Design with Microprocessors for Mech Engineers

Converting non-periodic to periodic
→ Duplicate the segment along t-axis
→ Expansion into even function OR odd-function

Lecture 1

Mechatronics: Electronic Control Systems in Mech Eng

Mechatronics Definition

-Stiffler: "Science that integrates mechanical devices with electronic controls"

-Bolton: "Integration of electronics, control engineering and mechanical engineering"

Mechatronics: Electronics in Products & Processes

-Bradley, et al: "An integrating theme within the design process [combining] electronic engineering, computing and mechanical engineering"

-Shetty & Kolk: "Methodology used for the optimal design of electromechanical products"

Mechatronics System Design

-Auslander & Kempf: "The application of complex decision making to the operation of physical systems"

Mechatronics: Mechanical System Interfacing

Measurement Systems

Transducer: a device to convert a physical quantity into a time varying voltage (called analog signal)

Signal Processor: a device to modify the analog signal

Recorder: a device to display or record the signal.

Gain / Proportional constant

Linearity:

$$V_{out}(t) - V_{out}(0) = \alpha(V_{in}(t) - V_{in}(0))$$

$V_{out}(t) = V_{in}(t) + 10t$
→ $V_{out}(t) - V_{out}(0) = (V_{in}(t) + 10t) - (V_{in}(0) + 10*0) = [V_{in}(t) - V_{in}(0)] + 10t$
∴ Non-linear

$V_{out}(t) = V_{in}(t) * V_{in}(t)$
→ $V_{out}(t) - V_{out}(0) = V_{in}(t)*V_{in}(t) - V_{in}(0)*V_{in}(0)$
∴ Mostly non-linear

Fundamental freq. :

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

Fourier Series:

$$F(t) = C_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} B_n \sin(n\omega_0 t)$$

Where,

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$$

$$B_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

$$C_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{A_0}{2}$$

Define:

$$C_n = \sqrt{A_n^2 + B_n^2}$$

$$\phi = -\arctan\left(\frac{B_n}{A_n}\right)$$

Then:

Fundamental term: n = 1
n-th harmonic term: nth term

$$F(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \phi)$$

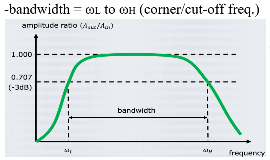
Three Characteristics of a Good Measurement System: Amplitude linearity, phase linearity and adequate bandwidth

Bandwidth & Freq. Response:

$$dB = 20\log_{10}\left(\frac{A_{out}}{A_{in}}\right)$$

-bandwidth ∝ fidelity

-bandwidth = ω_L to ω_H (corner/cut-off freq.)



- -3dB comes from

$$\frac{P_{out}}{P_{in}} = \left(\frac{A_{out}}{A_{in}}\right)^2 = \frac{1}{2}$$

Output amplitude: $A_i' = \left(\frac{A_{out}}{A_{in}}\right) A_i$

Mechatronics & Mechatronics System = Mechanical System + Electronic System + Control System + Computer

$$F(t) = C_0 + \sum_{n=1}^{\infty}(A_n\cos(n\omega_0 t) + B_n\sin(n\omega_0 t))$$

$$= C_0 + \sum_{n=1}^{\infty}\sqrt{A_n^2 + B_n^2}\left(\frac{A_n}{\sqrt{A_n^2+B_n^2}}\cos(n\omega_0 t) + \frac{B_n}{\sqrt{A_n^2+B_n^2}}\sin(n\omega_0 t)\right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n(\sin(\phi_n')\cos(n\omega_0 t) + \cos(\phi_n')\sin(n\omega_0 t))$$

$$= C_0 + \sum_{n=1}^{\infty} C_n(\sin(n\omega_0 t + \phi_n'))$$ for sine only

$$\phi_n' = \arctan\left(\frac{A_n}{B_n}\right), \sin(\phi_n') = \frac{A_n}{\sqrt{A_n^2+B_n^2}}, \cos(\phi_n') = \frac{B_n}{\sqrt{A_n^2+B_n^2}}$$

Lecture 2

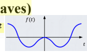
Periodic Function

$f(t+T) = f(t)$, T is period.

Even Function (Fully cosine waves)

$f(-t) = f(t)$ $A_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega_0 t)\,dt = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)\,dt$

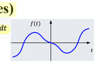
All Bn = 0



Odd Function (Fully sine waves)

$f(-t) = -f(t)$ $B_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_0 t)\,dt$

All C0 and An = 0



Complex Form

$$F(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

Where,

$$D_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}dt$$

Spectrum & Measurement

-important to estimate spectrum when choosing measurement

-ideal system replicates freq. of an input signal

-practical system limited

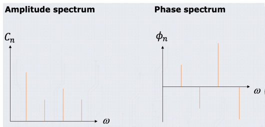
Error of Approximation

$$f(t) \cong \sum_{n=-N}^{N} D_n e^{jn\omega_0 t} = S_N(t)$$

$$\varepsilon(t) = f(t) - S_N(t)$$

$$MSE = \frac{1}{T}\int_0^T \varepsilon^2(t)dt$$

-MSE minimum when Dn = Fourier coefficients



Fourier Series Conditions:

Capacitor 1/jwc
Inductor jwL

-f(t) is a single-value function

-for any t_0, $\int_{t_0}^{t_0+T}|f(t)|\,dt < \infty$

-f(t) has a finite number of discontinuities

-f(t) has a finite number of maxima and minima

-f(t) amplitude function, fulfil 4 conditions

Insights

-how signals of diff. freq. represented in a signal

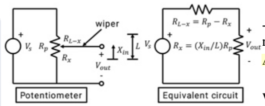
-easier & cost-eff. To characterise freq. content instead of time description of noise

