1. Introduction to Control Theory & Laplace Transform

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MA3005/MA3705 - Control Theory, Teaching Plan, Sem 1, 2025-26

Lecture Venue: LT1, Tuesdays, Time: 1:30 pm to 3:20 pm			Tutorials	
Lecture	**Lecture Dates (Tuesdays)**	**Lecture Topics**	**Week No**	**Tutorials**
1	12th Aug 2025	Introduction to Systems and Laplace Transformation, Block Diagrams	1	
2	19th Aug	Block Diagrams, Mathematical Modelling of Mechanical & Dynamic Systems	2	Tutorial 1
3	26th Aug	Mathematical Modelling of Mechanical & Dynamic Systems	3	Tutorial 2
4	2nd Sept 2025	Systems Response and Stability	4	Tutorial 3
5	9th Sept	Systems Responses : First-Order and Second-Order Systems	5	Tutorial 4
6	16th Sept	Basic Control Actions, Process Controllers	6	Tutorial 5
CA1	**23rd Sept**	**CA 1 – Quiz (20 %), 23rd Sept , Tuesday, 2:00 pm to 3:00 pm , Venue –LT 1. Detailed arrangement will be announced at a later.**	**7**	**CA1**

MA3005 Control Theory

The aims of this module are to:

- Introduce feedback systems and the concepts of block diagrams and transfer functions and different types of controllers
- Introduce the concept of stability and performance criteria of feedback systems
- Explain the concept of root locus and its application to classical control design
- Introduce frequency response and Bode diagrams-based analysis and design techniques

Having successfully completed the module, you will be able to:

- determine the step transient response of a system
- determine the frequency response of a system

General transferable skills

- Mathematical skills
- Graph plotting techniques, especially Bode diagrams
- Measurement and instrumentation techniques

Scope of the course

- Introductory course into control systems engineering
- Mainly concerned with “classical” control theory
- Restricted to linear time invariant systems.

Control Theory (classical, linear time invariant system)

physical system

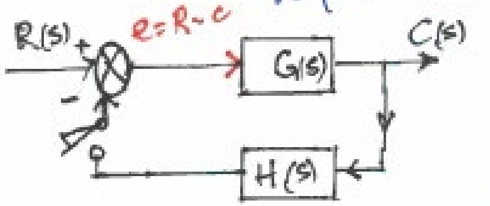
• Math Modelling → Transfer Function → Block Diagrams

- free body diagram

Math (differential eq - time domain) → (Laplace Transform $s = \sigma + j\omega$) → Partial Fraction → Inverse Transform ↓ time resp

System Response

- 1st order sys - τ
- 2nd order sys - ζ, ω_n, underdamp response, τ_1, τ_2 (2 time constant) overdamp response



Fwd TF

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} \quad (C/L\ TF)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+GH}$$

$1+GH=0$ → characteristic eq → Routh Hurwitz (stability analysis)

GH (O/L TF) → Polar Plot, Bode Plot] freq domain, → stability - gain margin - phase margin

→ Root Locus - 'K' varies $(0 \to \infty)$ in S plane

Test Signal → Time Response - R(s) - impulse input, step input, ramp input

→ Freq Response - R(s) - sinusoidal input; input/output • magnitude ratio • phase shift

Reference Textbook.

Modern Control Engineering, Katsuhiko Ogata, 4th Ed , or later, Prentice Hall.

Reference Books (Control Theory)

Introduction to Automatic Controls, H L Harrison, J G Bollinger, 2nd Ed, International Textbook Company, 1969.

System Modelling and Control, J Schwarzenbach, K F Gill, 2nd Ed, Edward Arnold, 1979.

Modern control system Theory and Design, S M Shinners, 1st Ed, John Wiley and Sons Inc, 1992.

Introduction to control system analysis and Design, F J Hale, 1st Ed, Prentice Hall, Inc, 1973.

Control Systems Engineering, Nise, 5th Ed, John Wiley 2008.

Feedback Control of Dynamic Systems, G F Franklin, JD Powell, AE Emami-Naeini, 5th Ed, Pearson International Edition ,2006.

Modern Control Systems, RC Dorf, RH Bishop, 11th Ed, Pearson International Edition ,2008.

Instrumentation and Control Systems, William Bolton, 3rd Ed, Elsever & Newnes.

Linear Control Systems Engineering, Morris Driels, International Eds, McGraw-Hill International, 1995.

Automatic Control Engineering, F H Raven, International Eds, McGraw-Hill International, 1995.

Automatic Control Systems, B C Kuo, 5th Ed, Prentice Hall Inc, 1987.

Feedback Control Systems, J V de Vegte, 1st Ed, Prentice Hall Inc, 1986.

Control System Technology, C J Chesmond, 1st Ed, Edward Arnold,1984.

Introduction to Control System Technology, RN Bateson, 3rd Ed, Merrill Publishing Company, 1989.

Control System Engineering, M E El-Hawary, 1st Ed, Reston Publishing Company, 1984.

1. ORIENTATION TO AUTOMATIC CONTROL

1.1 Historical Development

James Watt	1788	-	Flyball governor
Hurwitz	1875	-	Stability analysis
Routh	1884	-	Stability analysis
Minorsky	1922	-	Servomechanism
Nyquist	1932	-	Response of O/L to sin variation
Bode	1938	-	Freq - Ø characteristic - O/L system
Evans	1948	-	Root Locus

Basis of classical control theory.

1.2 Control Theory

Deals with dynamic response of system to commands or disturbances.

Classical
- T.F. concept with Analysis & Design in Laplace & Freq Domain
- More emphasis of physical understanding

Modern
- State Variable Concept with emphasis on matrix alegbra
- Mathematical technique

Open loop control

- control v/v adjusted to make output equal to input, but not readjusted to keep the 2 equal.
- O/L with certain safeguard very common, ie. guiding a process through a sequence of predetermined steps.

1.3 Types of Control System

Open-loop (O/L)
- no feedback

Close-loop (C/L *)
- with feedback
- error actuated

1.4 Scope of C/L Control

C/L Control

- Electrical
  - Voltage & current regulator
  - Amplifier
- Kinetic (Servos) (Mech)
  - Force (torque)
  - Position
  - Velocity
  - Acceleration
- Process
  - Temp
  - Pressure
  - Flow
  - Density

Closed loop control

- to improve performance, operator adjust v/v based on observation of system error, e.
- Feedback control system, automates this action.
- error used to adjust control v/v by means of actuator.

Feedback Control System has 2 categories

1. Regulator System - function is to maintain output constant, despite unwanted disturbance to the system. Input is seldom changed.

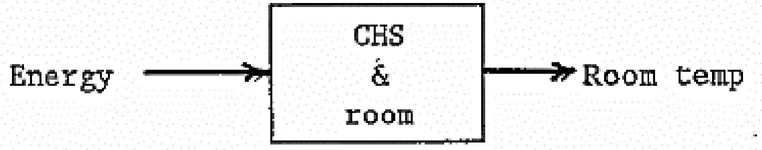
2. Follow-up System - keep output in close correspondence with input, which is always changing.

* AUTOMATION is essentially sequence-controlled mechanisation

1.5 Feedback Control System

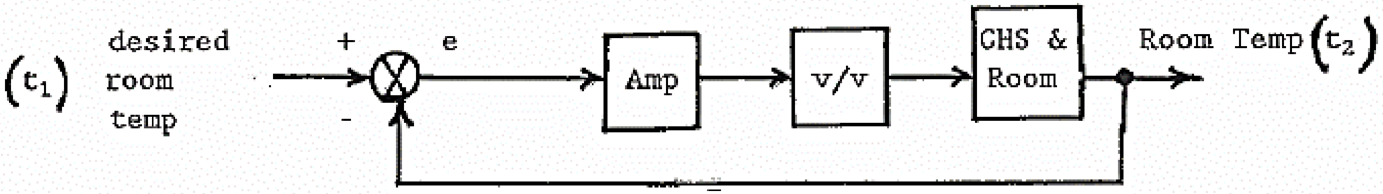
1.5.1 Regulator - maintain output constant, input seldom changed

- 1) O/L system - Manual - No thermostat

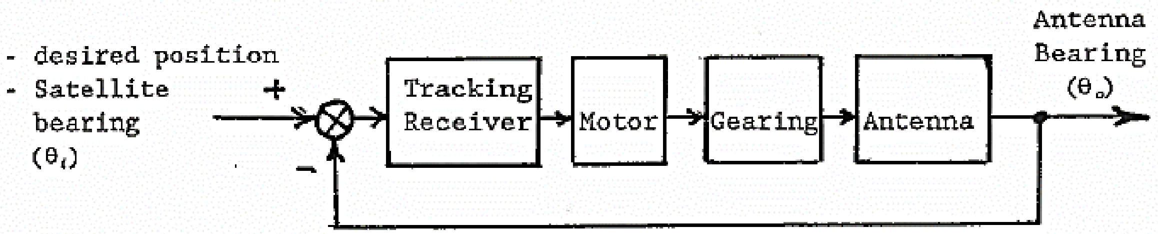


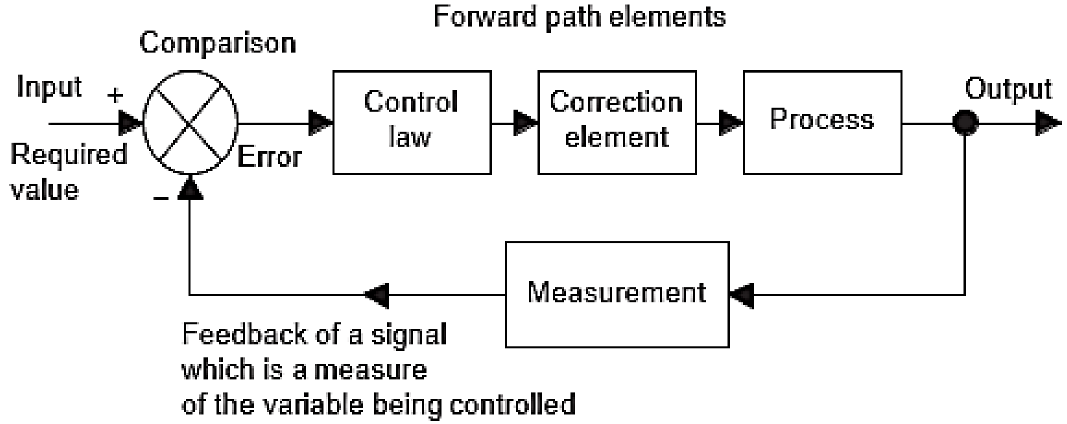
CHS - Central Heating system

- 2) C/L system & Automatic (c thermostat)



1.5.2 Follow-up System - set point frequently changed





Basic element of a control-loop control system

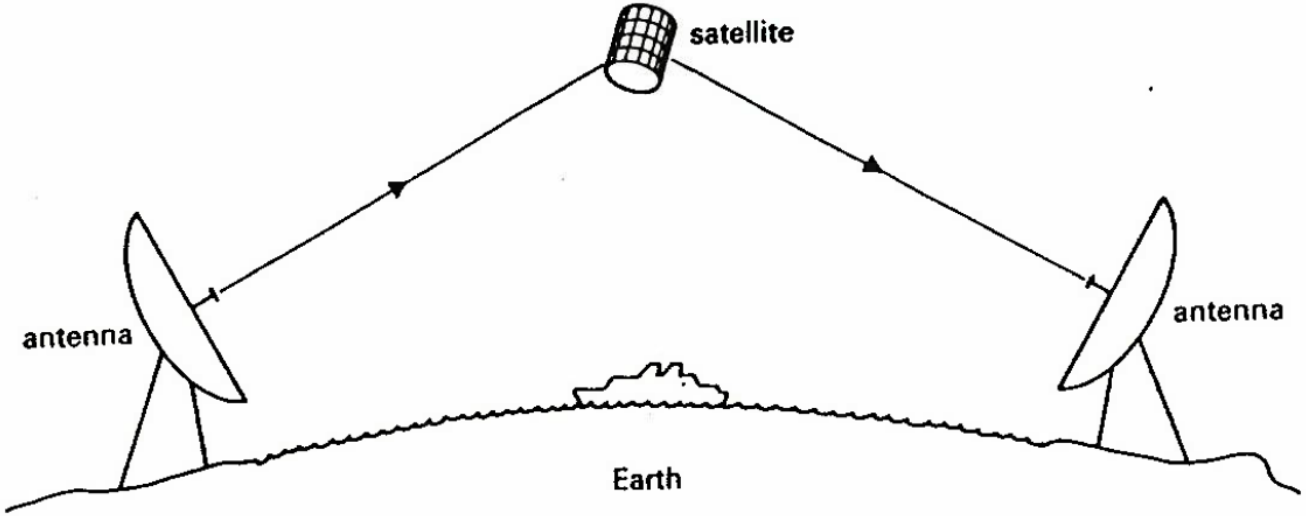
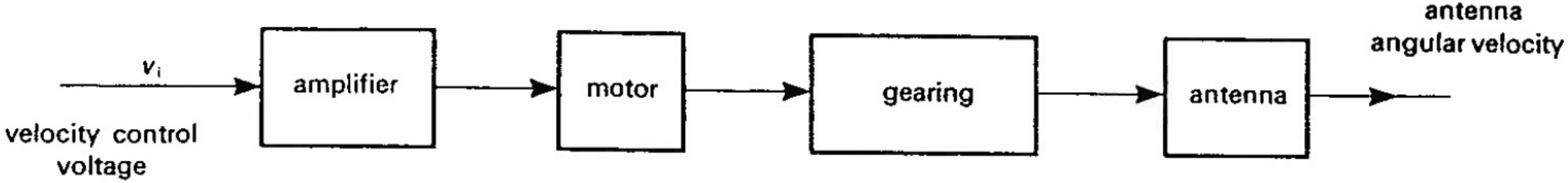


