inductance for L_1 will change as $L_0-\Delta L$)

(5 marks)

(ii)

$\frac{V_{out}}{V_{in}} = \frac{r_o + j\omega(L_o+\Delta L)}{2(r_o+j\omega L_o)} - \frac{1}{2}$

$\lim_{\omega\to 0} \frac{r_o + j\omega(L_o+\Delta L)}{2(r_o+j\omega L_o)} - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0$

$\lim_{\omega\to\infty} \frac{r_o + j\omega(L_o+\Delta L)}{2(r_o+j\omega L_o)} - \frac{1}{2}$

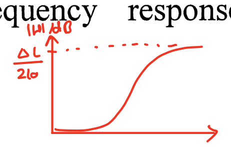
$= \lim_{\omega\to\infty} \frac{\frac{r_o}{\omega} + j(L_o+\Delta L)}{2(\frac{r_o}{\omega} + jL_o)} - \frac{1}{2}$

$= \frac{j(L_o+\Delta L)}{2jL_o} - \frac{1}{2}$

$= \frac{L_o+\Delta L}{2L_o} - \frac{1}{2}$

$= \frac{\Delta L}{2L_o}$ $\therefore$ HP

(ii) Sketch the frequency response of the amplitude $|V_{out}/V_{in}|$ between 100-100,000 Hz.



(5 marks)

(c) Considering the bridge as a filter, with input V_{in} and output V_{out}, determine whether it is a low-pass or high-pass filter and its cut-off frequency in the case of off-centered disk. High pass filter

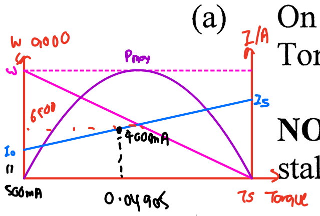
$|H| = \frac{H_o}{\sqrt{2}}$ $\quad \left|\frac{j\omega\Delta L}{2r_o+2j\omega L_o}\right| = \frac{\Delta L}{2\sqrt{2}L_o}$ $\quad \left|\frac{j\Delta L}{2\frac{r_o}{\omega}+2jL_o}\right| = \frac{\Delta L}{2\sqrt{2}L_o}$ $\quad \frac{\Delta L}{\sqrt{4\frac{r_o^2}{\omega^2}+4L_o^2}} = \frac{\Delta L}{2\sqrt{2}L_o}$

$\sqrt{4\frac{r_o^2}{\omega^2}+4L_o^2} = 2\sqrt{2}L_o$ $\quad 4\frac{r_o^2}{\omega^2}+4L_o^2 = 8L_o^2$ $\quad \frac{r_o^2}{\omega^2} = L_o^2 \Rightarrow \omega = \sqrt{\frac{r_o^2}{L_o^2}} = 14038\ rad/s$

(5 marks)

4. A DC motor driven at a nominal voltage V_0 = 10V, is designed to provide a rated torque 0.5 Kg.cm, at a rated speed of 6,500 rpm, drawing a rated current of 4,000 mA. At no load, the motor requires 500 mA of current and its speed reaches 9000 rpm.
ω_o

(a) On your answer book, draw on the same graph the Speed vs. Torque, Current vs. Torque and output Power vs. Torque responses.



NOTE: The rated values refer to one possible (suggested) operating point, not the stall point. You can assume linearity.

(8 marks)

(b) Determine, at the nominal voltage, the maximum power (in Watts) which can be delivered to a mechanical load as well as the torque, speed and efficiency at such optimal operating point.

$P_{max} = \frac{1}{4}(9000 \times \frac{1}{60} \times 2\pi)(0.17658) = 41.61W$

Current @ opt when τ_L,
$I = 6.8A$
$P = 10 \times 6.8 = 68W$

$0.5 kg\ cm \times 9.81 \times \frac{1}{100} = 0.04905 Nm$

(5 marks)

$I = \frac{4000-500}{0.04905}\tau + 500 = 71355.75\tau + 500$ $\quad \omega = \frac{9000-6500}{-0.04905}\tau + 9000 = -50968\tau + 9000$ $\quad \omega = 0 \Rightarrow \tau_s = 0.17658$ $\quad \omega = 471.23\ rad/s$

$\eta = \frac{41.61}{68} \times 100 = 61.2\%$

(c) Based on previous calculations and the information reported above, determine the friction coefficient of the bearings (assuming a linear model) and the amount of friction torque in the no-load conditions.

