NANYANG TECHNOLOGICAL UNIVERSITY

School of Mechanical and Aerospace Engineering

E3.1 DYNAMIC OR TUNED VIBRATION ABSORBERS (TVAS)

MECHANICS OF MACHINES LAB

Venue: N3-B1c-03

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Grade: ____________________________

NOTE: This title page should be attached to all required material for this experiment before submission.

E3.1 DYNAMIC OR TUNED VIBRATION ABSORBERS (TVAs)

SUMMARY

I) Area of Study
Mechanical vibrations

II) Learning Objectives

Upon successful completion of this experiment, the students will be able to:

1) Understand working principles of dynamic or Tuned Vibration Absorbers (TVAs)
2) Use TVAs to effectively reduce vibration of machineries operating at constant RPM.
3) Use vibration meters and tachometers, and make accurate vibration measurements.

a. Theoretical Principles and Concepts
Harmonic vibration of two degrees-of-freedom system
Resonance
Dynamic or tuned vibration absorbers

b. Experimental Techniques
Vibration isolation of a system by tuning a dynamic vibration absorber.

c. Instrumentation
Vibration meter
Digital tachometer
Frequency tachometer
Micrometer

Terminolgoies:

Natural Frequency: Natural frequency is the frequency at which a system naturally vibrates once it has been set into motion. In other words, natural frequency is the number of times a system will oscillate (move back and forth) between its original position and its displaced position, if there is no outside interference. For example, consider a simple beam fixed at one end and having a mass attached to its free end. If the beam tip is pulled downward, then released, the beam will oscillate at its natural frequency. Natural frequency is a property of the system, which depends on the system stiffness and its mass.

Circular Frequency: Circular frequency is shown with parameter, ω, which has units of rad/s or revolution per minute (RPM) and it is equal to, $\omega=2\pi f$, where f is frequency in Hertz (Hz).

E3.1 DYNAMIC VIBRATION ABSORBER

1 INTRODUCTION

All systems, which have mass and elasticity are capable of vibrating in one form or another. One of the important characteristics of such systems, under dynamic conditions, is the phenomenon of resonance. Resonance is the tendency of a system to oscillate at maximum amplitude at certain frequencies, known as the system's resonance frequencies (or resonant frequencies)

A machine supported by springs, may be characterised as a single-degree-of-freedom system, i.e. one single co-ordinate is sufficient to describe its motion. If the machine system is subjected to the excitation of a harmonic force, it will have a violent and objectionable vibration at a particular frequency, called natural frequency, when the frequency of the input harmonic force matches the machine's natural frequency. To overcome this difficulty, a dynamic or tuned vibration absorber (TVA) which is a spring mass system, can be attached to the machine system in order to eliminate the machine vibration. The use of the dynamic or tuned vibration absorber changes the single-degree-of-freedom system into a two-degree-of-freedom system.

2 OBJECTIVES

The objective of this experiment is to study how the vibration of a machine, particularly at resonance can be eliminated or minimised by means of a tuned vibration absorber. The tuned vibration absorber (tuned to a particular frequency) starts to vibrate at its tuned natural frequency; therefore absorbing the vibration energy and reducing the vibrations of the machine.

3. THEORY

3.1 Notation

m_1, m_2	Mass of machine, auxiliary mass
k_1, k_2	Isolator spring stiffness, vibration absorber spring stiffness
P_0	Harmonic force amplitude
ω	Harmonic forcing frequency
t	Time
x_1, x_2	Displacement of machine, displacement of auxiliary mass

3.2 Vibration of a machine coupled to an absorber

Consider a machine of mass $\boldsymbol{m_1}$ being coupled to an auxiliary mass m_2 and spring stiffness $\boldsymbol{k_2}$ as shown in Figure 1. The machine is subjected to a harmonic force $\boldsymbol{P_o \sin \omega t}$.