-diff. treatment of diff. parts of EM spectrum help you separate radio, tv and cell phone signals

Lecture 3

-system that is fast to respond + good to understand → good method, highly desired

Zero Order System



$$V_{out} = \frac{R_x}{R_p}V_s = \frac{V_s}{L}X_{in}$$

$X_{out} = KX_{in}$, K is gain/sensitivity

Characteristic Equation of Homogeneous equation of Linear system

$$\sum_{n=0}^{N} A_n S^n = 0$$

Homogeneous equation of Linear system:

$$\sum_{n=0}^{N} A_n \frac{d^n X_{out}}{dt^n} = 0$$

FOURIER SERIES REPRESENTATION OF SIGNALS

$$F(t) = C_0 + \sum_{n=1}^{\infty}(A_n\cos(n\omega_0 t) + B_n\sin(n\omega_0 t))$$

$$= C_0 + \sum_{n=1}^{\infty}\sqrt{A_n^2+B_n^2}\left(\frac{A_n}{\sqrt{A_n^2+B_n^2}}\cos(n\omega_0 t) + \frac{B_n}{\sqrt{A_n^2+B_n^2}}\sin(n\omega_0 t)\right)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n(\cos(\phi_n)\cos(n\omega_0 t) - \sin(\phi_n)\sin(n\omega_0 t))$$

$$= C_0 + \sum_{n=1}^{\infty} C_n\cos(n\omega_0 t + \phi_n)$$ for cosine only

$$\phi_n = -\arctan\frac{B_n}{A_n}, \cos(\phi_n) = \frac{A_n}{\sqrt{A_n^2+B_n^2}}, \sin(\phi_n) = -\frac{B_n}{\sqrt{A_n^2+B_n^2}}$$

Underdamped system (ζ<1, with 2 complex conjugate roots)

$$s_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$
$$s_2 = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$$

Two conjugate roots: $s_1 = \alpha + j\beta$, $s_2 = \alpha - j\beta$

$(C_1\sin(\beta t) + C_2\cos(\beta t))e^{\alpha t}$

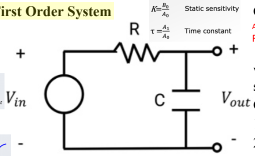
Homogeneous solution

$$x_h(t) = e^{-\zeta\omega_n t}\left[A\cos\left(\omega_n\sqrt{1-\zeta^2}t\right) + B\sin\left(\omega_n\sqrt{1-\zeta^2}t\right)\right]$$

First Order System

$K = \frac{B_0}{A_0}$ Static sensitivity

$\tau = \frac{A_1}{A_0}$ Time constant



$$RC\frac{dV_{out}}{dt} + V_{out} = V_{in}$$

$$\tau\frac{dX_{out}}{dt} + X_{out} = KX_{in}$$

$$X_{out} = X_{out}(0)e^{-t/\tau} + KA_{in}(1 - e^{-t/\tau})$$

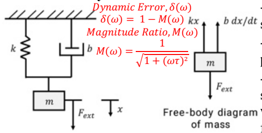
$$X_{out} = KA_{in}(1 - e^{-t/\tau})$$

-1 time constant → 63.2% τ = time constant

-4 time constants → >98% , assume steady state

Second Order System

Dynamic Error, $\delta(\omega)$
$\delta(\omega) = 1 - M(\omega)$
Magnitude Ratio, $M(\omega)$
$M(\omega) = \frac{1}{\sqrt{1+(\omega\tau)^2}}$



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_{ext}(t)$$

When $F_{ext} = 0$: $A_2\frac{d^2X_{out}}{dt^2} + A_1\frac{dX_{out}}{dt} + A_0X_{out} = B_0X_{in}$

When $\zeta = 0$,

$$x_h(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$$

When $\zeta < 1$, Under Damped

$$x_h(t) = e^{-\zeta\omega_n t}[A\cos(\omega_d t) + B\sin(\omega_d t)]$$
$$= Xe^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$$

When $\zeta > 1$, Over Damped

$$x_h(t) = e^{-\zeta\omega_n t}[A\cos(\omega_d t) + B\sin(\omega_d t)]$$

When $\zeta = 1$, Critically Damped

$$x_h(t) = (A + Bt)e^{-\omega_n t}$$

Lecture 4

Why Digital?

-more compact storage using diff. media

-more accurate

-large dataset

-use real-time control system

-enable data processing

-sampling rate high, accuracy high

TO SAMPLE FAST
Huge amt of data,
Need high speed hardware,
Large storage

-analog signal: continuous, generated via analog devices, not coded, original signal

-digital signal: discrete, sampled in a fixed interval, coded, sequential data array

Sampling

When encounting for sampling freq f_s → always take the MAX FREQ SIGNAL f_{max}

Shannon-Nyquist Theorem:

$$f_s > 2f_{max}$$

where, Sampling time → time btw 1st & next sample; Sampling rate/sampling freq → 1/sampling time

f_s is sampling rate, f_{max} is max. freq. component (Nyquist freq.)

Time interval:

$$\Delta t = 1/f_s$$

-under-sampled signal, less than $2f_s$, can result in wrong digital signal (aliasing)

Aliasing eqn.: Closest integer multiple of sampling rate (fs)

$$f_a = |f_s * i - f_n|$$ Signal being aliased (fn)

Where i is the integer multiple of f_s such that f_s * i is nearest to the signal being aliased (f_n)

-if u want waveform, sample at rate at least 10 times the Nyquist freq.