Figure 7 Using antennas to communicate through a satellite



'e 9 Open-loop velocity control of antenna

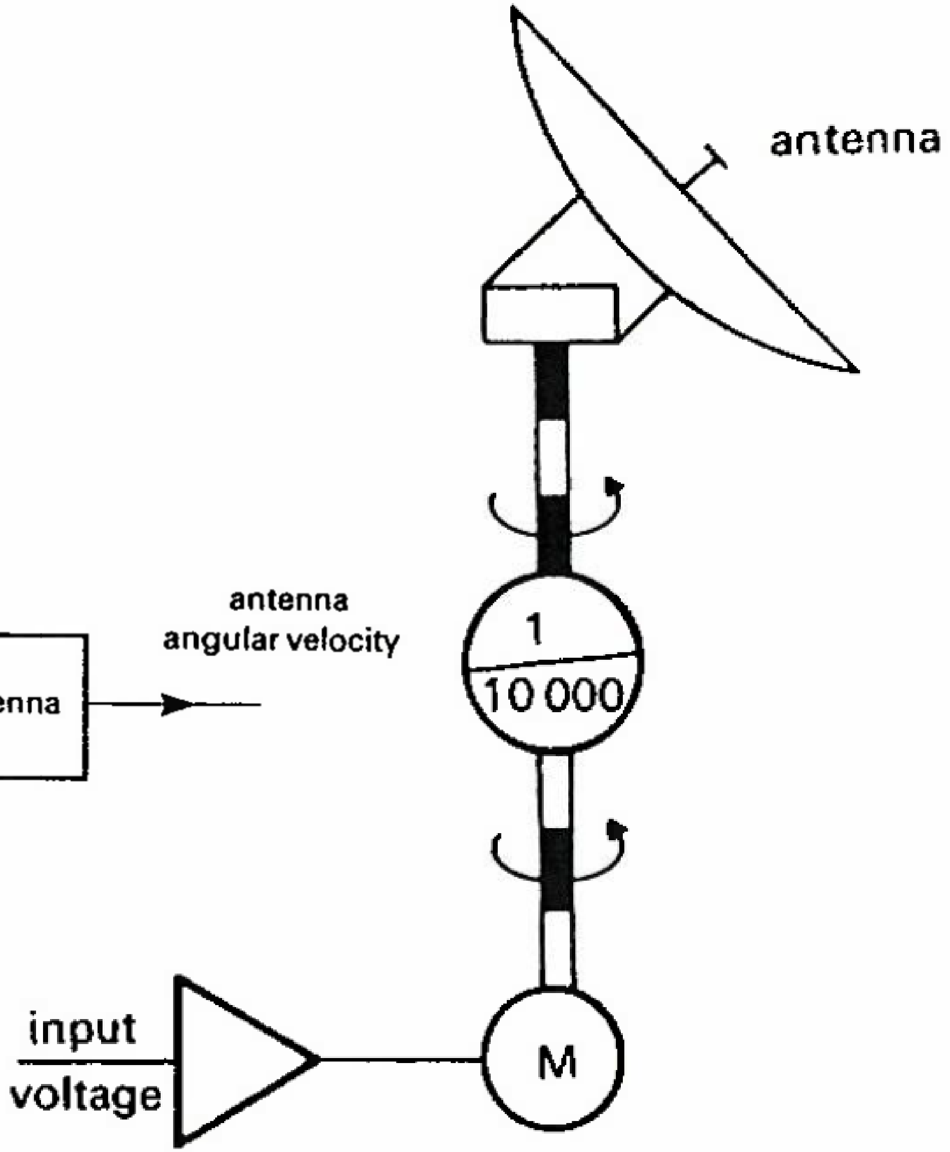
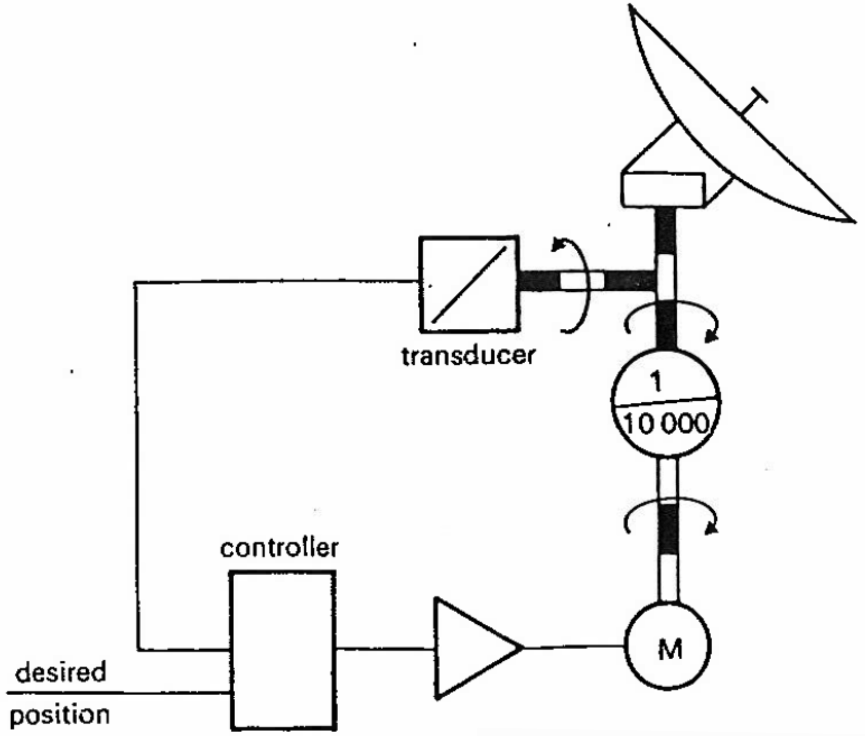


Figure 8 A diagram of the antenna drive system



A closed-loop antenna system

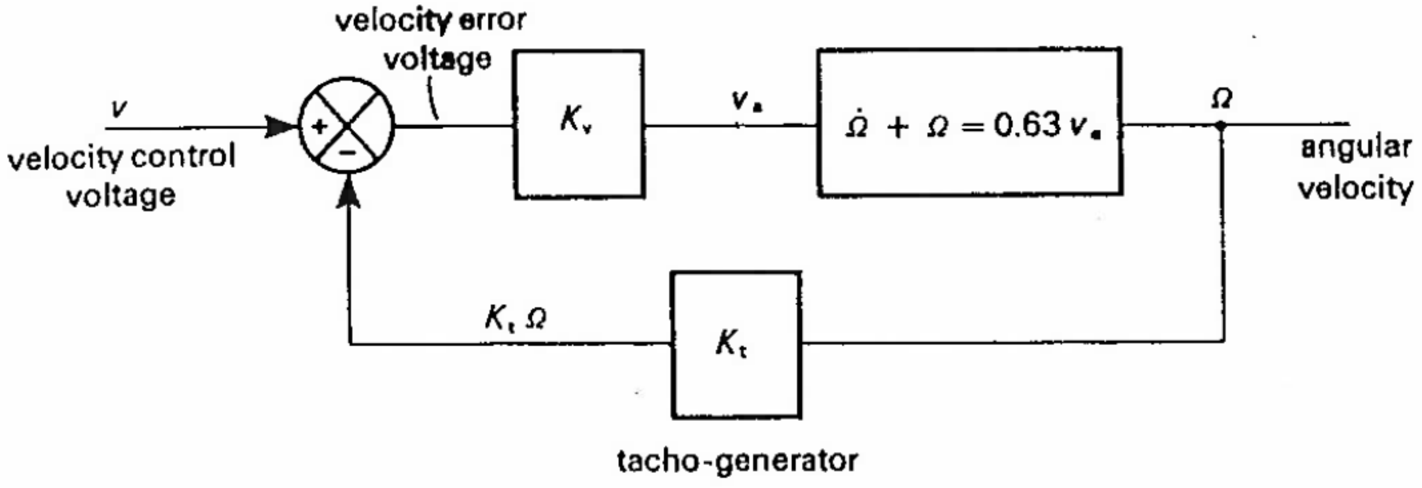


Figure 10 Motor velocity control loop

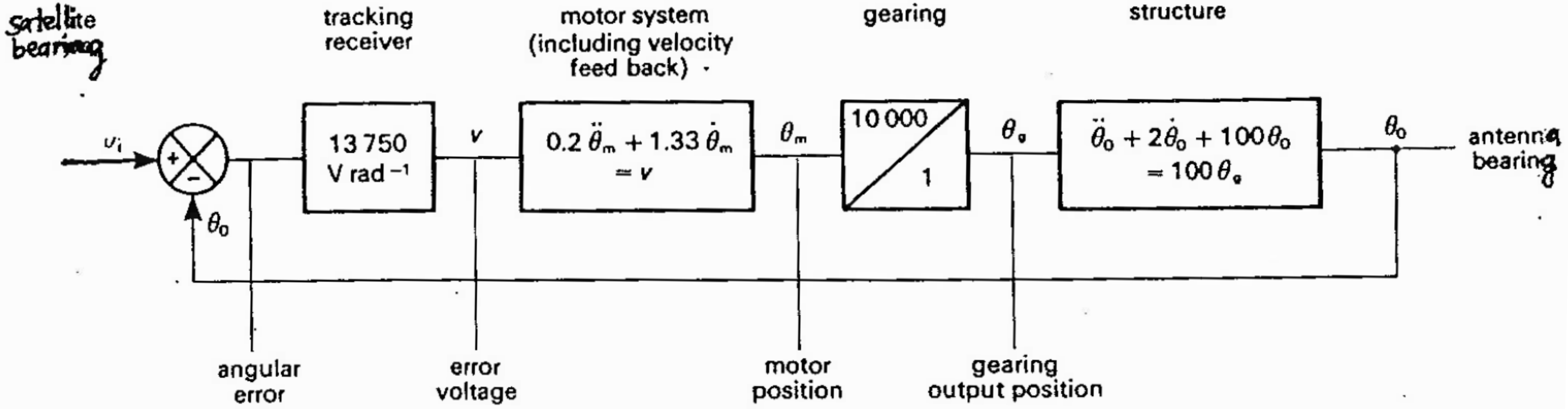


Figure 11 A block diagram of antenna control system

1.5 Application of Control Theory has 2 phases

1.5.1 Dynamic Analysis - determination of response of a plant to commands, disturbances and changes in plant parameter.

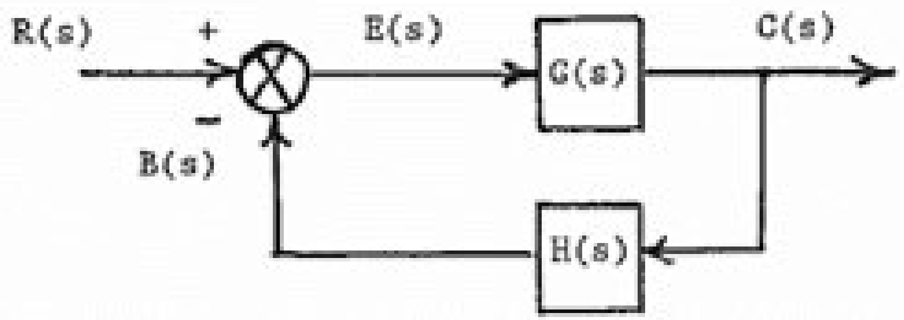
1.5.2 Control System design - If dynamic analysis is unsatisfactory and modification of plant is unacceptable design phase is necessary to select control elements needed to improve dynamic performance to acceptable level.

1.5.3 Methods of Analysis

1. Consider system performance in time domain by measuring output response for given input eg. step, ramp, sinusoidal.

2. Frequency domain - output response to sinusoidal input is considered in steady state only trasient allowed to subside, before measurement made.

- very common in practice to work in this domain even to the extent of writing specifications.



Requirement of Control Theory

1. Stability
2. Acccuracy - specified in terms of errors

Error → steady state (s.s.) - due to static friction as output ceases to move
→ transient - can reverse sign, overshoot due to energy (stored in inertia)

3. Speed of response

(stability & accuracy incompatiable & a good design is compromise of both)*

Eg: If system is subjected to sudden change of input

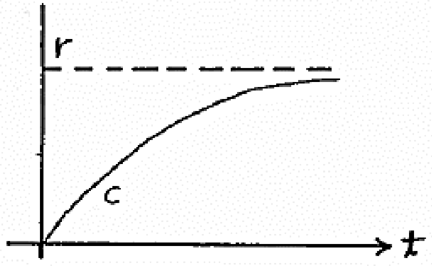
- transient period should be short and its response should not be excessively oscillatory.

- steady state error must be small

Transient response and steady state error characteristics can be improved using feedback and the motivation for feedback

- reduce effect of parameter variation
- reduce effect of disturbance input

- improve tranient response characteristics
- reducing steady state error



F. step response for low gain

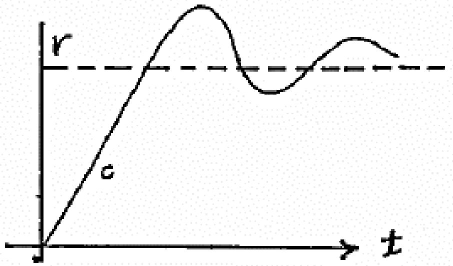


FIG 5: s.r. - High gain

- High accuracy and good stability are incompatible and a good design is a compromise between the two.

1.8 Alternatives for Systems Classification

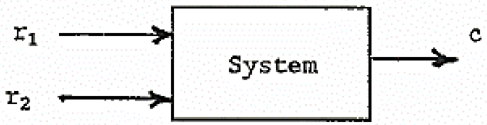
1.8.1 Stationary or time invariant system

- parameter do not vary with time
- output is independant of time
- coeff of describing d.e. - constant

e.g.: $\ddot{y} + 4x\dot{y} + 2x^2 = 3t$ – non linear

1.8.2 Linear system

- principle of superposition holds



$r_2 = 0$, Apply r_1 output - c_1

$r_1 = 0$, Apply r_2 output - c_2

Apply both -- $r_1 + r_2 = c_1 + c_2$ (output)

1.8.3 Lumped parameter

- physical characteristics are assumed to concentrate in one or more lumps & thus independant of spatial distribution.

E.g. bodies - assumed rigid & treated as point mass
spring - massless, temp-uniform
electrical leads - resistanceless

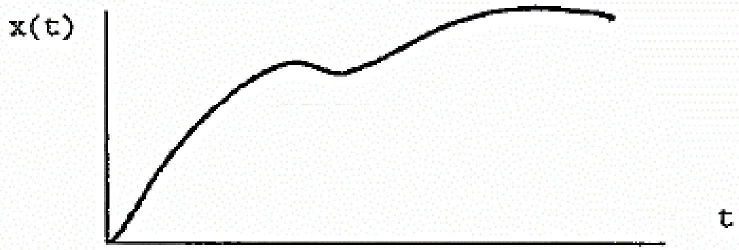
Distrubuted parameter

Bodies - elastic
springs - have distributed mass
electrical leads - distributed resistance
temperature - varying across a body

1.8.4 Deterministic (system or variable)

- future behaviour is predictable and repeatable within reasonable limits.
- if not, system-stochastic or random analysis-based on Probability Theory.

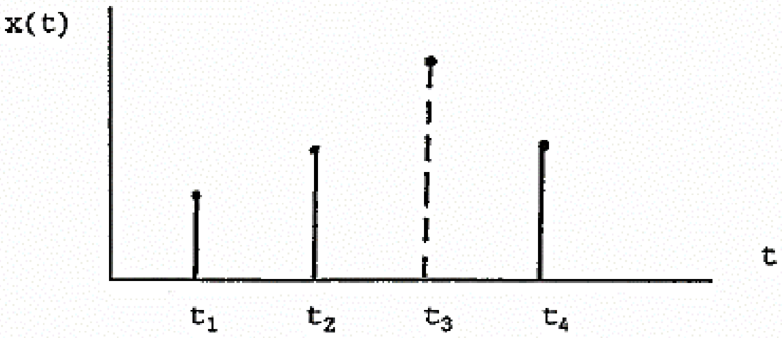
1.8.5 Continuous variable system



- all system variables are continuous function of time.
- describing equations are differential equation.

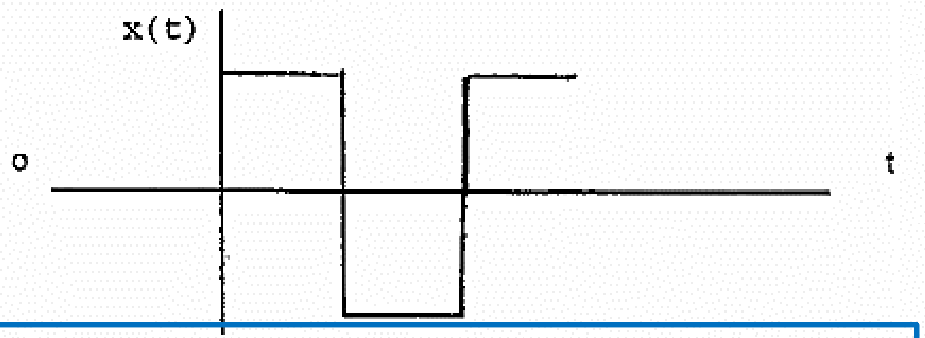
Single Variable

Discrete Variable System



- has one or more variable known only at particular instant of time.
- equations are difference equation.
- if time interval controlled-sampled data system.