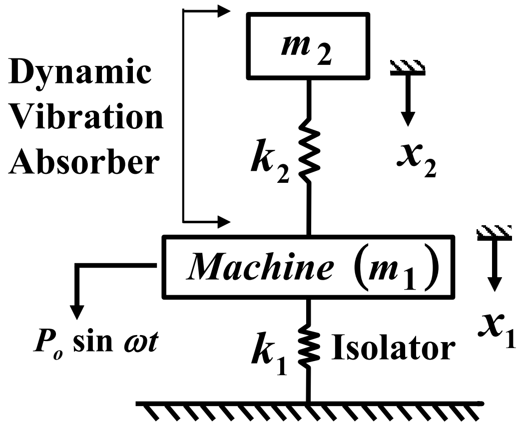

Figure 1 Dynamic or Tuned Vibration Absorber

The equations of motion are

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = P_o \sin \omega t \tag{1}$$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) = 0 \tag{2}$$

By assuming steady state solutions of the form

$$\left.\begin{aligned} x_1 &= X_1 \sin \omega t \\ x_2 &= X_2 \sin \omega t \end{aligned}\right\} \tag{3}$$

The amplitudes may be shown to be

$$\left.\begin{aligned} X_1 &= \frac{P_o (k_2 - m_2 \omega^2)}{\Delta} \\ X_2 &= \frac{P_o k_2}{\Delta} \end{aligned}\right\} \tag{4}$$

where: $\Delta = (k_1 + k_2 - m_1 \omega^2)(k_2 - m_2 \omega^2) - k_2^2$
and $\Delta = 0$ is the frequency equation

Thus, if a machine of mass m_1 and stiffness k_1, having a natural frequency of $\sqrt{\frac{k_1}{m_1}}$ is subjected to a harmonic force with a circular frequency ω, the resulting oscillation of that system can be reduced to zero by adding a second system (mass m_2 and stiffness k_2) tuned so that $\sqrt{\frac{k_2}{m_2}}$ becomes equal to ω, the frequency of the applied force i.e. $\left(k_2 - m_2 \omega^2\right) = 0$. The auxiliary system is called a Dynamic or Tuned Vibration Absorber.

By making the substitution

$$\omega_1^2 = \frac{k_1}{m_1} \qquad \omega_2^2 = \frac{k_2}{m_2}$$

into equation (4), it can be shown that the amplitude, X_1, is given by

$$\frac{X_1 k_1}{P_o} = \frac{1-\left(\frac{\omega}{\omega_2}\right)^2}{\left[1+\frac{k_2}{k_1}-\left(\frac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]-\frac{k_2}{k_1}} \tag{5}$$

From equation (5), if $\frac{\omega}{\omega_2} = 1$ or $\omega = \omega_2$

then $\frac{X_1 k_1}{P_o} = 0$ or $X_1 = 0$

The amplitude of the tuned vibration absorber system is $X_2 = -\frac{P_o}{k_2}$. The natural frequencies of the composite system are found when X_1 and X_2 are infinite. Since both X_1 and X_2 have the same denominator, these natural frequencies can be obtained be equating the denominator of equation (5) to zero, i.e.

$$\left(1+\frac{k_2}{k_1}-\left(\frac{\omega}{\omega_1}\right)^2\right)\left(1-\left(\frac{\omega}{\omega_2}\right)^2\right)-\frac{k_2}{k_1} = 0 \tag{6}$$

They lie on either side of $\omega = \omega_2$

3.3 Vibration of the absorber system

Consider the cantilever rod of length L, with a lumped mass M at its end, as shown in Figure 2.