Quantising & Coding

Pulse code modulation: sampling, quantisation, binary encoding

Methods:

Pulse amplitude modulation (PAM)

Ideal, natural, flat top

Sample and hold

An impulse at each sampling instant

A pulse of short width with varying amplitude

Before we sample, we must filter the signal to limit the maximum freq of the signal as it affects the sampling rate.

Filtering should not distort the signal.

Quantizing: Transformation of continuous analogue input into a set of discrete output states
Coding: Assignment of a digital code word or number of output states

Quantization Size/Zone Width:

Also known as Resolution

$$Q = (V_{max} - V_{min})/N$$
$$N = 2^n$$

Available in mkt: 8-bit, 12-bit, 14-bit, 16-bit

where, n is number of bits, N is number of states.

Error = Resolution
% Error = (Resolution / 0.5 Voltage-range) * 100

Quantizing & Encoding Process

1. Determine no. of zones & zone width

2. Assign midpoints of zones a value from 0 to N-1

3. Assign binary code to each zone (find no. of bits needed)

Quantising error:

-diff. btwn. actual and midpt.

-zones ↑, Q ↓, error ↓, bits required ↑, bitrate↑

Ideal waveform = Quantized waveform + errors

Lecture 5

-SST: charge carriers move thru a solid semicon. Material

-VTT: bulky tubes enclosed a gas at low pressure thru which electronics flowed

-VTT drawbacks: heavy power consumers, significant heat dissipation, large size, heavy weight, require frequent battery replacement for portable units

-SST strengths: small size, portable using a rechargeable battery, low reight, cool-running, energy saving

-SST→ John Bardeen, Walter Brattain, William Shockley

-VTT → Sir Joseph John Thomson

Why Op-Amps?

Choosing amp: size, cost, power assumption, accuracy, input/output Z, gain, bandwidth

-transducer signals too small

-transducer signals too noisy

-transducer signals contain wrong information and have DC offset due to design and installation of transducers

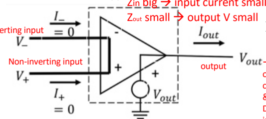
-low cost, versatile IC, consists of internal transistors, resistors & capacitors

-combine w/ ext. discrete components → wide variety of signal processing circuits

Impedance

$$Z_{in} = \frac{V_{in}}{I_{in}},\quad Z_{out} = \frac{\Delta V_{out}}{I_{in}}$$

Z_{in} big → input current small, Z_{out} small → output V small



Ideal Op-Amp:

-infinite input impedance, $I^+ = I^- = 0$

-infinite gain, $V^+ = V^-$

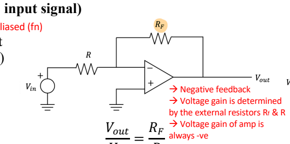
-zero output impedance, Vout independent of Iout

-Inverting input (-), non-inverting input (+)

- gain,

$$A_v = \frac{V_{out}}{V_{in}}$$ $V_{in} = V(+) - V(-)$

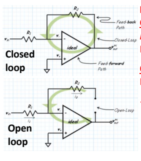
Inverting Amplifier (invert & amplify input signal)



→ Negative feedback
→ Voltage gain is determined by the external resistors Rf & R
→ Voltage gain of amp is always -ve

$$\frac{V_{out}}{V_{in}} = \frac{R_F}{R}$$

Ideal Model for Op Amp

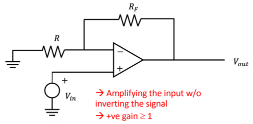


Closed Loop Configuration (closed loop" refers to loop formed by the feed-forward path and the feedback path of amplifier.) Feedback present, Stabilization of amp, Control of gain

Open Loop Configuration
Feedback absent, Considerable instability due to high gain, seldom used.

→ When choosing which Op Amp
- Input/output impedance
- Bandwidth
- Power rating
- Require only a single-output (unipolar) power supply
- Data sheets

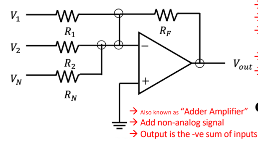
Non-Inverting Amplifier (amplifies input signal without inverting)



→ Amplifying the input w/o inverting the signal
→ +ve gain ≥ 1

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_F}{R}$$

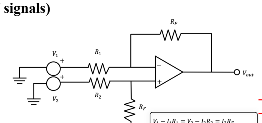
Summer Amplifier (add analog signals)



→ Also known as "Adder Amplifier"
→ Add non-analog signal
→ Output is the -ve sum of inputs

$$V_{out} = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2 + \frac{R_F}{R_N}V_N\right)$$

Difference Amplifier (subtract analog signals)

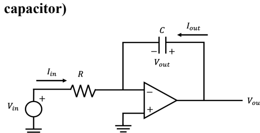


If $R_1 = R_2 = R$,

$$\boxed{V_{out} = \frac{R_F}{R}(V_2 - V_1)}$$

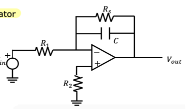
$$V_{out} = V_1 - (R_F + R_1)\left(\frac{V_2}{R_1} - \frac{V_2}{R_F+R_2}\frac{R_F}{R_1}\right)$$

Integrator (replace feedback resistor w/ capacitor)



$$V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(\tau)d\tau$$

Practical Integrator



Practical integrator
→ R_2 is an approx. of the parallel combination of R_1 & R_F to minimize DC offset due to input current bias

$$C\frac{dV_{out}}{dt} + \frac{V_{out}}{R_s} = I_{out} = -I_{in} = -\frac{V_{in}}{R_1}$$