Discontinuous Variable



1) $\ddot{x}+5t\dot{x}+6x=5y$

Lumped parameter, continuous, non stationery, SISO

2) $\ddot{x}+7\dot{x}+3x^2=2\dot{y}+3y$

Non linear, lumped parameter, stationery

3) $\frac{\partial^2 y}{\partial t^2}-10\frac{\partial^2 y}{\partial x^2}=0$

Distributed parameter, stationery, linear

4) $\ddot{x}+5\dot{x}+3x=2\dot{y}+3y+4z$

Multivariable, stationery, lumped parameter

5) $3\ddot{x}+30\dot{x}+21x=2y$

lumped parameter, SISO, linear, stationery

ALTERNATIVE FOR SYSTEM CLASSIFICATION

Static or Dynamic Systems

Static systems are composed of simple linear gains or nonlinear devices and described by algebraic equations, and dynamic systems are described by differential or difference equations.

Continuous-time or discrete-time systems

Continuous-time dynamic systems are described by differential equations, and discrete-time dynamic systems by difference equations.

Linear or nonlinear systems

Linear dynamic systems are described by differential (or difference) equations having solutions that are linearly related to their inputs. Equations describing nonlinear dynamic systems contain one or more nonlinear terms.

Lumped or distributed parameters

Lumped-parameter, continuous-time, dynamic systems are described by ordinary differential equations, and distributed-parameter, continuous-time, dynamic systems by partial differential equations.

Time-varying (non-stationary) or time-invariant stationary systems

Time-varying dynamic systems are described by differential (or difference) equations having one or more coefficients as functions of time. Time-invariant (constant-parameter) dynamic systems are described by differential (or difference) equations having only constant coefficients.

Deterministic or stochastic systems

Deterministic systems have fixed (nonrandom) parameters and inputs, and stochastic systems have randomness in one or more parameters or inputs.

Let F(s) be the Laplace Transform of f(t), written as

$\boxed{s = \sigma + j\omega}$

$$L\{f\} = F(s) = \int_0^{\infty} e^{-st} f(t)\, dt$$

Then

$$L\{\frac{df}{dt}\} = sF(s) - \boxed{f(o)}$$

$$L\{\frac{d^2f}{dt^2}\} = s^2F(s) - \boxed{sf(o) - \frac{df}{dt}(o)}$$

$$L\{\frac{d^nf}{dt^n}\} = s^nF(s) - \sum_{i=1}^{n} s^{n-i} \frac{d^{i-1} f}{dt^{i-1}}$$

Shift Theorems

$$L\{e^{-at} f(t)\} = F(s + a)$$

$$L\{f(t + a)\} = e^{as} F(s)$$

(1) Shift Theorem → in Automatic C.S → known as dead time in Process Industry → transport lag

(2) Convolution Theorem → product of 2 L.T to form L.T of Convolution Integral τ–dummy time variable

(3) Final Value Theorem → useful in determining steady state accuracy.

(4) Initial Value Theorem → useful in Inverse Transform when initial condition known to be zero

Convolution Theorems

$$L^{-1}\{X(s)\,Y(s)\} = \int_0^t x(t-\tau)y(\tau)\, d\tau$$

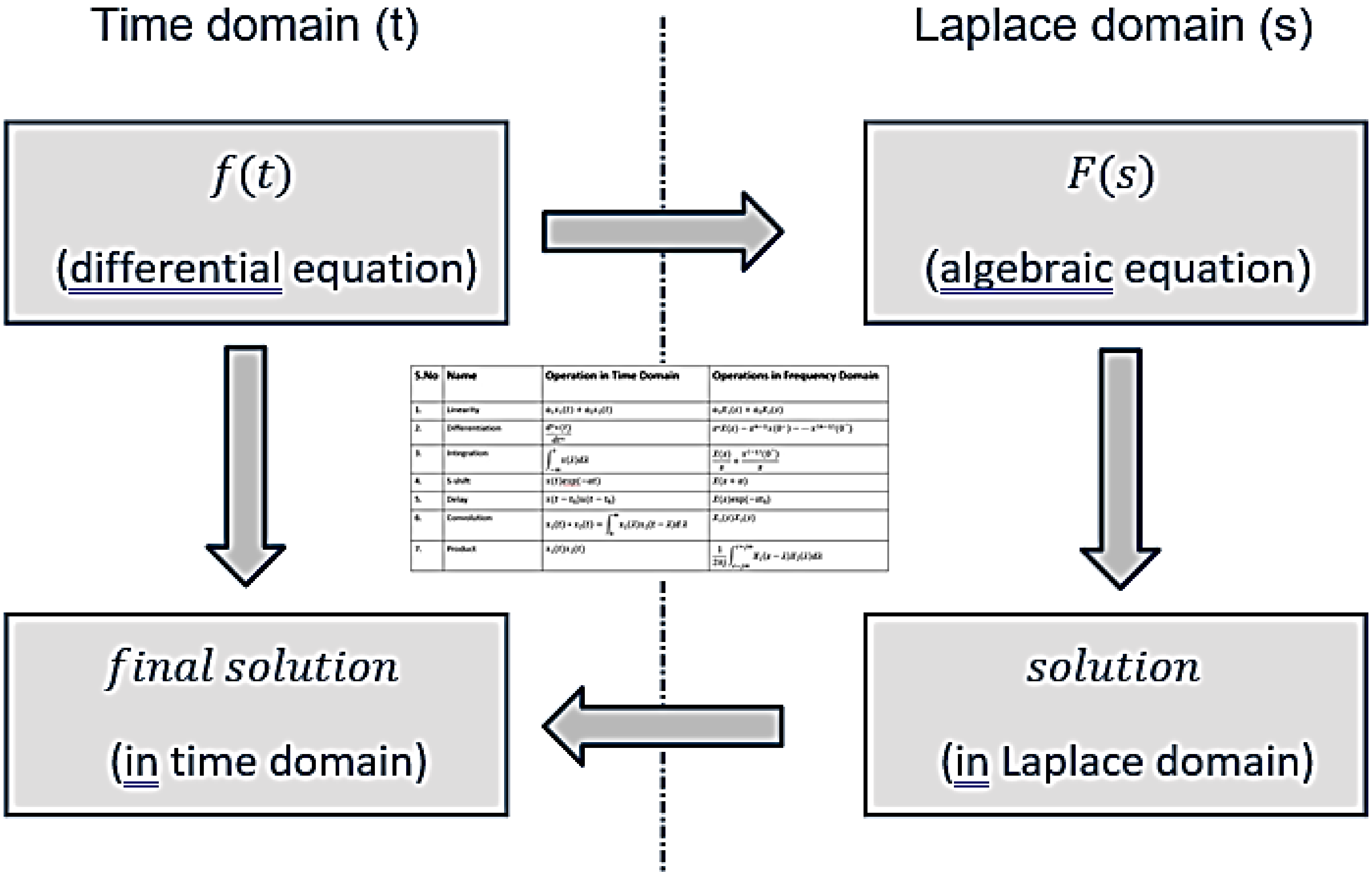
$$= \int_0^t y(t-\tau)x(\tau)\, d\tau .$$

Final Value Theorem $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Initial Value Theorem $\lim_{t\to 0+} f(t) = \lim_{s\to\infty} sF(s)$

Some basic Laplace Transforms

f(t) (0 for t < o)	F(s)
f(t) = const = u(t)	$^1/_s$
f(t) = at	$^a/_{s^2}$
f(t) = $at^n/n!$	$^a/_{s^{n+1}}$
f(t) = sin ωt	$\omega/s^2 + \omega^2)$
f(t) = cos ωt	$s/(s^2 + \omega^2)$
f(t) = t sin ωt	$2\omega s/(s^2 + \omega^2)^2$
f(t) = t cos ωt	$(s^2 - \omega^2)/(s^2 + \omega^2)^2$



S.No	Name	Operation in Time Domain	Operations in Frequency Domain
1.	Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
2.	Differentiation	$\frac{d^n x(t)}{dt^n}$	$s^nX(s) - s^{n-1}x(0^-) - \dots - x^{(n-1)}(0^-)$
3.	Integration	$\int_{-\infty}^{t} x(\lambda)d\lambda$	$\frac{X(s)}{s} + \frac{x^{(-1)}(0^-)}{s}$
4.	S-shift	$x(t)\exp(-at)$	$X(s + a)$
5.	Delay	$x(t - t_0)u(t - t_0)$	$X(s)\exp(-st_0)$
6.	Convolution	$x_1(t) * x_2(t) = \int_0^t x_1(\lambda)x_2(t - \lambda)d\lambda$	$X_1(s)X_2(s)$
7.	Product	$x_1(t)x_2(t)$	$\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X_1(s - \lambda)X_2(\lambda)d\lambda$

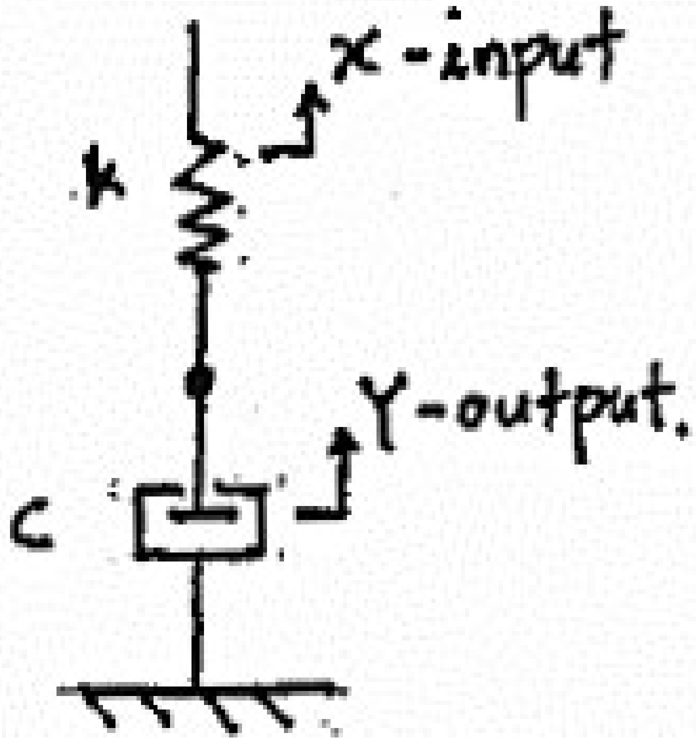
TABLE 10.1 Laplace Functions and Their Corresponding Time Functions

	Time function $f(t)$	Laplace Transform F (s)
1	A unit impulse	1
2	A unit step	$\frac{1}{s}$
3	t, a unit ramp	$\frac{1}{s^2}$
4	e^{-at}, exponential decay	$\frac{1}{s+a}$
5	$1-e^{-at}$, exponential growth	$\frac{a}{s(s+a)}$
6	te^{-at}	$\frac{1}{(s+a)^2}$
7	$t-\frac{1-e^{-at}}{a}$	$\frac{a}{s^2(s+a)}$
8	$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
9	$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$
10	$1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt}$	$\frac{ab}{s(s+a)(s+b)}$
11	$\frac{e^{-at}}{(b-a)(c-a)}+\frac{e^{-bt}}{(c-a)(a-b)}+\frac{e^{-ct}}{(a-c)(b-c)}$	$\frac{1}{(s+a)(s+b)(s+c)}$
12	$\sin \omega t$, a sine wave	$\frac{\omega}{s^2+\omega^2}$
13	$\cos \omega t$, a cosine wave	$\frac{s}{s^2+\omega^2}$
14	$e^{-at}\sin \omega t$, a damped sine wave	$\frac{\omega}{(s+a)^2+\omega^2}$
15	$e^{-at}\cos \omega t$, a damped cosine wave	$\frac{s+a}{(s+a)^2+\omega^2}$
16	$\frac{\omega}{\sqrt{1-\zeta^2}}e^{-\zeta\omega t}\sin \omega\sqrt{1-\zeta^2}t$	$\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$
17	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega t}\sin(\omega\sqrt{1-\zeta^2}t+\phi), \cos\phi=\zeta$	$\frac{\omega^2}{s(s^2+2\zeta\omega s+\omega^2)}$

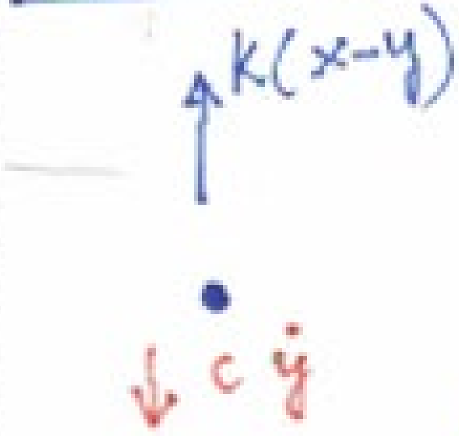
1. A system gives an output of $1/(s+5)$. What is the output as a function of time?
   The output is of the form given in Table 10.1 as item 4 with $a=5$. Hence the time function is e^{-5t} and thus describes an output which decays exponentially with time.

2. A system gives an output of $10/[s(s+5)]$. What is the output as a function of time?
   The nearest form we have in Table 10.1 to the output is item 5 multiplied by 2 to give $2\times a/[s(s+a)]$ with $a=5$. Thus the output, as a function of time, is $2(1-e^{-5t})$.