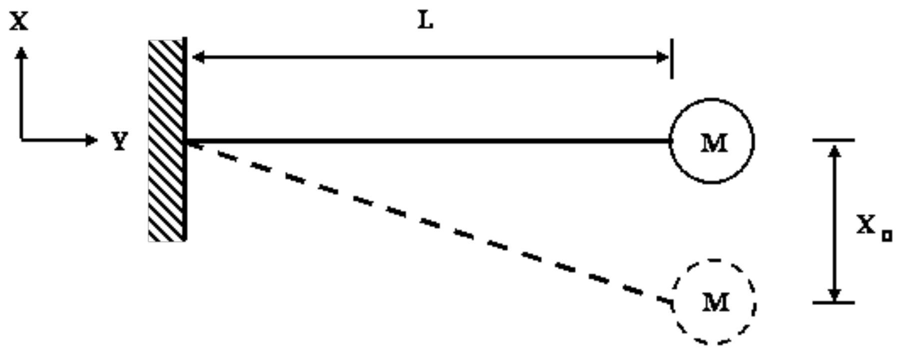

Figure 2 A cantilever rod with a lumped mass

Let x_o be the maximum deflection at its end, then the deflection, x, is given by

$$x = \frac{x_o}{2L^3}\left(3Ly^2 - y^3\right)\sin \omega t \tag{7}$$

The maximum kinetic energy

$$KE = \frac{1}{2}\omega^2\left(\int_o^L \rho A x^2 dy + Mx_0^2\right)$$

$$= \frac{1}{2}\omega^2\left[\frac{33\rho AL}{140} + M\right]x_0^2$$

The maximum strain energy of the above system is

$$SE = \frac{1}{2}\int_o^L EI\left[\frac{d^2x}{dy^2}\right]^2 dy = \frac{1}{2}EI\int_0^L\left(\frac{x_o}{2L^3}\right)^2(6L-6y)^2 dy$$

$$= \frac{1}{2}\left(\frac{3EI}{L^3}\right)x_o^2$$

Therefore, the natural frequency, ω_n, is given by letting $KE = SE$

$$\Rightarrow \quad \omega_n = \left[\frac{3EI}{\left(M + \frac{33\rho AL}{140}\right)L^3}\right]^{\frac{1}{2}} rad/s \tag{8}$$

3.4 Spring stiffness of the primary system

Figure 3 shows the primary system being deflected by a small angle θ from its equilibrium position by the load M. For static equilibrium, taking moment about C gives:

$$(Mg)L = k(L\theta)L + k(L\theta)L$$

$$\therefore \quad \theta = \left(\frac{g}{2kL}\right)M$$

But for small θ, $\quad y = L\theta$

$$\therefore \quad y = \left(\frac{g}{2k}\right)M \tag{9}$$

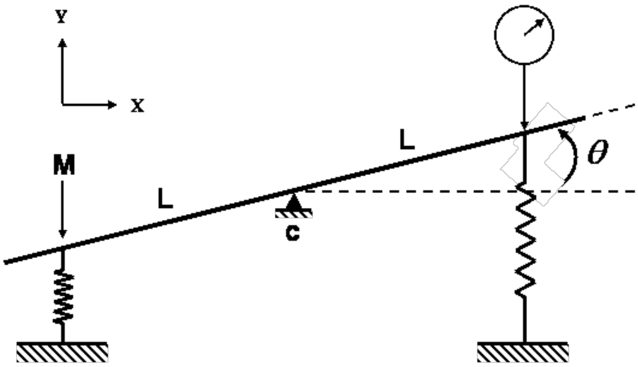

Figure 3 Compression spring being deflected under load M

4. EQUIPMENT

4.1 Equipment List

Tuned vibration absorber test rig, comprising a variable speed DC motor, a shaft supporting an unbalanced rotating disc, and a fin supported by two identical compressive springs, is given. The following equipments will be used throughout the test.