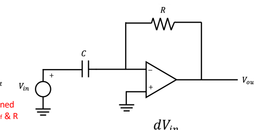
Current (i) for capacitor

$$\frac{dV_{out}}{dt} + \frac{V_{out}}{CR_s} = -\frac{V_{in}}{CR_1}$$

Should choose:

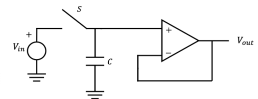
$$R_S > 10R_1,\quad R_2 = \frac{R_1R_s}{R_1+R_s}$$

Differentiator (input resistor of inverting op amp replaced by capacitor)



$$V_{out} = -RC\frac{dV_{in}}{dt}$$

Sample & Hold



When S is closed,

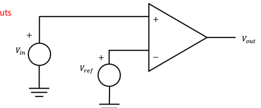
$$V_{out}(t) = V_{in}(t)$$

When S is open,

$$V_{out}(t - t_{sampled}) = V_{in}(t_{sampled})$$

→ Choose capacitor C w low leakage.
→ Extensively used in analog-to-digital (ADC) conversion.
→ Signal value must be stabilized while it is converted to a digital representation.
→ Consisting of voltage holding capacitor & voltage follower.
→ With switch closed.

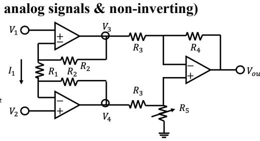
Comparator



$$V_{out} = \begin{cases} +V_{sat}, & V_{in} > V_{ref} \\ -V_{sat}, & V_{in} > V_{ref} \end{cases}$$

→Vsat = saturation voltage
→ To determine whether one signal is greater than other.
→ The comparator is an example of an op amp circuit where there is no -ve feedback & the circuit exhibits infinite gain.
→ Results in op amp saturates.
→ Saturation implies that the output remains at its most +ve or most -ve output value.

Instrumentation Amplifier (subtracting analog signals & non-inverting)



$$V_{out} = \frac{R_5(R_3+R_4)}{R_3(R_3+R_5)}V_4 - \frac{R_4}{R_3}V_3$$

→ Difference amplifier may be satisfactory for low-Z source, but its input Z is too low for high-output impedance source.
→ If the levels of the input signals are very low and the signals include noise, the difference amplifier is unable to extract a satisfactory difference signal.
→ Instrumentation amplifier is a solution for this problem.

→ Very high input impedance.
→ Large common mode rejection ratio (CMRR): Ratio of the difference mode gain to the common mode gain.
→ The difference mode gain is the amplification factor for the difference between the input signals.
→ The common mode gain is the amplification factor for the average of the two input signals.
→ For an ideal difference amplifier, the common mode gain is 0, implying an infinite CMRR.
→ Desirable to minimize the common node gain to suppress signals such as noise that are common to both inputs.
→ Capability to amplify low-level signals in a noise environment, often a requirement in differential-output sensor signal-conditioning application.
→ Consistent bandwidth over a large range of gains.

Lecture 6

Successive approx. for ADC

-pros: high spd. and good reliability, medium accuracy compared to other ADC types, good tradeoff btwn. spd. and cost, capable of outputting binary number in serial

-cons: higher resolution → ADC slow, speed limited to 5 MSPS

Flash ADC

-pros: v. fast, v. simple operational theory, spd. only limited by gate + comparator propagation delay

-cons: expensive, prone to glitches in output, each additional bit of resolution requires twice the comparators

Sigma-Delta ADC

-pros: high res, no need for precision components

-cons: slow, due to oversampling, only good for low bandwidth

Type	Speed (relative)	Cost (relative)
Dual-Slope	Slow	Med
Flash	Very Fast	High
Successive Approximation	Medium Fast	Low
Sigma-Delta	Slow	Low

relative quantization error is given by

$$\frac{\max e_Q}{|E|_{ADC-input}} = \frac{Q}{GA}$$ X100%

$$\text{Cutoff frequency}, \omega_c = \frac{1}{RC}rad/s$$

$RC = \frac{1}{\omega_c}s$; NOTE THAT $\omega_c = 2\pi f_c$

$$\text{Frequency response}, M(f) = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$$

$$\text{Static Sensitivity}, K = \frac{\Delta V_{out}}{\Delta V_{in}}$$

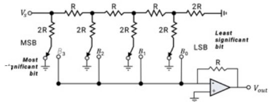
DAC

-finite word length

-loudest sounds here med room, normal sounds don't use entire range

-problems occur at low levels where sounds are represented by 1/2 bits, high distortions

-dithering adds low level broadband noise



WHAT IS MECHATRONICS?

First introduced in 1969 by Yasakawa Electric, Japan

Applied Mechatronics (Samill and Mrad)

"Interdisciplinary engineering field comprising the design and development of smart electromechanical systems."

Introduction to Mechatronics & Measurement Systems (Alciatore & Histand)

"The interdisciplinary field of engineering dealing with the design of products whose function relies on the integration of mechanical and electronic components coordinated by a control architecture."

Mechatronics (Harshama, Tomizuka & Fukuda)

"The synergistic integration of mechanical engineering with electronics and electrical systems with intelligent computer control in the design and manufacture of industrial products, processes, and operations."

Mechatronics - A Foundation Course (de Silva)

"The synergistic application of mechanics, electronics, and computer engineering in the development of electromechanical products and systems through an integrated design approach."

Mechatronics - Electromechanics and Contromechanics (Mill)

"Crossdisciplinary [field] ... that simultaneously involves mechanics, electronics, and computer-integrated electromechanical systems"

University Of Washington (US)

"The integrated study of the design of systems and products in which computation, mechanization, actuation, sensing, and control are designed together to achieve improved product quality and performance."

Rensselaer Polytechnic Institute (US): "The synergistic combination of mechanical engineering, electronics, control systems and computers, all integrated through the design process."