First Order System



Free body diagram (First Order System)



$$k(x-y) - c\dot{y} = 0 \quad \text{———} \quad (1)$$

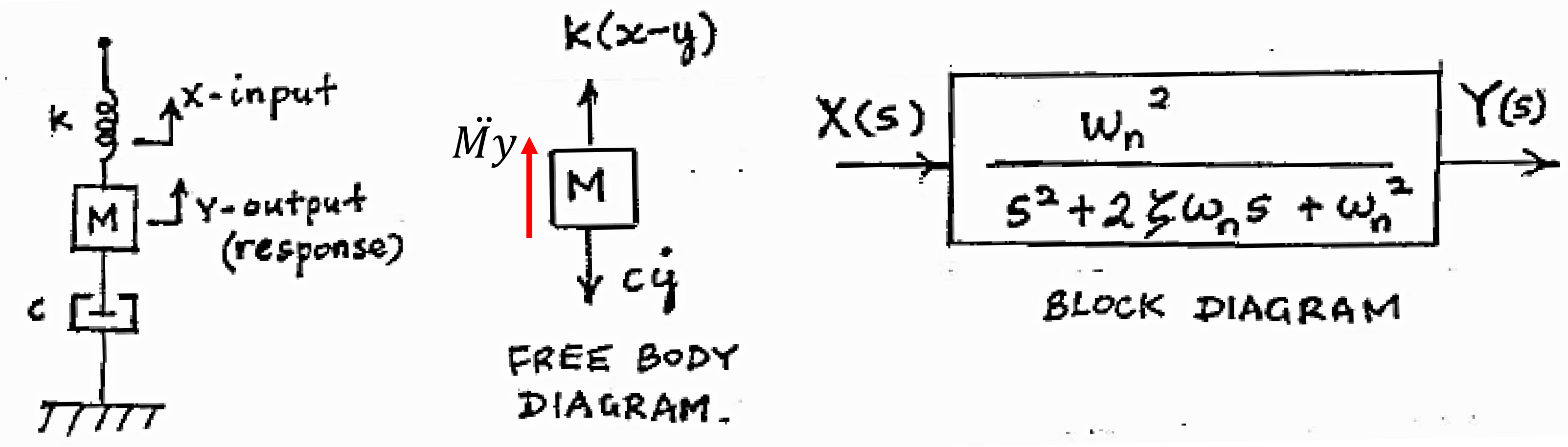
$$c\dot{y} + ky = kx \quad \text{———} \quad (2)$$

$$csY(s) + kY(s) = kX(s) \quad \text{———} \quad (3)$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{k}{cs+k} = \frac{1}{1+s\tau} \quad \text{———} \quad (4)}$$

$$* \quad \tau = c/k \ (\text{time constant})$$

Spring mass damper system (with an input) Translational system



$$m\ddot{y} = k(x-y) - c\dot{y}$$

Inertial force = external forces

$$m\ddot{y} + c\dot{y} + ky = kx$$

$$Ms^2Y(s) + CsY(s) + KY(s) = KX(s) = Force$$

$$\text{Hence } TF: \frac{Y(s)}{X(s)} = \frac{K}{Ms^2+Cs+k} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\text{where } \frac{k}{m} = \omega_n^2, \quad \frac{c}{m} = 2\zeta\omega_n$$

$$\frac{\theta(s)}{T_D(s)} = \frac{2.25}{s^2 + 0.45s + 2.25}$$

2nd order transfer function * $= \left[\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right]$

$\left[\begin{array}{l} \omega_n = \sqrt{2.25} \\ = 1.5 \\ 2\zeta\omega_n = 0.45 \\ \zeta = 0.15 \end{array}\right]$

Unit step input: $T_D(s) = \frac{1}{s}$

$$\theta(s) = \frac{1}{s}\left(\frac{2.25}{s^2 + 0.45s + 2.25}\right)$$

$-2a = 0.45, \quad a = -0.225$

$a^2 + b^2 = 2.25 \quad b = 1.48$

$$K(a+jb) = \left(\frac{2.25}{s}\right)_{s = -0.225 + j1.48}$$

$$= \frac{2.25}{1.497\angle -81.35^\circ} = 1.5\angle +81.35^\circ$$

$$\theta(t) = 1 - \frac{1.5}{1.48} e^{-0.225t} \sin(1.48t + 81.35^\circ)$$

$$= 1 - 1.01 e^{-0.225t} \sin(1.48t + 81.35^\circ)$$

OR

$$\theta(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n\sqrt{1-\zeta^2}\,t + \phi$$

$\cos\phi = \zeta$

$$= 1 - 1.01\, e^{-0.225t} \sin(1.48t + 81.35^\circ)$$

Step response of a second-order system for underdamped case, $(0 < \zeta < 1)$

- The general form of inverse transformation is

$$y(t) = \frac{1}{b}|K(a+jb)|e^{at}\sin(bt+\alpha) + K_1 e^{r_1 t}$$

- A pair of complex conjugate roots when multiplied together yields the following quadratic:

$$[s-(a+jb)][s-(a-jb)] = s^2 - 2as + a^2 + b^2$$

Laplace functions and their corresponding time functions

$\sin\omega t$, a sine wave	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$, a cosine wave	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin\omega t$, a damped sine wave	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$, a damped cosine wave	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t}\sin\omega\sqrt{1-\zeta^2}\,t$	$\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t}\sin(\omega\sqrt{1-\zeta^2}\,t+\phi), \cos\phi = \zeta$	$\frac{\omega^2}{s(s^2+2\zeta\omega s+\omega^2)}$

Percent overshoot

$$\%OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

① $F(s) = \frac{10}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$

$A = \frac{10}{(s+1)}\Big|_{s=0} = 10$

OR

$A(s+1) + Bs$

$(A+B)s + A = 10$

$A = 10$

$(A+B) = 0$

$B = -10$

$B = \frac{10}{(s)}\Big|_{s=-1} = -10$

$F(s) = \frac{10}{s} - \frac{10}{s+1}$

$f(t) = 10(1 - e^{-t})$

$\lim_{t \to \infty} f(t) = 10$

Final Value Theorem

$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$

$= \lim \frac{\cancel{s} \cdot 10}{\cancel{s}(s+1)} = 10$

$F(s) = \frac{10}{s(s+1)(s+10)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+10)}$

$A = \left[\frac{10}{(s+1)(s+10)}\right]_{s=0} = 1$

$B = \left[\frac{10}{s(s+10)}\right]_{s=-1} = -\frac{10}{9}$

$C = \left[\frac{10}{s(s+1)}\right]_{s=-10} = \frac{10}{90} = \frac{1}{9}$

$\therefore F(s) = \frac{1}{s} + \left[-\frac{10}{9}\frac{1}{(s+1)}\right] + \left[\frac{1}{9}\frac{1}{(s+10)}\right]$

$f(t) = 1 - \frac{10}{9}e^{-t} + \frac{1}{9}e^{-10t}$

Inverse Transform

3. WORKED EXAMPLE

A3.1

$$G(s)=\frac{S+1}{(S+2)^2(S+3)}=\boxed{\frac{C_1}{(S+2)^2}+\frac{C_2}{(S+2)}}+\frac{C_3}{S+3}$$

using heaviside theorem

$$\therefore\ C_1=\left[\frac{-2+1}{-2+3}\right]_{s=-2}=-1$$

$$C_2=\frac{d}{ds}\left[\frac{S+1}{S+3}\right]_{s=-2}=\frac{(S+3)-(S+1)}{(S+3)^2}\Big]_{-2}=2$$

$$C_3=\left[\frac{-3+1}{(-3+2)^2}\right]_{s=-3}=-2$$

$\therefore$ Inverse Transform : $G(t)=2e^{-2t}-te^{-2t}-2e^{-3t}$

$$=\boxed{\frac{C_1}{(S+2)^2}+\frac{C_2}{(S+2)}}+\frac{C_3}{S+3}$$

A3.2

$$G(s)=\frac{S+2}{S^3+8S^2+19S+12}=\frac{S+2}{(S+3)(S+4)(S+1)}$$

Expand into partial fraction & use Heaviside Theorem.

$$\frac{c1}{(s+3)}+\frac{c2}{(s+4)}+\frac{c3}{(s+1)}$$

$$\therefore\ C_1=\frac{-3+2}{(-3+4)(-3+1)}=\frac{1}{2}$$

$$C_2=\frac{-4+2}{(-4+3)(-4+1)}=-\frac{2}{3}$$

$$C_3=\frac{-1+2}{(-1+3)(-1+4)}=\frac{1}{6}$$

$$\therefore\ G(t)=\frac{1}{2}e^{-3t}-\frac{2}{3}e^{-4t}+\frac{1}{6}e^{-t}$$

To factorise:

$$S^3+8S^2+19S+12$$

Test : whether (s+1) is a factor, subs s = -1 in the polynomial above, and if result is 0, then (s+1) is a factor

$$\frac{d}{dx}\left(\frac{u}{v}\right)=v\frac{du}{dx}-u\frac{dv}{dx}\ /\ v^2$$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

A3.3 $\ddot{x}+3\dot{x}+2x=5U_s(t)$

$U_s(t)$ - step function

$U_s(t)=1, t>0$

$0, t<0$

Taking L.T on both sides

$$= S^2X(s) \boxed{-Sx(0)} \boxed{-\dot{X}(0)} + 3SX(s)$$

$$\boxed{-3X(0)} + 2X(s) = \frac{5}{S}$$

Initial Condition

$\boxed{X(0)=-1}$

$\boxed{\dot{X}(0)=2}$

Rearranging and substitute initial conditions

$$X(s)=\frac{-S^2-S+5}{S(S+1)(S+2)}=\frac{5}{2S}-\frac{5}{S+1}+\frac{3}{2(S+2)}$$

$$\therefore\ X(t)=\frac{5}{2}-5e^{-t}+\frac{3}{2}e^{-2t}\quad (t>0)$$

$$\lim_{t\to\infty} X(t) = \lim_{S\to 0} SX(s) = \frac{5}{2}$$

X(s) [s^2 +3s + 2] + s -2 +3 =5/s

$$X(s) = \frac{\frac{5}{s}-s-1}{s^2+3s+2}$$

$$= \frac{-S^2-S+5}{S(S+1)(S+2)}$$

$$F(s)=\frac{4}{S^2(S+1)(S+2)}=\frac{C_2}{S^2}+\boxed{\frac{C_1}{S}+\frac{K_1}{S+1}}+\frac{K_2}{S+2}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\mathrm{v}\frac{du}{dx}-u\frac{dv}{dx}\ /\ v^2$$

$$C_2=\left[\frac{4}{(S+1)(S+2)}\right]_{s=0}=2$$

$$C_1=\frac{d}{ds}\left[\frac{4}{S^2+3S+2}\right]=\left[\frac{-4(2S+3)}{(S^2+3s+2)^2}\right]_{s=0}=-3$$

$$K_1=\left[\frac{4}{S^2(S+2)}\right]_{s=-1}=4$$

$$K_2=\left[\frac{4}{S^2(S+1)}\right]_{s=-2}=-1$$

$$\therefore F(s)=\frac{\overset{2}{\cancel{3}}}{S^2}-\frac{3}{S}+\frac{4}{S+1}-\frac{1}{S+2}$$

Inverse L.T $\quad \therefore \; F(t)=4e^{-t}-e^{-2t}+2t-3$

$$\frac{C_2}{S^2}+\frac{C_1}{S}+\frac{K_1}{S+1}+\frac{K_2}{S+2}$$

$$Y(s) = \frac{75}{(s^2 + 4s + 13)(s + 6)}$$

Solution. Equating coefficients to obtain the values of a and b for the quadratic yields $-2a = 4$, or $a = -2$, and $a^2 + b^2 = 13$, or $b = \sqrt{13 - 4} = 3$. Evaluation of $K(a + jb)$ gives

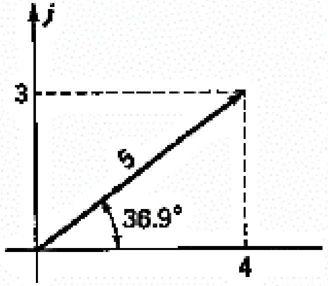
$$K(a + jb) = \left[(s^2 - 2as + a^2 + b^2)\frac{A(s)}{B(s)}\right]_{s=a+jb}$$
$$= \left(\frac{75}{s + 6}\right)_{s=-2+j3} = \frac{75}{4 + j3} \tag{6.25}$$

As shown in Fig. 6.2, the vector whose real part is 4 and whose imaginary part is 3 may be expressed in polar form as

$$4 + j3 = 5\underline{/36.9^\circ}$$

Hence, Eq. (6.25) becomes

$$K(a + jb) = \frac{75}{5\underline{/36.9^\circ}} = 15\underline{/-36.9^\circ}$$



Vector representation for $(4 + j3)$.

Thus

$$|K(a + jb)| = 15$$

and

$$\alpha = \measuredangle K(a + jb) = -36.9^\circ$$

The general form of the inverse transformation is

$$y(t) = \frac{1}{b}|K(a + jb)|e^{at} \sin(bt + \alpha) + K_1 e^{r_1 t}$$

Evaluation of K_1 gives

$$K_1 = \lim_{s \to -6} \frac{75}{s^2 + 4s + 13} = \frac{75}{25} = 3$$

Thus the desired result is

$$y(t) = 5e^{-2t} \sin(3t - 36.9^\circ) + 3e^{-6t} \tag{6.26}$$

Application of the relationship $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$ in which $\alpha = 3t$ and $\beta = -36.9^\circ$ yields the alternate form

$$y(t) = e^{-2t}(4 \sin 3t - 3 \cos 3t) + 3e^{-6t} \tag{6.27}$$

This form of the result may be obtained directly. The response term due to a pair of complex conjugate roots $a \pm jb$ may be written in the form

$$y(t) = \frac{|K(a + jb)|}{b} e^{at} \sin(bt + \alpha) = \frac{e^{at}}{b}|K(a + jb)|(\cos\alpha \sin bt + \sin\alpha \cos bt)$$
$$= \frac{e^{at}}{b}(A \sin bt + B \cos bt) \tag{6.28}$$

where $A = |K(a + jb)| \cos\alpha$
$B = |K(a + jb)| \sin\alpha$

Figure 6.3 shows the vector $K(a + jb)$. Note that the horizontal component is $A = |K(a + jb)| \cos\alpha$, and the vertical component is $B = |K(a + jb)| \sin\alpha$. For the preceding example $|K(a + jb)| = 15$ and $\alpha = -36.9^\circ$, hence $A = 15 \cos(-36.9^\circ) = 12$ and $B = 15 \sin(-36.9^\circ) = -9$. For $a = -2$ and $b = 3$, application of Eq. (6.28) yields for the response due to the complex conjugate roots

$$e^{-2t}(4 \sin 3t - 3 \cos 3t)$$

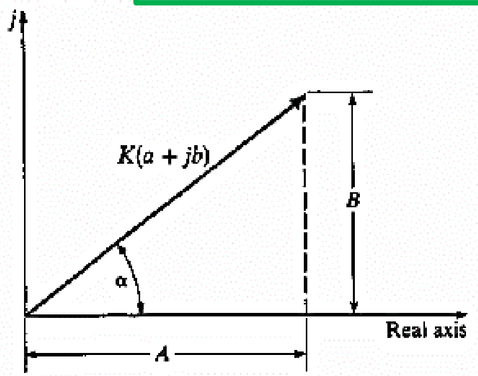
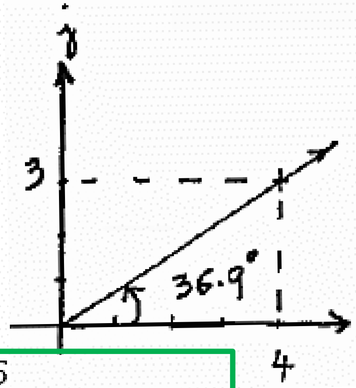


FIGURE 6.3
Horizontal component A and vertical component B of vector $K(a + jb)$.