- Rion vibration meter
- Ono Sokki digital tachometer
- Koyo frequency tachometer
- Micrometer (dial indicator)
- Various masses (1-6 kg)
- Steel rod with end mass

(a)

(b)

Figure 4 (a) Rion vibration meter and (b) Ono Sokki digital tachometer

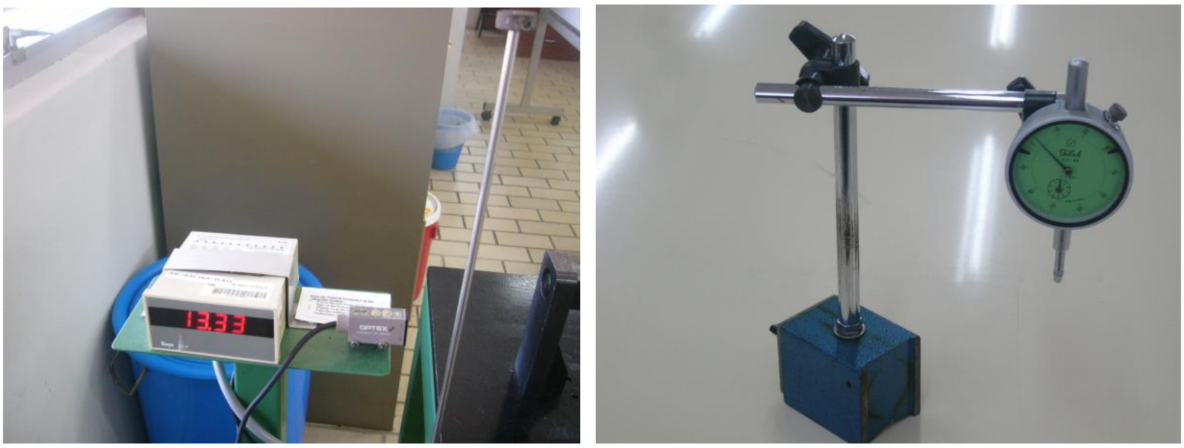

Figure 5 (a) Koyo Frequency tachometer (b) Dial indicator

Figure 6 Steel rod with end mass (TVA)

4.2 Equipment technical data

Compression spring stiffness k	=	11200 N/m
Absorber rod diameter	=	6 mm
Absorber rod length	=	410 mm
Young's Modulus of steel	=	201 GPa
Density of steel	=	7860 kg/m^3
Lumped Mass	=	23 gm

4.3 Experimental Setup

Figure 7 shows the experimental set-up. In this test set-up, a variable DC motor is rotating a disk. The speed of the DC motor can be controlled. A bolt and a nut are added to the disk to create the imbalance. As the DC motor rotates the imbalanced disk, a periodic disturbance input is created. This periodic disturbance input causes the bar which is sitting on two compression springs to vibrate. When the input disturbance frequency (motor speed) matches the natural frequency of the bar supported on two compression springs, resonance occurs.

Figure 7 Experimental test set-up without the TVA

Figure 8 Experimental test set-up without the TVA

5. EXPERIMENT PROCEDURE

5.1 Operating manual

- E3.1 Operating Manual
- Rion Vibration Meter User Manual

5.2 Spring stiffness of the primary system

a) Set up the micrometer (or dial indicator) at point B, as shown in Figure 3.

b) Place 1 kg load at A and measure the deflection on the micrometer. Gradually increase the loads in 1 kg steps up to 6 kg. Record the corresponding deflections.
c) Repeat the experiment in (b) with the micrometer placed at A and loading at B.

5.3 Dynamic displacement of the primary system

a) Place the magnetic transducer at point B.
b) Increase the speed of the d.c. motor gradually and record its speed. Record the corresponding displacement from the vibration meter.
c) Take a complete set of readings for a range of motor speeds, up to 1800 *rev*/min. Closer spacing of readings in the region of resonance should be taken.

5.4 Dynamic displacement of the composite system

a) Tighten the steel rod with lumped mass onto the beam supporting the rotating disc.
b) Take a full set of readings as 5.3.

5.5 Natural frequency of the absorber system

a) Tighten the steel rod with its lumped mass onto the frame supporting the rig.
b) Displace the rod from its equilibrium position and let go.
c) Observe the ensuing oscillations and take the reading on the frequency tachometer.

6. RESULTS

6.1 Plot the graph of spring deflection against load and determine the average spring stiffness for the system.
6.2 Plot the dynamic displacement against speed curves for the primary system and the composite system.