Loughborough University (UK): "Mechatronics is a design philosophy that utilizes a synergistic integration of mechanics, Electronics and Computer Technology (or IT) to produce enhanced products, processes or systems."

Johannes Kepler University Linz (AUSTRIA): "Technical systems operating mechanically with respect to at least some kernel functions but with more or less electronics supporting the mechanical parts decisively."

University Of California At Berkeley (US): "A flexible, multi-technological approach in the integration of Mechanical Engineering, Electronics, Computer Engineering, and Information Sciences"

Mechatronics (Journal): "The synergistic combination of precision mechanical engineering, electronic control and systems thinking in the design of products and manufacturing processes"

ME Magazine: "The synergistic use of precision engineering, control theory, computer science, and sensor and actuator technology to design improved products and processes." "simply the application of the latest techniques in precision mechanical engineering, controls theory, computer science, and electronics to the design process to create more functional and adaptable products."

Industrial Research and Development Advisory Committee of the European Community: "Synergetic combination of precision engineering, electronic control and systems thinking in the design of products and manufacturing processes."

INPUT

Physical quantity to be measured is called input to a measurement system.

OUTPUT

Transducer transforms the input into a form compatible with the processor to be processed then becomes the output measurement system.

DIFFERENCE

Usually, the input is different from the output of a measurement system.

CHARACTERISATION

A good measurement system is characterised by: phase linearity, amplitude linearity, and adequate bandwidth.

COMPLEX FORM OF FOURIER SERIES

$e^{jn\omega_0 t} = \cos(n\omega_0 t) + j\sin(n\omega_0 t)$ $\cos(n\omega_0 t) = \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}$

$e^{-jn\omega_0 t} = \cos(n\omega_0 t) - j\sin(n\omega_0 t)$ $\sin(n\omega_0 t) = \frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}$

$D_0 = A_0/2$ $D_n = \frac{A_n - jB_n}{2}$, $D_{-n} = \frac{A_n + jB_n}{2}$

$A_n\cos(n\omega_0 t) + B_n\sin(n\omega_0 t) = D_n e^{jn\omega_0 t} + D_{-n}e^{-jn\omega_0 t}$

Amplitude Linearity

1) If not amplitude linearity, it is difficult to interpret the output

2) Linear response of a measurement system usually holds for a limited range of the input rate.

3) An ideal measurement system exhibits amplitude linearity for any input amplitude & input rate.

Conversion from Non-periodic to Periodic

Offset the original pattern along the time axis by a distance nT (τ<T)

[EVEN FUNCTION] Mirroring the original one against an axis t = τ.

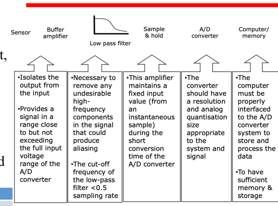
[ODD FUNCTION] Mirroring the original one against the time axis and then the axis t= τ.

Characteristics Equation	$\sum_{n=0}^{N} A_n S^n = 0$	$A_N \neq 0$	1	Single real root: $s_1 = r$	C_1e^{rt}
			2	Double real roots: $s_1 = s_2 = r$	$(C_1+C_2t)e^{rt}$
			2	Two different real roots: $s_1 \neq s_2$	$C_1e^{s_1t} + C_2e^{s_2t}$
Homogeneous Equation	$\sum_{n=0}^{N} A_n\frac{d^nX_{out}}{dt^n} = 0$	Root	2	Two conjugate roots: $s_1 = \alpha + j\beta$, $s_2 = \alpha - j\beta$	$(C_1\sin(\beta t) + C_2\cos(\beta t))e^{\alpha t}$
		Root	N	General solutions for Homogeneous Equation	

Linear System	Homogeneous Equation
$\tau\frac{dX_{out}}{dt} + X_{out} = KX_{in}$	$\tau s + 1 = 0$

General Solutions of Linear System

Root	$r = -1/\tau$
General solutions for Homogenous Equation	$X_{out} = C_0e^{-t/\tau}$ (Where C_0 is a constant determined later by applying initial conditions)
A particular or steady state solution resulting from the Step Input $X_{in} = A_{in}$	$X_{out} = KA_{in}$
General solutions for Linear System	$X_{out} = X_{out} + X_{out} = C_0e^{-t/\tau} + KA_{in}$



$\Delta V = V_{ref} / 2^n$

- ΔV: Resolution
- n: Number of bits in digital output
- 2^n: Number of states
- V_{ref}: Reference voltage range

The Conversion Time

→ Setting time depends on design of convertor, method for conversion, speed of the components used.

→ Analog signal changes continuously (uncertainty when conversion occurs), causing corresponding uncertainty in the digital value.

→ If there is no sample & hold amp on the A/D input, then this is important.

The Aperture Time

Duration of the time window & is associated w any error in the digital output due to changes in the input during this time. The r/s btw the aperture time and the uncertainty in the input amplitude.

$$\Delta V(t) = \frac{dV(t)}{dt}\Delta T_a$$

In general, the time response of a 1st order system is
$x(t) = a + b\,e^{-t/\tau}$ where τ is the time constant.

[C] = [K] - 273.15
[F] = 1.8*[C] + 32

** short circuit → V = 0

voltage → torque
current → speed
inductance → inertia
resistor → damping
capacitor → compliance
electrical power → mechanical power
mechanical parallel → electrical series

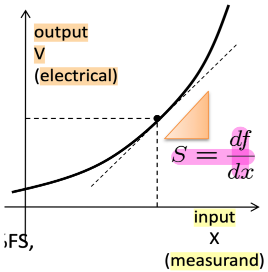
Sensors
-transducer: convert one form of energy to another
-sensors: produce (electrical) output signal for sensing a physical phenomenon

Classification
-analog vs digital
-passive (no ext. power, draw from input signal) vs active
-null (deflection due to measured quantity is balanced) vs deflection
-subject of measurement: mechanical, optical, thermal etc.