A3.5 $$Y(s)=\frac{75}{(S^2+4S+13)(S+6)}$$



Equating coeff to obtain α and b

$\therefore\ -2\alpha=4, \qquad \therefore\ \alpha=-2$

$\alpha^2+b^2=13, \qquad \therefore\ b=3$

$$\therefore\ K(\alpha+jb)=\left(\frac{75}{S+6}\right)_{s=-2+j3}=\frac{75}{4+j3}=\frac{75}{5\angle 36.9}$$

Thus $|K(\alpha+jb)|=15, \quad \alpha=\angle K(\alpha+jb)=-36.9°$

General Form of Inverse L.T is

$$y(t)=\frac{1}{b}|K(\alpha+jb)|\,e^{\alpha t}\sin(bt+\alpha)+K_1e^{r_1t}$$

$$K_1=\lim_{S=-6}\frac{75}{S^2+4S+13}=\frac{75}{25}=3$$

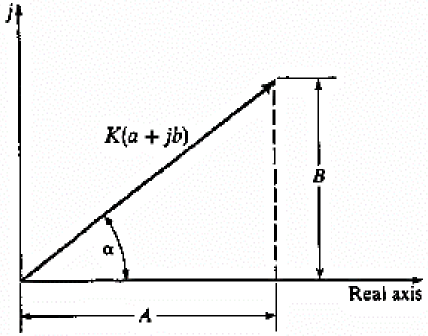
$$\therefore\quad y(t)=5e^{-2t}\sin(3t-36.9°)+3e^{-6t}$$

OR $$\boxed{y(t)=e^{-2t}(4\sin 3t-3\cos 3t)+3e^{-6t}}$$

$$y(t)=\frac{|K(a+jb)|}{b}e^{at}\sin(bt+\alpha)=\frac{e^{at}}{b}|K(a+jb)|(\cos\alpha\sin bt+\sin\alpha\cos bt)$$

$$=\frac{e^{at}}{b}(A\sin bt+B\cos bt) \qquad (6.$$

where $A=|K(a+jb)|\cos\alpha$
$B=|K(a+jb)|\sin\alpha$



Another method for obtaining the response due to complex conjugate roots results from writing $Y(s)$ in the form

$$Y(s)=\frac{75}{(s^2+4s+13)(s+6)}=\frac{As+B}{(s+2)^2+3^2}+\frac{K_1}{s+6}$$

Evaluation of the constants yields $A=-3$, $B=6$, and $K_1=3$, whence

$$\boxed{Y(s)=4\frac{3}{(s+2)^2+3^2}-3\frac{s+2}{(s+2)^2+3^2}+\frac{3}{s+6}}$$

Thus the desired result is

$$y(t)=5e^{-2t}\sin(3t-36.9°)+3e^{-6t} \qquad (6.26)$$

Application of the relationship $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ in which $\alpha=3t$ and $\beta=-36.9°$ yields the alternate form

$$\boxed{y(t)=e^{-2t}(4\sin 3t-3\cos 3t)+3e^{-6t}} \qquad (6.27)$$

$e^{-at}\sin\omega t$, a damped sine wave $\longleftarrow$ $\dfrac{\omega}{(s+a)^2+\omega^2}$

$e^{-at}\cos\omega t$, a damped cosine wave $\longleftarrow$ $\dfrac{s+a}{(s+a)^2+\omega^2}$

$[s-(a+jb)][s-(a-jb)] \leftarrow$ complex conjugate roots.

$= s^2-2as+a^2+b^2$

$\therefore\ s^2+4s+13, \quad -2a=4, \quad a=-2$

$a^2+b^2=13, \quad b=3$

A3.5. Laplace Transform

$$F(s) = \frac{-3s+6}{(s+2)^2+3^2} + \frac{3}{s+6}$$

$$= \frac{-3(s+2)}{(s+2)^2+3^2} + \frac{3}{s+6}$$

$$= \frac{12}{(s+2)^2+3^2} - \frac{3(s+2)}{(s+2)^2+3^2} + \frac{3}{s+6}$$

$$= 4\frac{3}{(s+2)^2+3^2} - 3\frac{(s+2)}{(s+2)^2+3^2} + \frac{3}{s+6}$$

$$y(t) = e^{-2t}(4\sin 3t - 3\cos 3t) + 3e^{-6t}$$

Another method for obtaining the response due to complex conjugate roots results from writing $Y(s)$ in the form

$$Y(s) = \frac{75}{(s^2+4s+13)(s+6)} = \frac{As+B}{(s+2)^2+3^2} + \frac{K_1}{s+6}$$

Evaluation of the constants yields $A = -3$, $B = 6$, and $K_1 = 3$, whence

$$Y(s) = 4\frac{3}{(s+2)^2+3^2} - 3\frac{s+2}{(s+2)^2+3^2} + \frac{3}{s+6}$$

Thus the desired result is

$$y(t) = 5e^{-2t}\sin(3t - 36.9°) + 3e^{-6t} \tag{6.26}$$

Application of the relationship $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ in which $\alpha = 3t$ and $\beta = -36.9°$ yields the alternate form

$$y(t) = e^{-2t}(4\sin 3t - 3\cos 3t) + 3e^{-6t} \tag{6.27}$$

$e^{-at}\sin\omega t$, a damped sine wave	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$, a damped cosine wave	$\frac{s+a}{(s+a)^2+\omega^2}$

$[s-(a+jb)][s-(a-jb)]$ ← complex conjugate roots.

$= s^2 - 2as + a^2 + b^2$

$\therefore s^2+4s+13, \quad -2a = 4, \quad a = -2$

$a^2+b^2 = 13, \quad b = 3$

A3.5. Laplace Transform

$$F(s) = \frac{-3s+6}{(s+2)^2+3^2} + \frac{3}{s+6}$$

$$= \frac{-3(s+2)}{(s+2)^2+3^2} + \frac{3}{s+6}$$

$$= \frac{12}{(s+2)^2+3^2} - \frac{3(s+2)}{(s+2)^2+3^2} + \frac{3}{s+6}$$

$$= 4\,\frac{3}{(s+2)^2+3^2} - 3\,\frac{(s+2)}{(s+2)^2+3^2} + \frac{3}{s+6}$$

$$y(t) = e^{-2t}\left(4\sin 3t - 3\cos 3t\right) + 3e^{-6t}$$

$\sin \omega t$, a sine wave	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$, a cosine wave	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin \omega t$, a damped sine wave	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$, a damped cosine wave	$\frac{s+a}{(s+a)^2+\omega^2}$

3. A system has a transfer function of $1/(s+2)$. What will be its output as a function of time when it is subject to a step input of 1 V?

The step input has a Laplace transform of $(1/s)$. Thus:

$$R(s) \to \boxed{G} \to C(s)$$

$$\text{Output}(s) = G(s) \times \text{Input}(s) = \frac{1}{s+2} \times \frac{1}{s} = \boxed{\frac{1}{s(s+2)}}$$

The nearest form we have in Table 10.1 to the output of $1/[s(s+2)$ is item 5 as $\boxed{½ \times 2/[s(s+2)]}$. Thus the output, as a function of time, is $½(1 - e^{-2t})$ V.

4. A system has a transfer function of $4/(s+2)$. What will be its output as a function of time when subject to a ramp input of 2 V/s?

The ramp input has a Laplace transform of $(2/s^2)$. Thus:

$$R(s) \to \boxed{G} \to C(s)$$

$$\boxed{\text{Output}(s) = G(s) \times \text{Input}(s) = \frac{4}{s+2} \times \frac{2}{s^2} = \frac{8}{s^2(s+2)}}$$

The nearest form we have in Table 10.1 to the output is item 7 when written as $\boxed{4 \times 2/[s^2(s+2)]}$. Thus the output, as a function of time, is $\boxed{4[t - (1 - e^{-2t})/2] = 4t - 2(1 - e^{-2t})\ \text{V}}$.

5	$1 - e^{-at}$, exponential growth	$\dfrac{a}{s(s+a)}$
6	te^{-at}	$\dfrac{1}{(s+a)^2}$
7	$t - \dfrac{1 - e^{-at}}{a}$	$\dfrac{a}{s^2(s+a)}$

5. A d.c. motor drive system has a transfer function of $5/(s+5)$ and drives a load which has a transfer function of $1/(s+1)$. What will be the output of the system when the motor has a unit step input.

The overall transfer function will be:

$$G(s) = \frac{5}{s+5} \times \frac{1}{s+1} = \frac{5}{(s+5)(s+1)}$$

The step input has a Laplace transform of $(1/s)$. Thus:

$$\text{Output}(s) = G(s) \times \text{Input}(s) = \frac{5}{(s+5)(s+1)} \times \frac{1}{s} = \boxed{\frac{1}{s} + \frac{b}{(s+5)} + \frac{a}{(s+1)}}$$

This is similar to item 10 in Table 10.1 so the output as a function of time is:

$\boxed{b = 1/4 \;,\; a = -5/4}$

$$\boxed{\text{Output}(t) = 1 - \frac{1}{1-5}e^{-5t} + \frac{5}{1-5}e^{-1t} = 1 + \frac{1}{4}e^{-5t} - \frac{5}{4}e^{-t}}$$

10	$1 - \frac{b}{b-a}e^{-at} + \frac{a}{b-a}e^{-bt}$	$\boxed{\frac{5}{(s+5)(s+1)} \times \frac{1}{s}}$	$\frac{ab}{s(s+a)(s+b)}$

a=5, b=1

7. Determine the partial fractions of:

$$\frac{3s+1}{(s+2)^3}$$

This will have partial fractions of:

$$\frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)^3}$$

Then, for the partial fraction expression to equal the original fraction, we must have:

$$\boxed{\frac{3s+1}{(s+2)^3} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)^3}}$$

and so consequently have:

$$\boxed{3s+1 = A(s+2)^2 + B(s+2) + C = A(s^2+2s+1) + B(s+2) + C}$$

Equating s^2 terms gives $\boxed{0 = A.}$ Equating s terms gives $\boxed{3 = 2A + B}$ and so $\boxed{B = 3.}$ Equating the numeric terms gives $\boxed{1 = A + 2B + C \text{ and so } C = -5.}$ Thus:

$$\frac{3s+1}{(s+2)^3} = \boxed{\frac{3}{(s+2)^2}} \boxed{- \frac{5}{(s+2)^3}}$$

8. Determine the partial fractions of:

$$\frac{2s+1}{(s^2+s+1)(s+2)}$$

This will have partial fractions of:

$$\boxed{\frac{As+B}{s^2+s+1}+\frac{C}{s+2}}$$

Thus we must have:

$$\frac{2s+1}{(s^2+s+1)(s+2)}=\frac{As+B}{s^2+s+1}+\frac{C}{s+2}$$

and so:

$$\boxed{2s+1=(As+B)(s+2)+C(s^2+s+1)}$$

With $s=-2$ then $-3=3C$ and so $\boxed{C=-1.}$ Equating s^2 terms gives $0=A+C$ and so $\boxed{A=1.}$ Equating s terms gives $2=2A+B+C$ and so $\boxed{B=1.}$ As a check, equating numeric terms gives $1=2B+C$. Thus:

$$\frac{2s+1}{(s^2+s+1)(s+2)}=\boxed{\frac{s+1}{s^2+s+1}}\boxed{-\frac{1}{s+2}}$$

The transfer function that relates the output response, *C(s)* to the input command, *R(s)*, in a control system is given as:

$$\frac{C(s)}{R(s)} = \frac{(S+0.5)}{(S+2)(s+5)}$$

(i) Using partial fractions, determine the output response c(t) from the system when it is subjected to a unit step input.

(4 marks)

(ii) Determine the steady state value of the system response when it is subjected to a unit step input.

2c(i) $\frac{C(s)}{R(s)} = \frac{s+0.5}{(s+2)(s+5)}$, $R(s) = \frac{1}{s}$

$$C(s) = \frac{s+0.5}{s(s+2)(s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+5}$$

$$A = \left.\frac{s+0.5}{(s+2)(s+5)}\right|_{s=0} = 0.05 = \frac{1}{20}$$

$$B = \left.\frac{s+0.5}{s(s+5)}\right|_{s=-2} = \frac{1}{4}$$

$$C = \left.\frac{s+0.5}{s(s+2)}\right|_{s=-5} = -\frac{3}{10}$$

$$\therefore c(t) = \frac{1}{20} + \frac{1}{4}e^{-2t} - \frac{3}{10}e^{-5t} \quad (4)$$

2c(ii) $C_{ss} = \lim_{s \to 0} s \cdot \left[\frac{s+0.5}{(s+2)(s+5)}\right]\frac{1}{s} = \frac{1}{20} = 0.05$

(c) A system has an output y related to the input x by the differential equation :

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = x$$

(i) Determine the transfer function, $\frac{Y(s)}{X(s)}$.

(3 marks)

(ii) Using partial fractions, determine the output $y(t)$ from the system when it is subjected to a unit step input.

$$Y(s)\left[s^2 + 5s + 6\right] = X(s)$$

$$G = \frac{Y(s)}{X(s)} = \frac{1}{s^2+5s+6}$$

3

Unit step input ; $X(s) = 1/s$

$$Y(s) = \left(\frac{1}{s^2+5s+6}\right)\frac{1}{s} = \frac{1}{s(s+2)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3} = \frac{1}{s(s+2)(s+3)}$$

Using heaviside theorem,

$$A = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

$$B = -\frac{1}{2} \times 1 = -\frac{1}{2}$$

$$C = -\frac{1}{3} \times -1 = \frac{1}{3}$$

2

$$Y(s) = \frac{1}{6s} + \frac{1}{3(s+3)} - \frac{1}{2(s+2)}$$

$$y(t) = \frac{1}{6} + \frac{1}{3}e^{-3t} - \frac{1}{2}e^{-2t}$$

2

9. A thermocouple has a transfer function linking its voltage output V and temperature input of:

$$G(s) = \frac{30 \times 10^{-6}}{10s + 1}\ \mathrm{V/^\circ C}$$

Determine the response of the system when it is suddenly immersed in a water bath at 100°C.
The Laplace transform of the output is:

$$V(s) = G(s) \times \text{input}\ (s)$$

The sudden immersion of the thermometer gives a step input of size 100°C and so the Laplace transform of the input is $100/s$. Thus:

$$V(s) = \frac{30 \times 10^{-6}}{10s + 1} \times \frac{100}{s} = \frac{30 \times 10^{-4}}{10s\,(s + 0.1)} = 30 \times 10^{-4}\,\frac{0.1}{s\,(s + 0.1)}$$

The fraction element is of the form $a/s(s + a)$ and so the output as a function of time is:

$$V = 30 \times 10^{-4}(1 - e^{-0.1t})\ \mathrm{V}$$

5	$1 - e^{-at}$, exponential growth	$\frac{a}{s(s+a)}$

6. Determine the partial fractions of:

$$\frac{s+4}{(s+1)(s+2)}$$

The partial fractions will be of the form:

$$\frac{A}{s+1}+\frac{B}{s+2}$$

Then, for the partial fraction expression to equal the original fraction, we must have:

$$\boxed{\frac{s+4}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}=\frac{A(s+2)+B(s+1)}{(s+1)(s+2)}}$$

and consequently:

$$s+4=A(s+2)+B(s+1)$$

$$\frac{s+4}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$$

$$\boxed{\begin{aligned}A + B &= 1 \ldots\ldots\ldots\ldots \text{eq1}\\ 2A + B &= 4 \ldots\ldots\ldots\ldots \text{eq2}\end{aligned}}$$

This must be true for all values of s. The procedure is then to pick values of s that will enable some of the terms involving constants to become zero and so enable other constants to be determined. Thus if we $\boxed{\text{let } s=-2}$ then we have

$$\boxed{(-2)+4=A(-2+2)+B(-2+1)}$$

$\boxed{\text{and so } B=-2.}$ If we now $\boxed{\text{let } s=-1}$ then

$$\boxed{(-1)+4=A(-1+2)+B(-1+1)}$$

and so $\boxed{A=3.}$ Thus

$$\frac{s+4}{(s+1)(s+2)}=\frac{3}{s+1}-\frac{2}{s+2}$$

7. STEADY-STATE ANALYSIS AND CLASSIFICATION OF SYSTEMS

Once the absolute stability of a closed-loop control system has been investigated to ensure that the system possesses a steady state the next step is to analyse its steady-state performance or static accuracy, since it is desirable that the ultimate response of the system should be equal to the reference or input signal. Thus, the error in steady state is a measure of the system's steady-state performance.

This error will depend upon the nature of the open-loop transfer function G(s) and the type of input signal that the control system has to follow and also upon the effect of disturbance signals. To avoid investigating the numerous variations that can exist, it is possible to classify the steady-state behaviour depending upon the system type number, which leads to a definition of error constants.

Consider now the unity-feedback system of figure 7, where G(s) is the open-loop transfer function and K is the gain of the controller. It is assumed that other forms of control system are put into the form of figure 7.