7. DISCUSSION

a) Comment on the spring stiffnesses obtained experimentally.
b) What are the resonant frequencies of the primary and composite systems? Hence, comment on the effectiveness of the dynamic absorber.
c) Calculate the theoretical natural frequency of the absorber and compare it with the experimental result.
d) Any other comments.

8. REFERENCES

Den Hartog, J.P. 1956. Mechanical Vibrations. 4th Ed., New York, McGraw-Hill.

Timoshenko, S., Young, D.H. and Weaver Jr., W., 1974. Vibration Problems in Engineering. 4th Ed., New York, John Wiley.

Thomson, W.T., 1988. Theory of Vibration with Applications. 3rd Ed. New Jersey, Prentice Hall.

Hunt, J.B., 1979. Dynamic Vibration Absorbers. Bury St. Edmunds, MEP Ltd.

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EXPERIMENT E3.1: DYNAMIC OR TUNED VIBRATION ABSORBER

LOG SHEET

Name: Trevor Chiang Kai Teng

Date: 8 March 2022 Time: AM / PM

Submit To: Prof Shu Dong Wei

Spring stiffness of primary system

Dial gauge at point A			Dial gauge at point B		
Micrometer δ_o(mm) = 0			Micrometer δ_o(mm) = 0		
M (kg)	δ (mm)	$\delta - \delta_o$(mm)	M (kg)	δ (mm)	$\delta - \delta_o$(mm)
1	0.42	0.42	1	0.36	0.36
2	0.84	0.84	2	0.725	0.725
3	1.28	1.28	3	1.12	1.12
4	1.7	1.7	4	1.530	1.530
5	2.165	2.165	5	1.950	1.950
6	2.63	2.63	6	2.390	2.390

Plot a graph from the above table.

Dynamic displacement of primary system

ω (rpm)	x (mm)	ω (rpm)	x (mm)
514	0.02	1098	1.28
557	0.03	1142	1.28
599	0.04	1201	0.84
660	0.070	1256	0.60
713	0.09	1290	0.48
746	0.1	1362	0.370
795	0.14	1410	0.330
868	0.22	1452	0.3
901	0.25	1507	0.270
941	0.3	1546	0.25
1004	0.53	1625	0.23
1045	0.61	1738	0.2

Dynamic displacement of composite system

ω (rpm)	*x* (mm)	ω (rpm)	*x* (mm)
564	0.04	1253	0.53
635	0.07	1300	1.27
716	0.21	1355	1.27
755	1.28	1429	0.86
808	0.05	1490	0.54
875	0.08	1562	0.39
910	0.1	1621	0.33
988	0.13	1687	0.29
1041	0.12	1758	0.26
1096	0.15	1824	0.24
1138	0.2	1889	0.21
1185	0.29	2056	0.2

Plot a graph from the above tables.

Natural frequency of absorber system.

No.	ω_n (Hz)
1	15.57
2	15.57
3	15.47
Average:	15.54

Discussion

a) Comment on the spring stiffness of the system.

Since $y = \left(\frac{g}{2k}\right)m$

*(Graph's at the last page)

Gauge at Point A, $\frac{g}{2k} = \frac{y}{m}$ = Gradient = 0.4413 mm/kg (Taken from graph)

$\therefore k_A = 11114.89\,N/m$

Gauge at Point B, $\frac{g}{2k} = \frac{y}{m}$ = Gradient = 0.4067 mm/kg (Taken from graph)

$\therefore k_B = 12060.49\,N/m$.

Average Experimental $k = \frac{k_A + k_B}{2} = 11587.69\,N/m$

As observed above, the experimental k value is relatively close to the given theorectical value of K = 11200 N/m. Therefore, the values of K obtained are sufficiently accurate.

b) What are the resonant frequencies of the primary and composite systems? Hence comment on the vibration absorber's effectiveness.