$n = \frac{\omega T_L}{2v}$ $\quad T_L = \frac{0.6119(10)(0.5)}{300\pi} = 3.24\times10^{-3} Nm$ $\quad T_L = b_L\omega_L \Rightarrow b_L = \frac{T_L}{\omega_L} = 3.44\times10^{-6}$

(7 marks)

(d) Compute the *efficiency* of the motor when operating at the rated speed and when operating at maximum power transfer conditions.

(5 marks)

n :

END OF PAPER

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2016-2017

MA2011 - MECHATRONICS SYSTEM INTERFACING

April/May 2017 Time Allowed: 2½ hours

INSTRUCTIONS

1. This paper contains **FOUR (4)** questions and comprises **THREE (3)** pages.

2. Answer **ALL** questions.

3. All questions carry equal marks.

4. This is a **RESTRICTED OPEN-BOOK** examination. One double sided A4 reference sheet is allowed.

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1. A first-order instrument is used to measure a periodic signal.

(a) What is the magnitude ratio of the instrument in terms of time constant (τ) and angular frequency (ω)? What is the dynamic error?

Magnitude Ratio, $M(\omega) = \frac{1}{\sqrt{1+(\omega\tau)^2}}$ Dynamic error, $\delta(\omega) = 1 - M(\omega)$

(6 marks)

(b) If δ is a dynamic error that the measurement system can be tolerated, determine the maximum frequency (ω_{max}) of a periodic input that can be measured.

$M(\omega) = 1 - \delta(\omega)$ $1+(\tau\omega)^2 = \frac{k}{[1-\delta(\omega)]^2}$ $\omega = \frac{1}{\tau}\sqrt{\frac{k}{[1-\delta(\omega)]^2} - 1}$

$\frac{k}{\sqrt{1+(\tau\omega)^2}} = 1 - \delta(\omega)$

(7 marks)

(c) Assuming that a periodic signal has a single frequency f=50 Hz with sensitivity K=1, estimate the range of the time constant τ given the output amplitude of the signal varies from 50 and 100 units, and the dynamic error to be less than 1%. Discuss the relationship between the time constant, system time response, and dynamic error.

(12 marks)

2. Identify three pairs of operational amplifiers with opposite functions.

(a) Show the three pairs of amplifiers with their names, functions, schematic diagrams and equations.

(15 marks)

(b) Compare each pair of amplifiers.

(10 marks)

3. A capacitive sensor and its relative bridge measurement circuit are represented in Figure 1. The sensor consists of three rectangular and conductive plates of similar length L (along the x-axis) and width W. One moving plate (grey colour in Figure 1) can slide back and forth, along the x-axis, on top of two fixed plates (white colour in Figure 1(a)). When the grey plate is in its 'zero' position (x=0), the system is in a symmetric configuration, i.e. it overlaps equally with both fixed plates (half of the area overlaps). Assume that the capacitance between two plates is proportional to the overlapping area, with a maximum value C0 when their area fully overlaps.

Electrically, this system of capacitances is in a bridge configuration, with two similar resistors R connected to the fixed plates as in Figure 1. The bridge is driven by an AC voltage source V_{in} and the output $V_{out} = V_b$ - V_a is voltage difference between the two fixed plates, as shown in Figure 1(b).