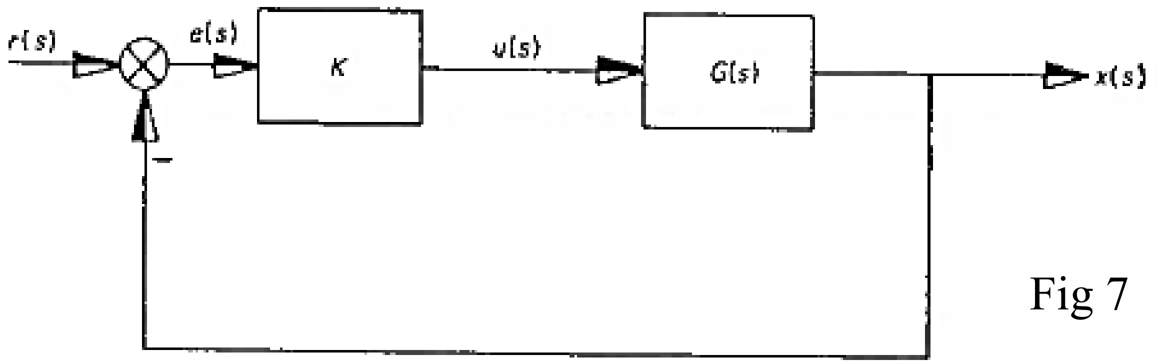


Fig 7

7.1 Classification of Systems

The forward transference $KG(s)$ of the control system in figure 7 will in general be expressed as

$$KG(s) = K\frac{K_G(s+a_1)(s+a_2)\ldots(s^2+bs+c)\ldots}{s^l(s+d_1)(s+d_2)\ldots(s^2+es+f)\ldots} \tag{7.5}$$

$$= \frac{K\sum_{k=0}^{m} A_k s^k}{s^l \sum_{k=0}^{n-l} B_k s^k}, \quad n \geq m+1 \tag{7.6}$$

where the order of the system is defined as the highest power of s in the denominator - that is, n: the rank of the system is defined as the difference between the highest power of s in the denominator and that in the numerator - that is, $n-m \geq 1$; and the class (or type number) of the system is defined as the power of the factor s in the denominator - that is, l.

Example A7.1

State the order, rank and type number of the systems with open-loop transfer function

$$(i) \qquad G(s) = \frac{s+2}{s^4+3s^3+3s^2+s}$$

$$(ii) \qquad G(s) = \frac{1}{s^3(s+2)(s+1)}$$

$$(iii) \qquad G(s) = \frac{s^2+s+1}{(s+2)(s^2+s+4)}$$

(i) In this case the order is 4, the rank is 4 - 1 = 3 and the type number is 1 since the denominator can be written as $s(s^3+3s^2+2s+1)$.

(ii) Here the order is 5, the rank is 5 and the type number is 3 since s^3 is a factor.

(iii) Finally the order is 3, the rank is 1, and the type numebr is 0.

Another method of describing frequency response of systems and their stability. The method uses *Nyquist diagrams*; in these diagrams the gain and the phase of the open-loop transfer function are plotted as polar graphs for various values of frequency. The transfer function of a basic closed-loop control system

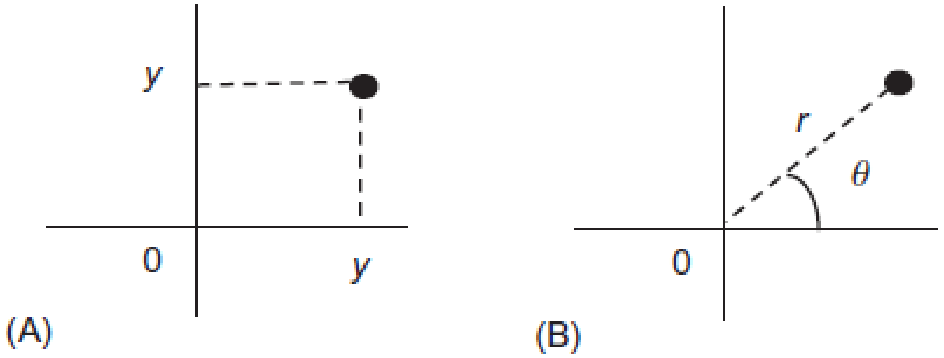
$$\boxed{\text{Transfer function of closed-loop system} = \frac{G(s)}{1 + G(s)H(s)}}$$

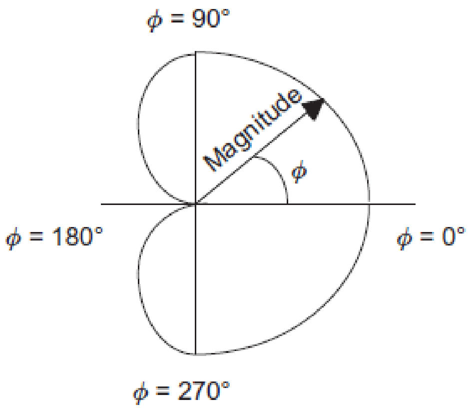
where $G(s)$ is the transfer function of the forward path and $H(s)$ that of the feedback path. It is the term $G(s)H(s)$ which is termed the *open-loop transfer function*, it effectively being the transfer function of a closed-loop system if the feedback path from the feedback element is broken. This open-loop transfer function is of significance in determining whether a system will be stable

With Cartesian graphs the points are plotted according to their x and y coordinates from the origin; with polar graph the points are plotted from the origin according to their radial distance from it and their angle to the reference axis

THE POLAR PLOT

The *polar plot* of the frequency response of a system is the line traced out as the frequency is changed from 0 to infinity by the tips of the phasors whose lengths represent the magnitude, i.e. amplitude gain, of the system and which are drawn at angles corresponding to their phase ϕ



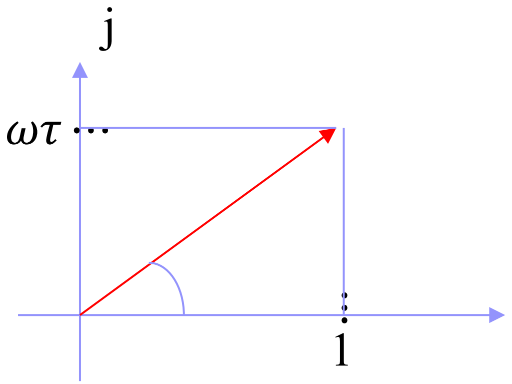


(A) Cartesian graph with points specified by x and y values, (B) polar graph with points specified by r and θ values.

Polar plot with the plot as the line traced out by the tips of the phasors as the frequency is changed from zero to infinity.

Determine the Nyquist diagram for a first-order system with an open-loop transfer function of $1/(1+\tau s)$. The frequency response is:

$$\boxed{\frac{1}{1+j\omega\tau}} = \frac{1}{1+j\omega\tau} \times \frac{1-j\omega\tau}{1-j\omega\tau} = \frac{1}{1+\omega^2\tau^2} - j\frac{\omega\tau}{1+\omega^2\tau^2}$$



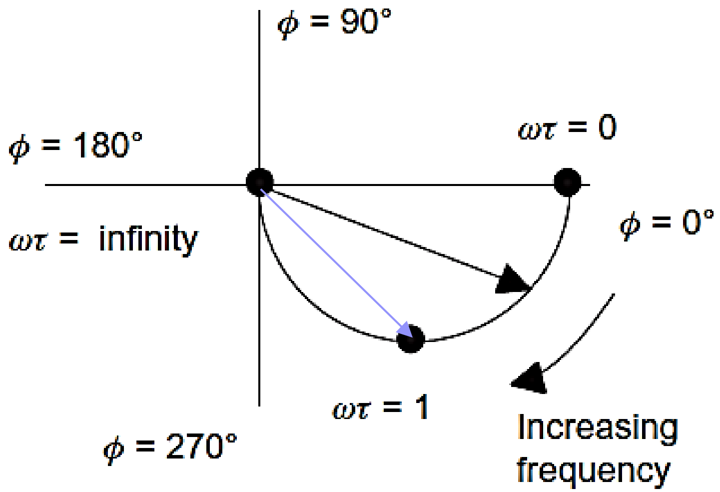
The magnitude is thus:

$$\boxed{\text{Magnitude} = \frac{1}{\sqrt{1+\omega^2\tau^2}}}$$

and the phase is:

$$\boxed{\text{Phase} = -\tan^{-1}\omega\tau}$$

At zero frequency the magnitude is 1 and the phase 0°. At infinite frequency the magnitude is zero and the phase is $-90°$. When $\omega\tau = 1$ the magnitude is $1/\sqrt{2}$ and the phase is $-45°$. Substitution of other values leads to the result shown in Figure of a semicircular plot.

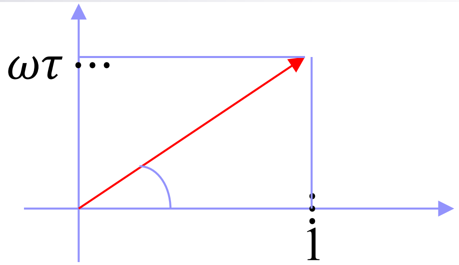


Nyquist diagram for a first-order system.

$s = \cancel{\sigma} + j\omega$

$$\boxed{GH = \frac{s+2}{s(s+10)}}$$

System type 1



Subst $s = j\omega$. → frequency domain

Rationalisation

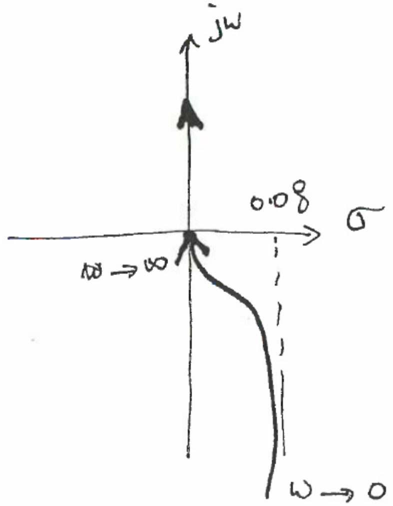
$$GH = \frac{j\omega+2}{j\omega(j\omega+10)} \cdot \frac{(-j\omega+10)}{(-j\omega+10)}$$

$$= \frac{\omega^2+20 + j(10\omega - 2\omega)}{j\omega(\omega^2+100)} \cdot \frac{j\omega}{j\omega}$$

$$\boxed{GH = \frac{8\omega - j(\omega^2+20)}{\omega(\omega^2+100)}} \quad (1)$$

Use, $\omega \to 0$, $Re = 8/100$, $Im \to \infty$

$\omega \to \infty$, $Re \to 0$, $Im \to \infty$



Polar Plot

POLAR Co-od $|GH| = \frac{\sqrt{(\omega^2+20^2)^2 + 64\omega^2}}{\omega(\omega^2+100)}$, $\angle GH = \tan^{-1}\left(\frac{\omega}{2}\right) - 90^\circ - \tan^{-1}\left(\frac{\omega}{10}\right)$

CARTESIAN Co-od $Real(GH) = \frac{8}{\omega^2+100}$, $Im(GH) = -\frac{\omega^2+20}{\omega(\omega^2+100)}$

$$\boxed{* \; G(s) = \frac{s+2}{s^2(s+10)}}$$

System Type : 2

subst $s = j\omega$ (freq domain)

$s = \cancel{\sigma} + j\omega$

Rationalisation

$$G(j\omega) = \boxed{\frac{j\omega+2}{-\omega^2(j\omega+10)}} \cdot \frac{(-j\omega+10)}{(-j\omega+10)} = -\frac{20+\omega^2+j8\omega}{\omega^2(\omega^2+100)}$$

CARTESIAN co-ordinates

$$Re[G(j\omega)] = -\frac{20+\omega^2}{\omega^2(\omega^2+100)}, \quad Im[G(j\omega)] = -\frac{8\omega}{\omega^2(\omega^2+100)}$$

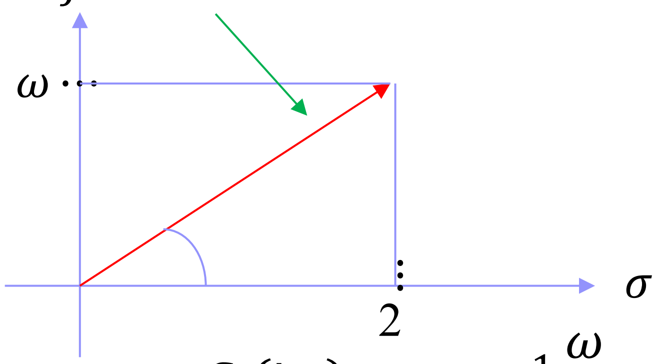
POLAR coordinates

$$\boxed{|G(j\omega)| = \frac{\sqrt{\omega^2+4}}{\omega^2\sqrt{\omega^2+100}}}, \quad \measuredangle G(j\omega) = \tan^{-1}\frac{\omega}{2} - 180^\circ - \tan^{-1}\frac{\omega}{10}$$

- Low freq region: $\omega \to \epsilon$ $\quad \measuredangle G(\epsilon) > -180^\circ$
- High freq region: $\omega \to \infty$ $\quad \measuredangle G(\infty) \to -180^\circ$

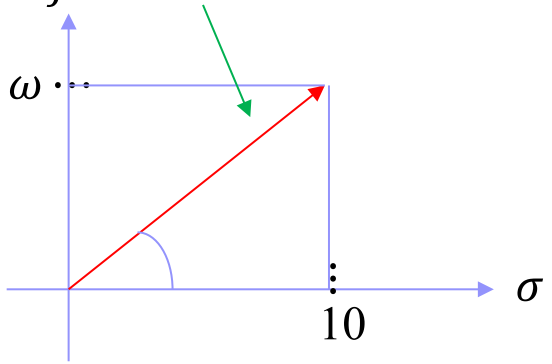
$|G(\omega \to \infty)| \to 0$

$|G(j\omega| = \sqrt{\omega^2+4})$



$$\measuredangle G(j\omega) = \tan^{-1}\frac{\omega}{2}$$

$|G(j\omega| = \sqrt{\omega^2+100})$



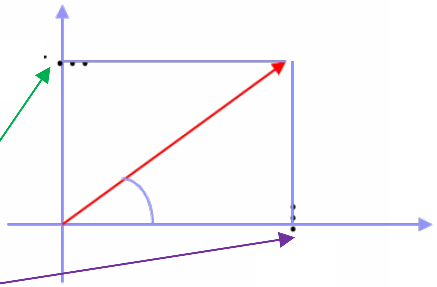
$$\measuredangle G(j\omega) = \tan^{-1}\frac{\omega}{10}$$

Frequency Response Example 1

Determine the magnitude and phase of the output from a system when subject to a sinusoidal input of $2 \sin 3t$ if it has a transfer function of $G(s) = 2/(s+1)$.