Primary System: $\omega = \frac{(1098+1142)}{2}$ (At peak of graph at $x \approx 1.28$)

$= 1120rpm$ (18.67Hz)

Composite System: $\omega = 755rpm$ (12.58Hz) (At peak of graph at $x = 1.28$)

$\omega = \frac{(1300+1355)}{2}$

$= 1327.5rpm$ (22.125Hz) (At peak of graph at $x \approx 1.28$)

The vibration absorber is effective for a machine that runs approximately at 1120rpm as it decreases its vibrations to an amplitude of 0.179mm. (through interpolation)

Discussion

c) Calculate the theoretical natural frequency of the vibration absorber and compare it with the experimental value. Comment on your result.

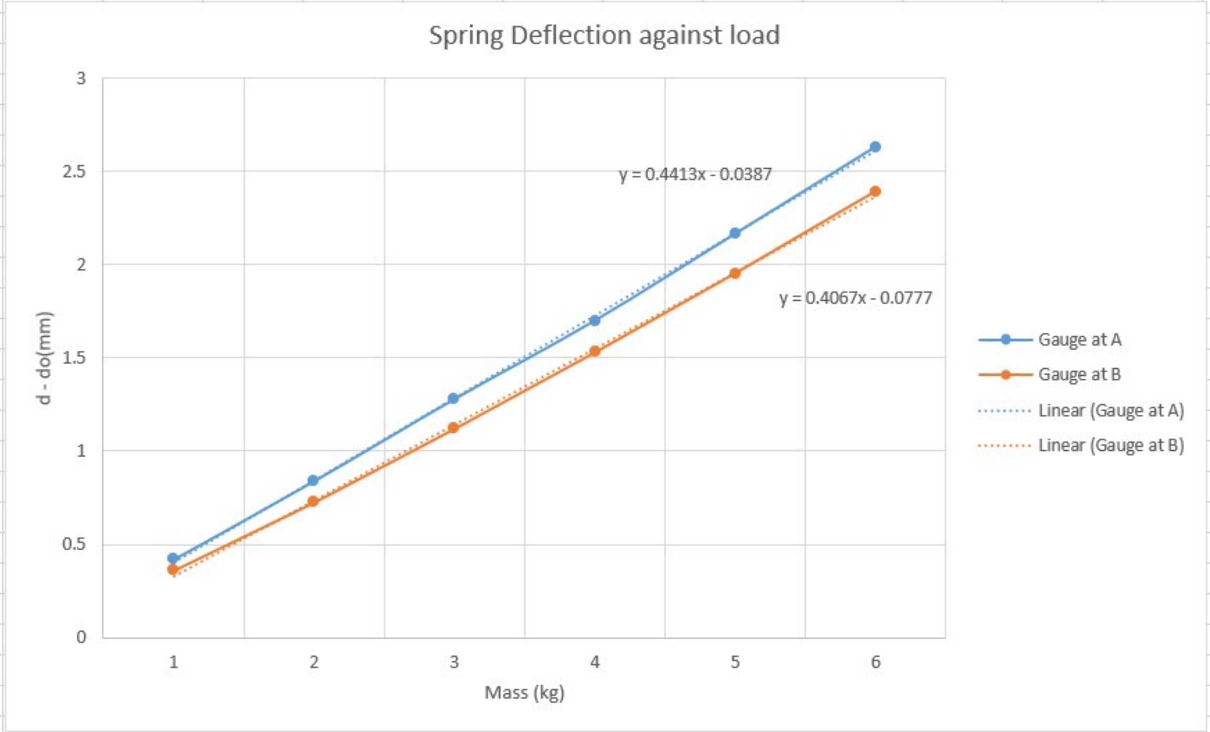
$$\omega_n = \left[\frac{3EI}{\left(M + \frac{33\rho AL}{140}\right)L^3} \right]^{1/2} \text{ rad/s}$$

$$= \sqrt{\frac{3 \times (201\times10^9)\left(\frac{0.006}{2}\right)^4\left(\frac{\pi}{4}\right)}{\left(\frac{23}{1000} + \frac{33(7860)\left(\frac{0.006^2\pi}{4}\right)(0.41)}{140}\right)(0.41)^3}}$$