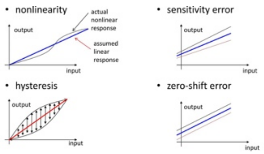
Instrumentation Systems
-sensing module: electro + mechanical/thermal/optical/pyro/piezo
-conversion module: analog → digital
-pre-processing: variable manipulation module
-data transmission: wired/wireless, over the web
-presentation/storage: to final user

Basic Concepts

$$v_{out} = f(x)$$



-I/O characteristic function/response: input→ stimulus or measurand (temp, pressure, strain)
Output → electrical signal (voltage, current, freq, phase)
-sensitivity S: output variation/input variation, S = df/dx
-resolution: minimum change of the measurand that can be reliably detected, limited by noise, bit-conversion
-accuracy: difference of measurement from true value, %FS, full scale
-repeatability: how well a system/device can reproduce an outcome in unchanged situation



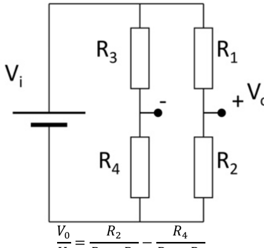
Resistance Temperature Detector (RTD)
-based on changes of resistance w/ temp.
-metal wire on insulating support → eliminate mechanical strain
-encasing→minimize environment influence
-linear for limited range:

$$\frac{R}{R_0} = 1 + \alpha(T - T_0)$$

temp. coefficient

Platinum
→ ±0.3% over the range 0-200 [C]
→ ±1.2% over the range 200-800 [C]

Wheatstone Bridge



$$\frac{V_0}{V_i} = \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}$$

Bridge balance:

$$V_0 = 0 \Leftrightarrow R_1R_4 = R_2R_3$$

Thermistors
-ceramic like semicons.
-resistance decreases w/ temp.
-high sensitivity, ruggedness, fast time-response

$$R = R_0 e^{\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)}$$ temp. in KELVIN!!!

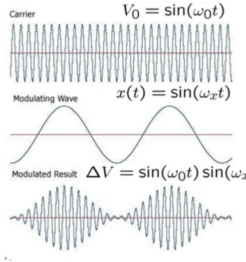
Resistive Sensors (Potentiometer, pot)
-3 terminal electro-mech. device based on a conductive wiper against fixed resistive element
-many varieties: rheostats, trimmers, volume control
-precision potentiometers: manually/digitally tuneable

Linear Variable Differential Transformer (LVDT)
-type of transformer
-measures linear displacement
-variable coupling via sliding ferromagnetic core
-differential voltage:

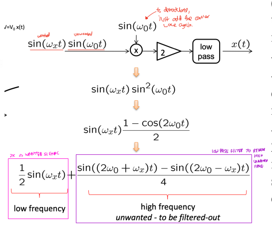
$$V_{out} = \Delta V = V_2 - V_1 \approx x(t)V_0$$

LVDT: Amplitude Modulation

$$\Delta V \approx \sin(\omega_0 t)\sin(\omega_x t)$$



LVDT: Amplitude Demodulation

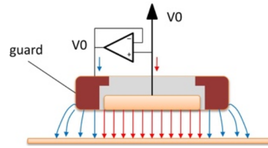


Capacitive Sensor (proximity sensor)
-C=Q/V
-ideal case: infinite parallel plates

Gauss' Law: $Q = \iint_\Sigma \epsilon_0\epsilon_r \boldsymbol{E} d\boldsymbol{S}$

$$C = \frac{Q}{V} = \frac{\epsilon_0\epsilon_r S}{d}$$

Guard Electrode
-limit field-fringing effects



AC Interfacing
-AC bridge, AC driving, modulation

Hall Effect
-Lorentz Force:

$$\vec{F} = q\vec{v} \times \vec{B}$$

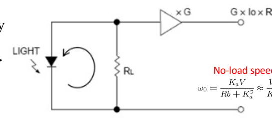
-Lorentz's Law:

$$\vec{F} = (\vec{\imath} \times \vec{B})L \quad F = |\vec{F}| = BiL$$

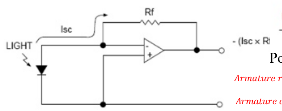
-used for contactless proximity sensor, current sensor

Light Detectors
→ Phototransistors
1. Transmissive: photo-interrupter
2. Reflective: resistive load
3. Current-voltage op amp

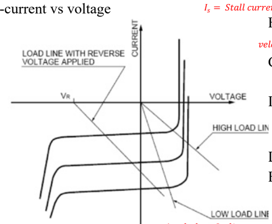
-photo-resistors
-photo-diodes: load resistance



-photo-diodes: op-amp

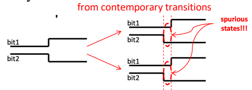


-current vs voltage

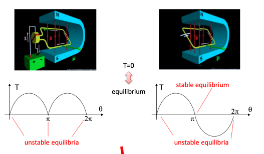


Encoders Resolution = $\frac{\text{Angle btw 2 slits}}{\text{No. of gear teeth}}$
-digital encoders: convert motion (linear/rotary) → digital pulses (optical transmitter/receiver pairs – glass/plastic material photographically patterned)
-Hall effect sensors + magnetic rings/bars
-incremental encoders w/ 2 Tx/Rx pairs – encoding steps + direction; simple design, needs a 'reset' position
min
Quadrature signals (1/4 cycle out of phase)
-absolute encoders n Tx/Rx pairs for coding 2^n sectors; more expensive; spurious states may arise from contemporary transitions solved using gray code as only bit changes every time