(c) $Ia = \frac{Vin}{R + Ca} = \frac{Vin}{R + \frac{1}{j\omega Ca}}$

$Ib = \frac{Vin}{R + \frac{1}{j\omega Cb}}$

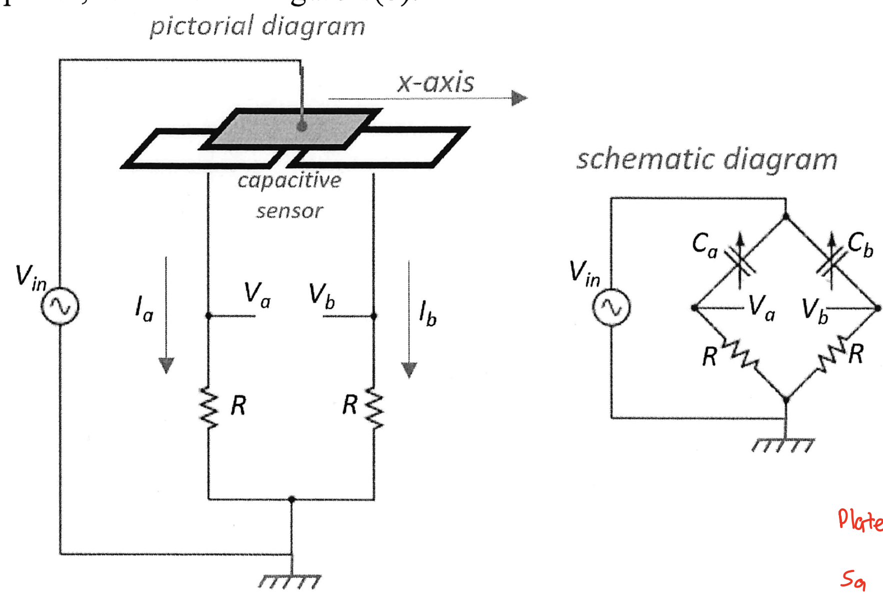


Plate area = LW

$Sa = W(\frac{L}{2}+x)$

$Sb = W(\frac{L}{2}-x)$ #

(a) (b)

Figure 1

(a) Determine the overlapping areas $S_a(x)$ and $S_b(x)$ between the moving plate and, respectively, the left and right plates, as a function of the position x of the moving plate, considering at most a displacement $L/2$ from the zero position.

(5 marks)

(b) Considering that the capacitance between two plates is directly proportional to the overlapping area, with maximum capacitance C0 when the areas fully overlap and with null capacitance whenever two plates are not overlapping, derive analytical expressions and draw, superimposing in the same graph, the capacitances $C_a(x)$ and $C_b(x)$, as functions of the moving plate position x.

(5 marks)

$Ca \propto Sa$

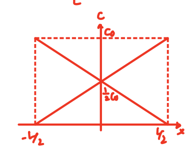
$Cb \propto Sb$

$Ca = Co(\frac{L}{2}+x)$

$Ca = \frac{Co}{L}x + \frac{1}{2}Co$

$Cb = Co(\frac{L}{2}-x)$

$= -\frac{Co}{L}x + \frac{1}{2}Co$



(c) Considering an AC driving input, at generic frequency ω and considering similar resistances R on both sides of the bridge, derive currents $I_a(x)$ and $I_b(x)$ on each side of the bridge.

(7 marks)

(d) Assuming very small displacements ($|x| << L$), determine amplitude and phase of the output voltage $V_{out} = V_b - V_a$, as a function of the moving plate position x.

(8 marks)

4. The characteristics of a commercial DC gearless motor, as found from the datasheet, are listed in the following table:

Nominal Voltage [V]	**Rated Torque [Kg.cm]**	**Rated Speed [rpm]**	**Rated Current [mA]**	**No-Load Speed [rpm]**	**No-Load Current [mA]**	**Rated Output [W]**
12	0.7	5700	5500	7000	900	41.3

(a) On your answer book, draw on the same graph the Speed vs. Torque, Current vs. Torque and output Power vs. Torque responses.

NOTE: the datasheet does not provide stall torque and stall current. The rated values refer to one possible (suggested) operating point. You can assume linearity to derive such values.

(8 marks)

(b) Determine, at the nominal voltage, the maximum power (in Watts) which can be delivered to a mechanical load as well as the torque, speed and efficiency at such optimal operating point.

(5 marks)

(c) Based on previous calculations and the information reported above, determine the friction coefficient of the bearings (assuming a linear model) and the amount of friction torque under no-load conditions.

(7 marks)

(d) Compute the *efficiency* of the motor when operating at the rated speed and when operating at maximum power transfer conditions.

(5 marks)

End of Paper