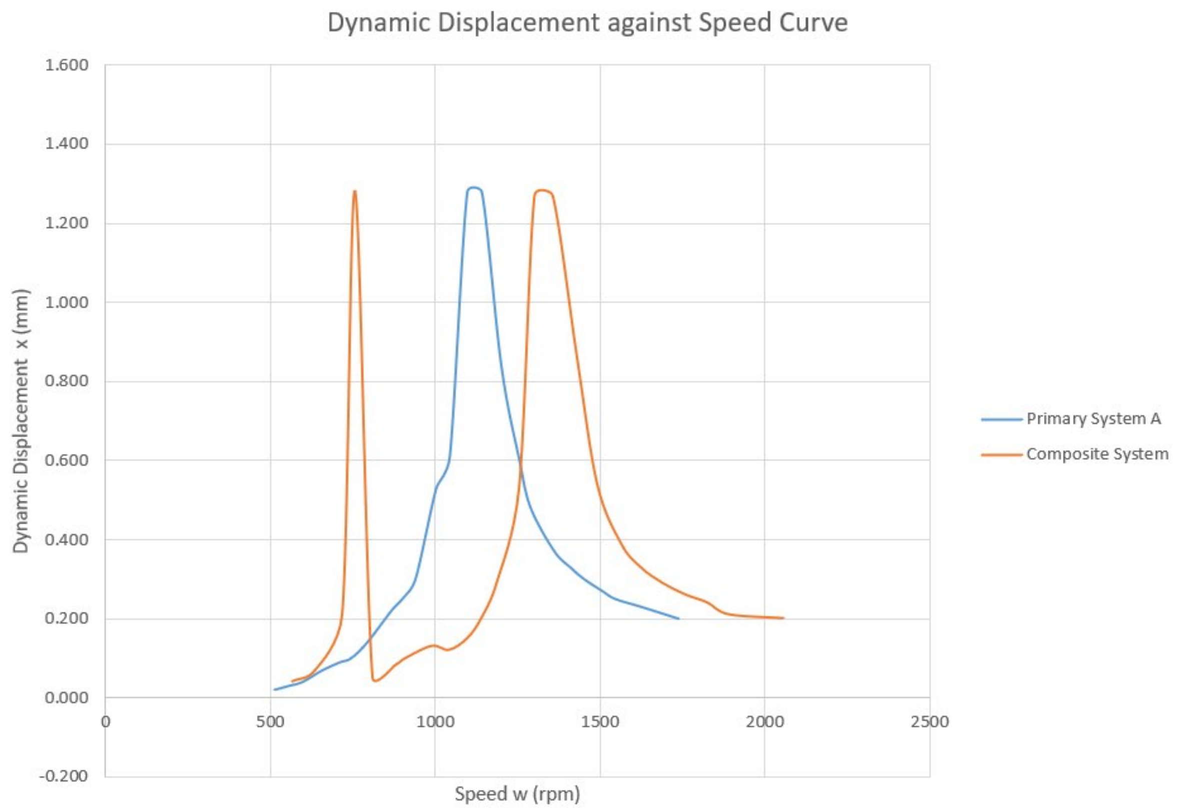
$= 111.87$ rad/s

$= 17.8$ Hz

∴ The theorectical natural frequency is higher than the experimental value. The reason for this is that there is environmental factors that affect the experimented values.



Spring Deflection against load
y = 0.4413x - 0.0387
y = 0.4067x - 0.0777
d - do(mm)
Mass (kg)
Gauge at A
Gauge at B
Linear (Gauge at A)
Linear (Gauge at B)



Dynamic Displacement against Speed Curve
1.600
1.400
1.200
1.000
0.800
0.600
0.400
0.200
0.000
-0.200
0
500
1000
1500
2000
2500
Dynamic Displacement x (mm)
Speed w (rpm)
Primary System A
Composite System