CAVEAT: spurious states may arise from contemporary transitions



• with commutation • without commutation



Motors
-EM induction using LHR:

$$emf = \mathcal{E} = -\frac{d\Phi}{dt},\ \Phi \triangleq \int_\Sigma \vec{B}d\vec{\Sigma}$$

-Faraday's Law:
emf = electro-motive-force
Φ = magnetic flux
Σ = surface whose boundary coincides w/ the coil

DCM Structure
-stator: external, fixed
-rotor: internal, rotates
-stator and rotor fields always orthogonal → maximum torque
-w/ commutation: maintains unstable equilibria → constant motion
-w/o commutation: there is a stable equilibria → motion ceases
-more poles, more constant torque i.e., independent of rotor positions

Equations
Armature Eqn:

$$V = Ri + L\frac{di}{dt} + e$$

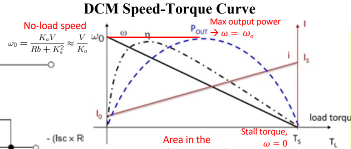
Mechanical eqn:

$$J\dot{\omega} + b\omega = T_e - T_L$$

Coupling:

$$\begin{cases} T_e = K_t i \\ e = K_e\omega \end{cases}; \quad T_e\omega = ei;\ K_e = K_t \triangleq K_a$$

DCM Speed-Torque Curve



Power: $P_{out} = \omega T_L$

Armature resistance, $R = \frac{V}{I_s}$

Armature constant, $K_a = \frac{T_s}{I_s}$

$$P_{out}^{max} = \frac{1}{4}\omega_0 T_s$$

I_s = Stall current, T_s: Stall torque

Friction load:

velocity, $v = r\omega$ $\qquad T_L = b_L\omega_L$

Constant torque: Nonlinear T_L = $f\omega$

$$T_L = const.$$

Inertial:

$$T_L = I\frac{d\omega}{dt} = 0$$ @ steady state

$$\boxed{\omega = \frac{K_a V - RT_L}{Rb + K_a^2}}$$

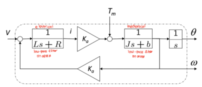
Loads can be a combination of above 3 ↑
Efficiency:

$$\eta = \frac{power\ out}{power in} = \frac{\omega T_L}{VI}$$ Form 2 linear eqn to find ω & I from the graph.

Driving DCM
-avoid power amp. bcuz large power dissipation + over-heating of amp.
-continuously switch the motor on and off, Pulse Width Modulation (PWM)
-DCM is a 2nd order low pass filter

DCM: block-box modeling

$$\begin{cases} V = Ri + L\frac{di}{dt} + K_a\omega \\ J\frac{d\omega}{dt} + b\omega = T_m + K_a i \end{cases} \Rightarrow \begin{cases} V - K_a\omega = (R + Ls)i \\ (Js + b)\omega = T_m + K_a i \end{cases}$$

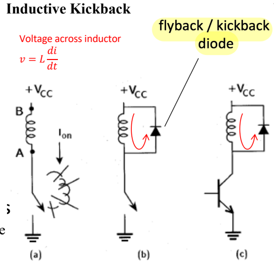


$$\omega = \frac{K_a V - (R + Ls)T_m}{(Ls + R)(Js + b) + K_a^2}$$

$Duty\ Cycle = \frac{T_{ON}}{T}$;
$T_{ON} = Time\ on, T = Period$

Inductive Kickback

Voltage across inductor
$v = L\frac{di}{dt}$



Steady-state I_{on} cannot immediately go to 0 when the switch is opened. Changing current induces a voltage across the inductor, making potential at A > B, causing the switch to blow up. Kickback/flyback diode protects the switch (physical/transistor) from blowing up.

DC Brushless Motors
-stator field rotating
-rotor field given by permanent magnet
-keep rotor and stator field orthogonal to maximize output torque
-detect rotor position via encoders (Hall effect sensors)
-select appropriate switches to determine desired torque

Strain Gauges
-determine safe loading conditions of mechanical structures

Electrical Resistance Strain Gauges
-thin metal (constantan) foil, patterned onto plastic backing material, bonded onto mech. structures, stress derived using solid mech.
-strain, S=dL/L, +ve if tensile, -ve if compressive
-Poisson's ratio: $\upsilon = \frac{lateral\ strain}{axial\ strain}$ $\upsilon_{steel} = 0.3$, $\upsilon_{rubber} = 0.5$
-resistance, $R = \rho\frac{L}{A}$ A = width x height
-rectangular conductor:

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \left(\frac{dw}{w} + \frac{dh}{h}\right)$$

-axial strain, S =dL/L
-lateral strain:

$$\frac{dw}{w} = \frac{dh}{h} = -\upsilon\frac{dL}{L} = -\upsilon S$$

Combining above eqns.,

$$\frac{dR}{R} = \frac{d\rho}{\rho} + S + 2\upsilon = \left(\frac{d\rho}{\rho}\frac{1}{S} + 1 + 2\upsilon\right)S$$

$$\frac{dR}{R} = \mathcal{G}S$$

FYI:

$$piezoresistivity = \frac{d\rho}{\rho}\frac{1}{S}$$

Transverse Sensitivity

$$dR = R_0\mathcal{G}S$$

dR → change in resistance
R_0 → resistance
S → strain
$\mathcal{G}$ → gauge factor

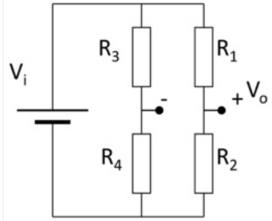
-the larger R_0, the larger dR
-long and thin wires allow R_0, wires must be aligned w/ axial strain S^a
-serpentine wires
-end loops: aligned w/ transverse axis, made thicker to reduce sensitivity to S^t

Materials
-best materials: constantan, ferrous alloys
-typical strain ranges S: 1-10^4 μS, G: 2
-challenge: detecting small resistance changes

• gauge factor:

$$\boxed{\mathcal{G} := \frac{dR}{R}\frac{1}{S} = \frac{1}{R}\frac{\partial R}{\partial S} = \frac{d\rho}{\rho}\frac{1}{S} + 1 + 2\upsilon}$$

Wheatstone bridge: 1st order approx.:



$$\frac{dV_o}{V_i} = \frac{1}{4}\left(\frac{dR_2}{R_2} - \frac{dR_1}{R_1} + \frac{dR_3}{R_3} - \frac{dR_4}{R_4}\right)$$

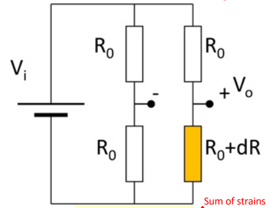
top of positive terminal
across R2 (or wtv it may be)

$$\frac{V_o}{V_i} = \frac{1}{4}\mathcal{G}(S_{R2} - S_{R1} + S_{R3} - S_{R4})$$

btm of positive terminal

2-wire connection → temp dependent, unbalancing effects
3-wire connection → balanced bridge, attenuated gauge factor

Quarter-bridge



$$\frac{dV_o}{V_i} \approx \frac{1}{4}\frac{dR}{R_0} = \frac{1}{4}\mathcal{G}S$$

Sum of strains S_a, S_b, S_t

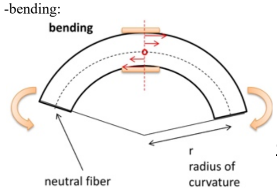
Poisson Gauge

Mounted in close thermal contact & bonded

$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(1 + \upsilon)S$$

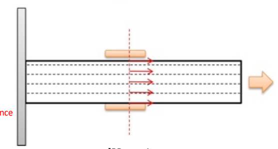
Dummy gauge
mounted in close thermal contact but not bonded

Half Bridge
-two active strain gauge
-bending:



$$\frac{dV_o}{V_i} = \frac{1}{2}\mathcal{G}S^b$$

-axial loading:



$$\frac{dV_o}{V_i} = \frac{1}{2}\mathcal{G}S^a$$

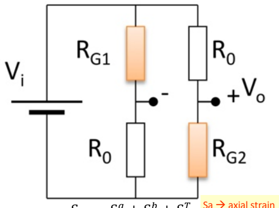
Apparent Strain:
-combinations of different mechanical loading
-e.g. beam under bending and axial loading:
Top of beam:

$$S_1 = S^a + S^b$$

Bottom of beam:

$$S_1 = S^a - S^b$$

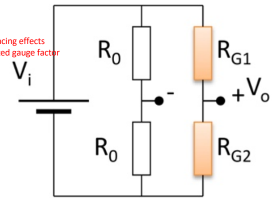
Different electrical configuration:



$$S_{G1} = S^a + S^b + S^T$$
$$S_{G2} = S^a - S^b + S^T$$

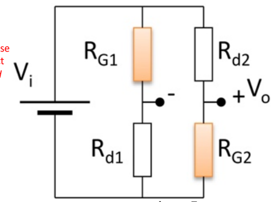
Sa → axial strain
Sb → bending strain
St → temp. strain

$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(S_{G1} + S_{G2}) = \frac{1}{2}\mathcal{G}(S^a + S^T)$$



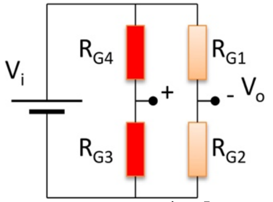
$$S_{G1} = S^a + S^b + S^T$$
$$S_{G2} = S^a - S^b + S^T$$

$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(-S_{G1} + S_{G2}) = \frac{1}{2}\mathcal{G}S^b$$



$$S_{G1} = S^a + S^b + S^T$$
$$S_{G2} = S^a - S^b + S^T$$
$$S_{d1} = S_{d2} = S^T$$

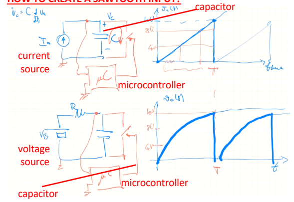
$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(S_{G1} - S_{d1} + S_{G2} - S_{d2}) = \frac{1}{2}\mathcal{G}S^a$$



$$S_{G1} = S_{G3} = S^a + S^b + S^T$$
$$S_{G2} = S_{G4} = S^a - S^b + S^T$$

$$\frac{dV_o}{V_i} = \frac{1}{4}\mathcal{G}(S_{G1} - S_{G2} + S_{G3} - S_{G4}) = \mathcal{G}S^b$$

HOW TO CREATE A SAWTOOTH INPUT?



Strain, $S = \frac{\Delta L}{L}$
ΔL = extension,
L = original length

$E = \frac{\sigma}{S}$, $\sigma = \frac{Force}{Area}$
σ = Stress,
S = Strain

Strain, $S = \frac{F_e}{AE}$
F_e = Force,
A = Area,
E = Young's Modulus

+ve → compression
-ve → extension

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Choose u in this order: LIATE
Logs
Inverse
Algebraic
Trig
Exponential