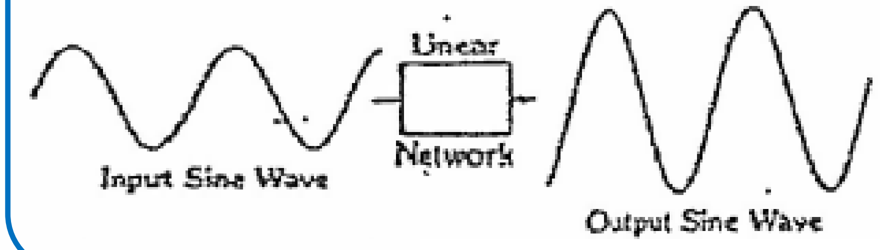
The frequency response function is obtained by replacing s by $j\omega$:

$$G(j\omega) = \frac{2}{j\omega + 1}$$



Multiplying top and bottom of the equation by $(-j\omega + 1)$ gives:

$$G(j\omega) = \frac{2}{j\omega + 1} \times \frac{-j\omega + 1}{-j\omega + 1} = \frac{-j2\omega + 2}{\omega^2 + 1} = \frac{2}{\omega^2 + 1} - j\frac{2\omega}{\omega^2 + 1}$$

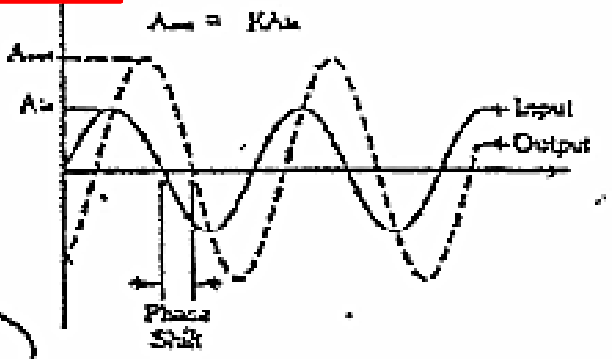


The magnitude $a + jb$ is $\sqrt{(a^2 + b^2)}$ and is thus for $G(j\omega)$:

$$\boxed{|G(j\omega)| = \sqrt{\frac{2^2}{(\omega^2+1)^2} + \frac{2^2\omega^2}{(\omega^2+1)^2}} = \frac{2}{\sqrt{(\omega^2+1)}}}$$

and the phase angle is given by:

$$\boxed{\tan\phi = -\omega}$$



For the specified input we have $\omega = 3$ rad/s. The magnitude is thus:

$$|G(j\omega)| = \frac{2}{\sqrt{(\omega^2+1)}} = \frac{2}{\sqrt{3^2+1}} = 0.63$$

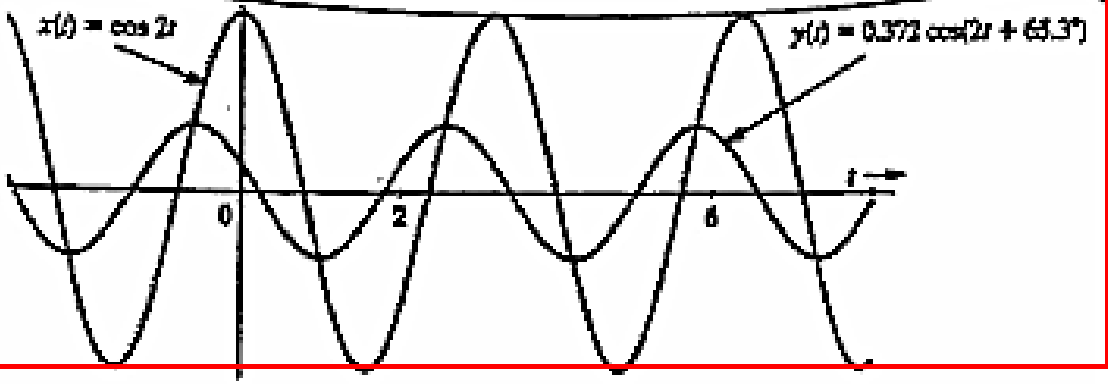
Linear network response to a sine wave input

and the phase is given by $\tan\phi = -3$ as $\phi = -72°$. This is the phase angle between the input and the output. Thus, the output is the sinusoidal signal of the same frequency as the input signal and described by $2 \times 0.63 \sin(3t - 72°) = 1.26 \sin(3t - 72°)$.

For input $x(t)=\cos 2t$, we have:

$$|H(j2)| = \frac{\sqrt{2^2+0.01}}{\sqrt{2^2+25}} = 0.372 \qquad \Phi(j2) = \tan^{-1}\left(\frac{2}{0.1}\right) - \tan^{-1}\left(\frac{2}{5}\right) = 65.3^\circ$$

Therefore $$y(t) = 0.372\cos(2t + 65.3^\circ)$$



- Find the frequency response of a system with transfer function:

$$H(s) = \frac{s+0.1}{s+5}$$

- Then find the amplitude and phase response $y(t)$ for inputs:
  (i) $x(t)=\cos 2t$ and (ii) $x(t)=\cos(10t-50^\circ)$

- Then find the amplitude and phase response $y(t)$ for inputs:
  (i) $x(t)=\cos 2t$ and (ii) $x(t)=\cos(10t-50^\circ)$

- Substitute $s=j\omega$

$$H(j\omega) = \frac{j\omega+0.1}{j\omega+5}$$

$$|H(j\omega)| = \frac{\sqrt{\omega^2+0.01}}{\sqrt{\omega^2+25}} \qquad \Phi(j\omega) = \angle H(j\omega) = \tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

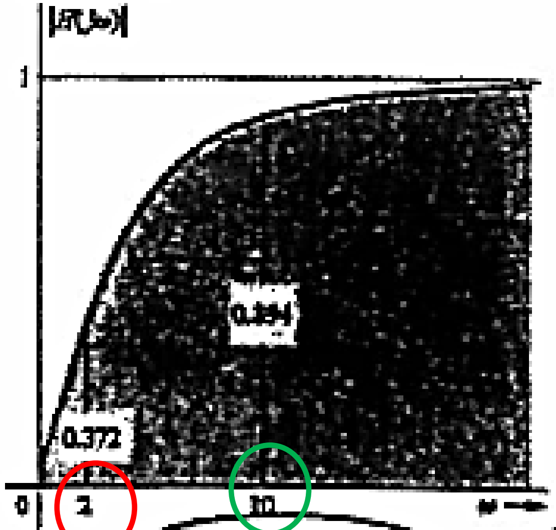
- For input $x(t)=\cos(10t-50^\circ)$, we will use the amplitude and phase response curves directly:

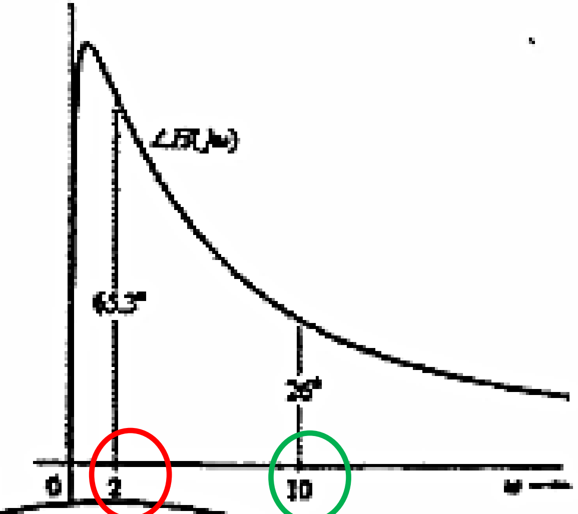
$$|H(j10)| = 0.894$$

$$\Phi(j10) = \angle H(j10) = 26^\circ$$

- Therefore

$$y(t) = 0.894\cos(10t - 50^\circ + 26^\circ) = 0.894\cos(10t + 24^\circ)$$





for input $x(t)=\cos 2t$ and $x(t)=\cos(10t-50^\circ)$

Obtain a polar plot for a system with transfer function

$$G(s) = \frac{1}{(1+2s)(s^2+s+1)}$$

Letting $s = j\omega$

Cartesian coordinates

$$G(j\omega) = \frac{1}{(1+j2\omega)(1-\omega^2+j\omega)}$$

$$= \frac{1}{1-3\omega^2+j(3\omega-2\omega^3)}$$

$$G(j\omega) = \frac{(1-3\omega^2)-j\omega(3-2\omega^2)}{1+3\omega^2-3\omega^4+4\omega^6}$$

Now insert a range of numerical values of frequency ω rad/second

e.g. for $\omega = 1$ $\quad G(j\omega) = -0.4 - j0.2$

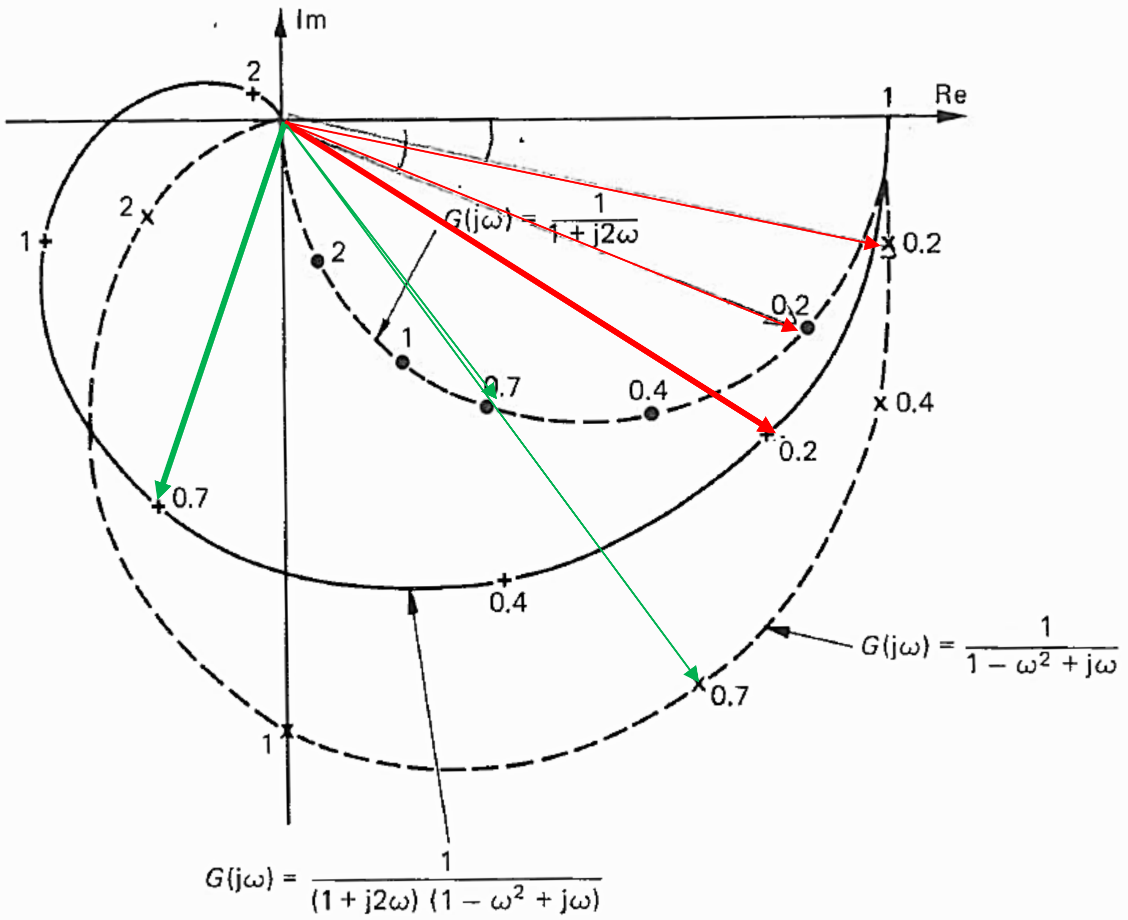
$\omega = 0.5$ $\quad G(j\omega) = +0.154 - j0.77$

etc.

Polar coordinates

Tabular evaluation of harmonic response

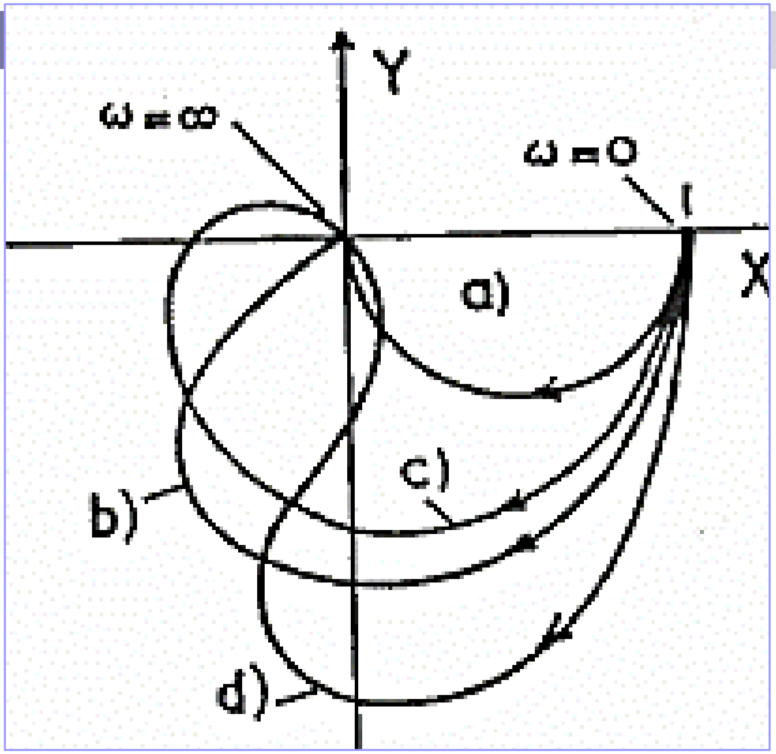
Frequency (rad/second)	0	0.2	0.4	0.7	1	2
$\left\lvert \frac{1}{1+j2\omega} \right\rvert = \frac{1}{\sqrt{(1+4\omega^2)}}$	1	0.928	0.781	0.581	0.447	0.243
$\left\lvert \frac{1}{1-\omega^2+j\omega} \right\rvert = \frac{1}{\sqrt{\{(1-\omega^2)^2+\omega^2\}}}$	1	1.020	1.075	1.155	1.000	0.277
$\lvert G(j\omega) \rvert$	1	0.947	0.840	0.671	0.447	0.067
$\angle\left(\frac{1}{1+j2\omega}\right) = \tan^{-1} 2\omega$ (degrees lag)	0	21.8	38.7	54.5	63.4	76.0
$\angle\left(\frac{1}{1-\omega^2+j\omega}\right) = \tan^{-1} \frac{\omega}{1-\omega^2}$ (degrees lag)	0	11.8	25.5	53.9	90.0	146.3
$\angle G(j\omega)$ (degrees lag)	0	33.6	64.2	108.4	153.4	222.3

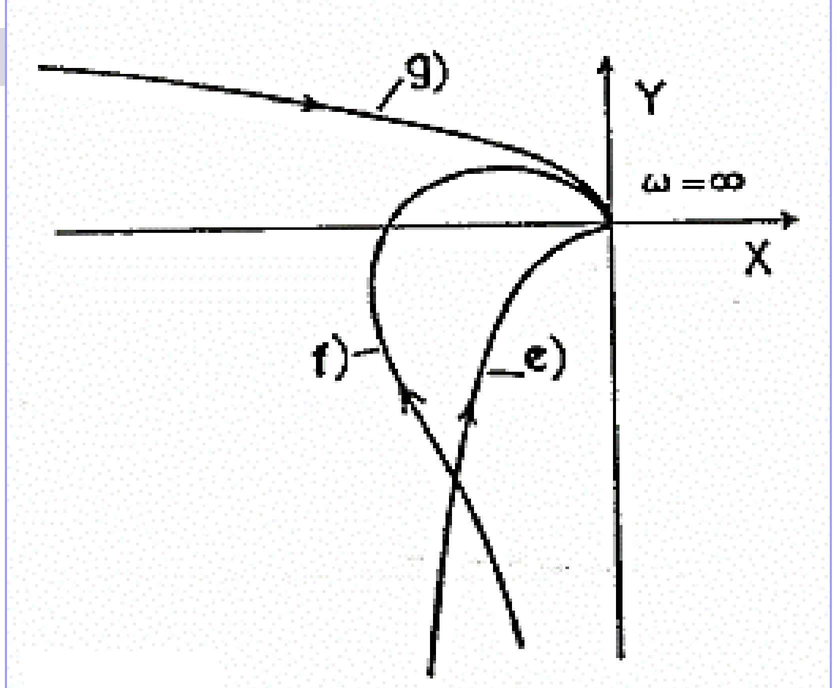


Polar plot for system with transfer function $G(s) = \frac{1}{(1+2s)(s^2+s+1)}$

Sketches of Polar plots of different

- system type number,
- order of the system
- rank of the system





Typical Nyquist diagrams for:

System type 0

(a) $G(s) = \dfrac{1}{1 + sT}$;

(b) $G(s) = \dfrac{1}{1 + bs + as^2}$;

(c) $G(s) = \dfrac{1}{(1 + sT)(1 + bs + as^2)}$;

(d) $G(s) = \dfrac{1 + sT}{1 + bs + as^2}$;

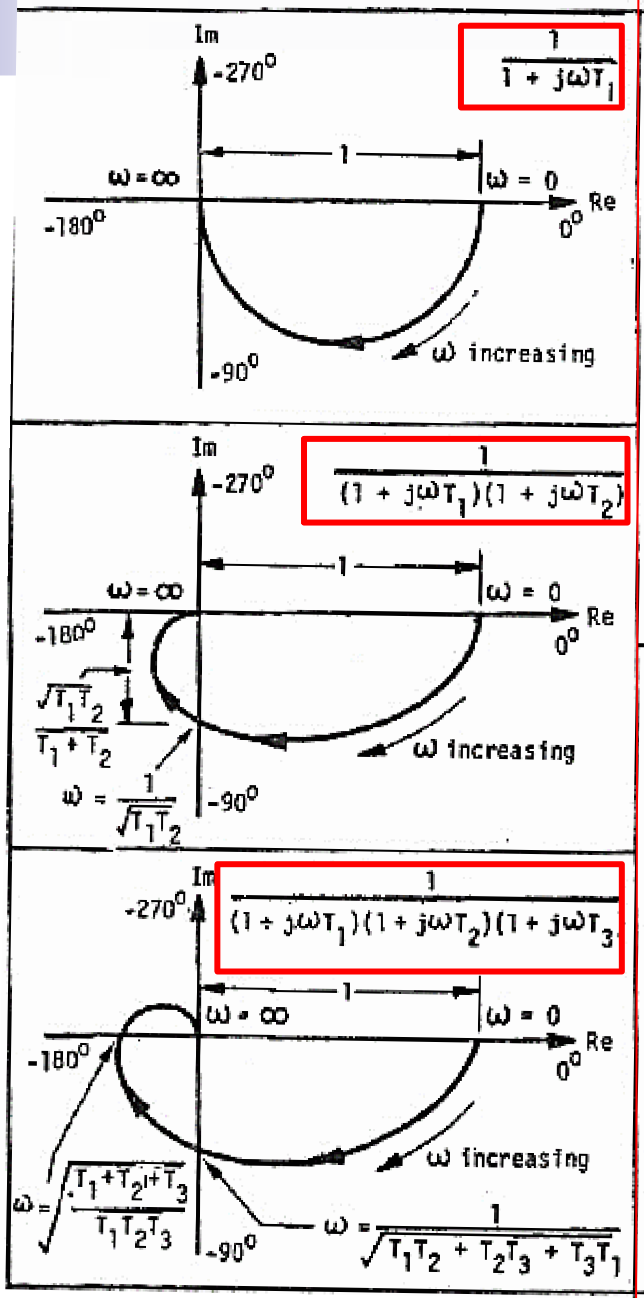
System type 1

(e) $G(s) = \dfrac{1}{s(1 + sT)}$;

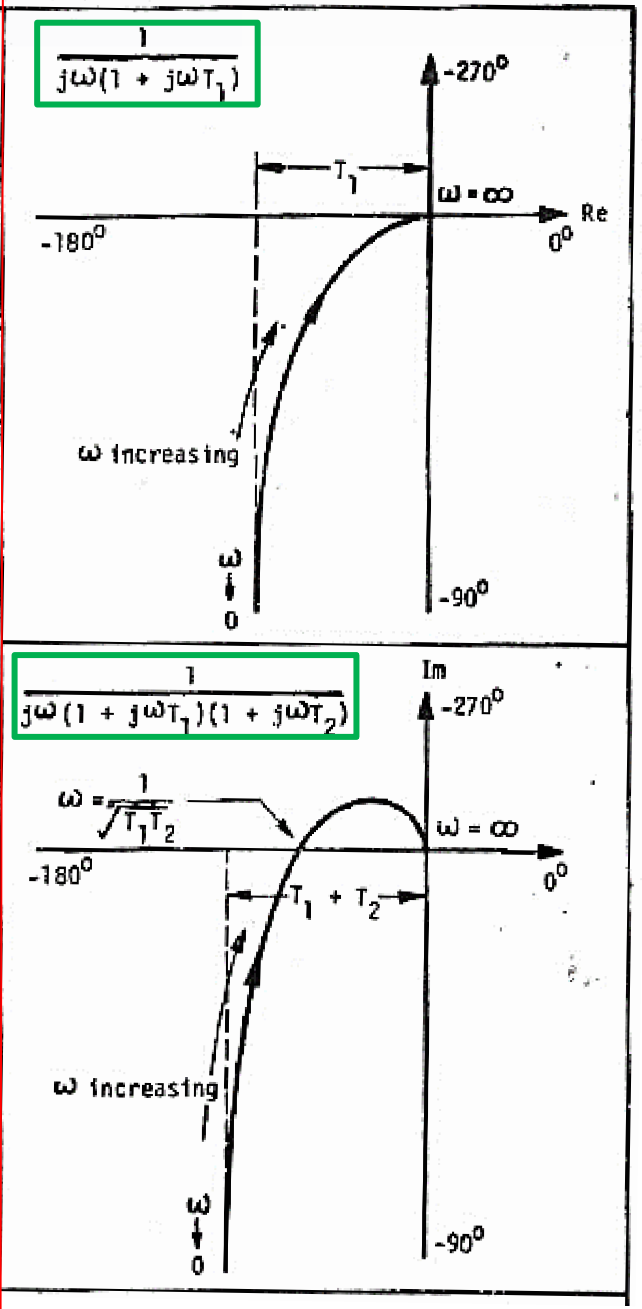
(f) $G(s) = \dfrac{1}{s(1 + bs + as^2)}$;

System type 2

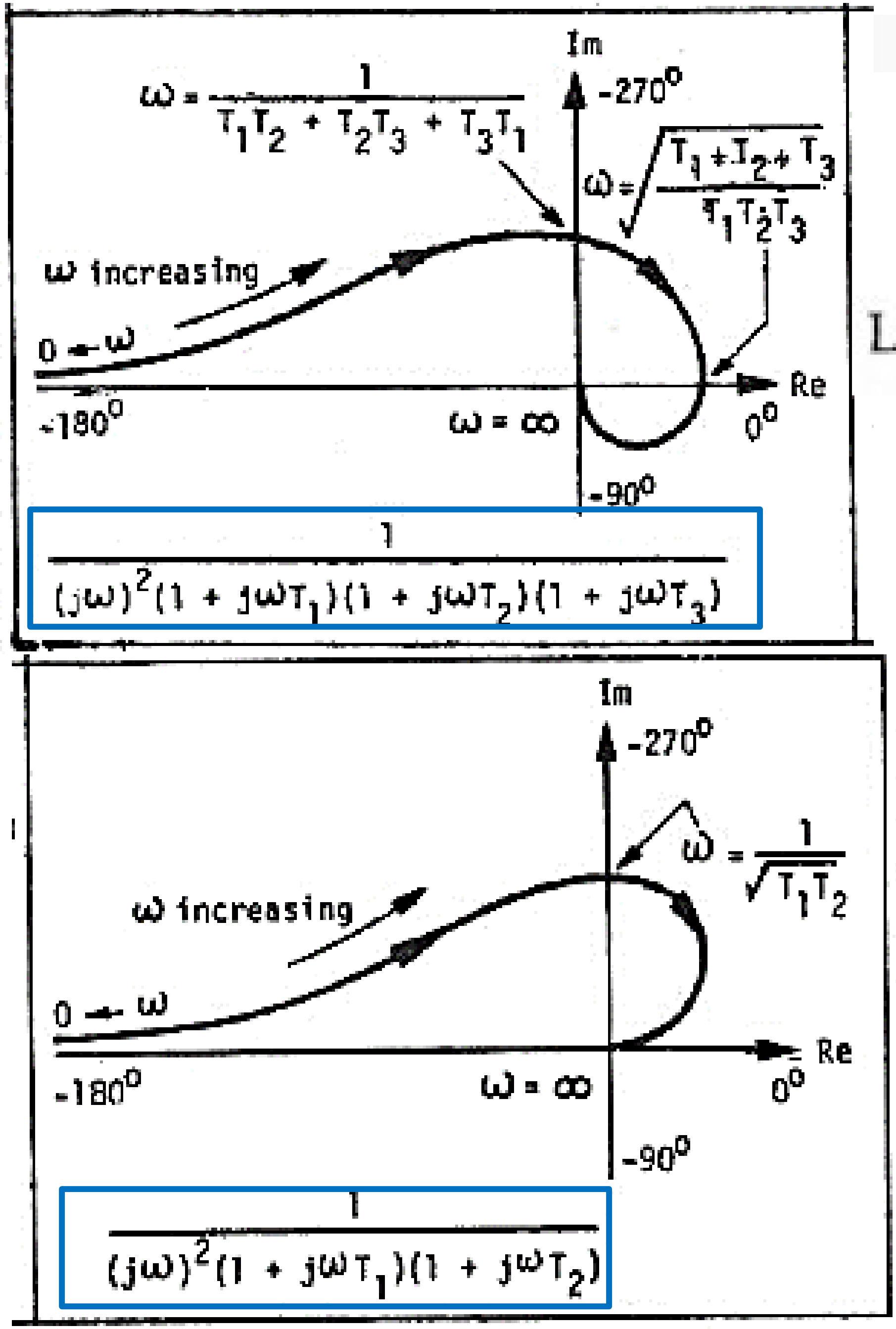
(g) $G(s) = \dfrac{1}{s^2(1 + sT)}$



System Type 0

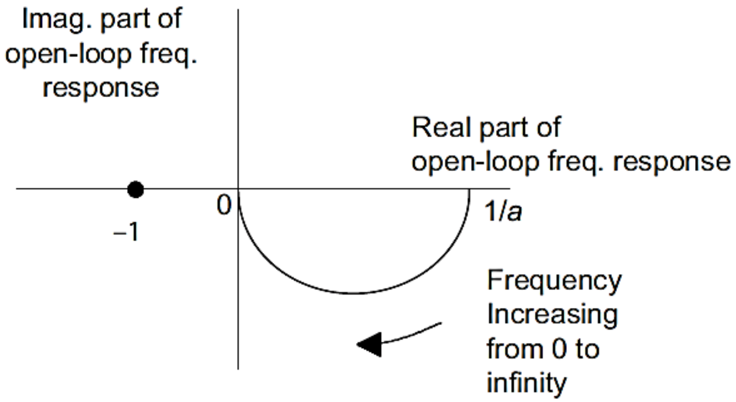


System Type 1

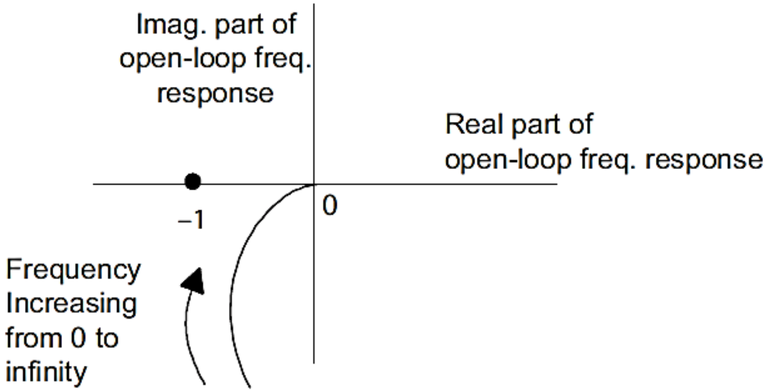


Letting $s = j\omega$

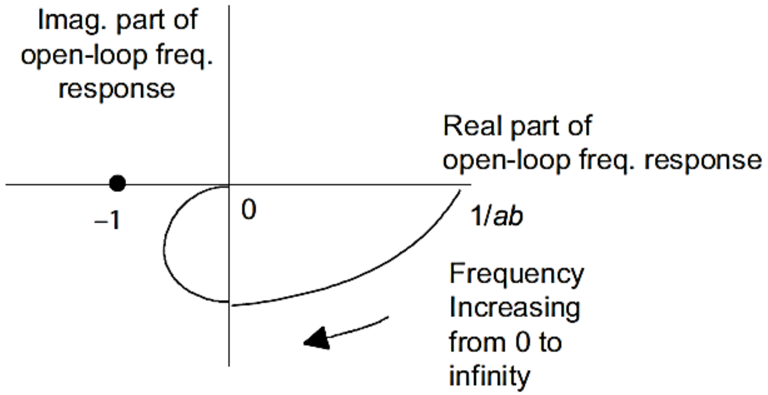
System Type 2



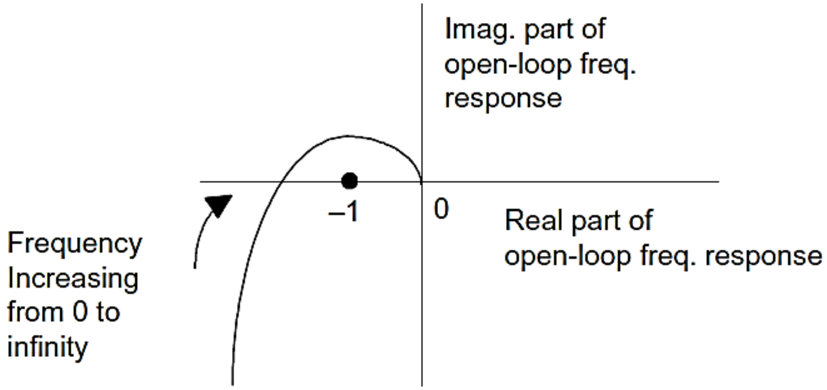
$G(s)H(s) = K/(s+a)$, stable for all values of $K > 0$.



$G(s)G(s) = K/s(s+a)$, stable for all values of $K > 0$.

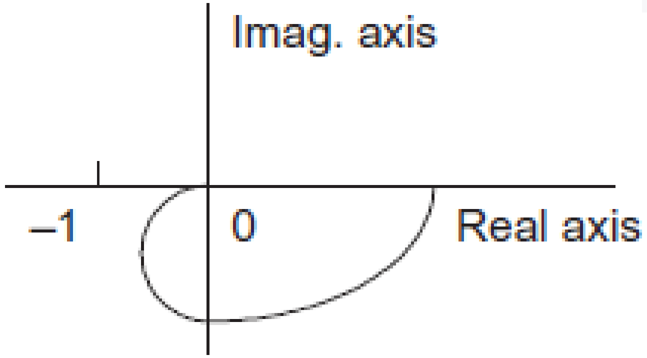


$G(s)H(s) = K/(s+a)(s+b)$, stable for all of $K > 0$.

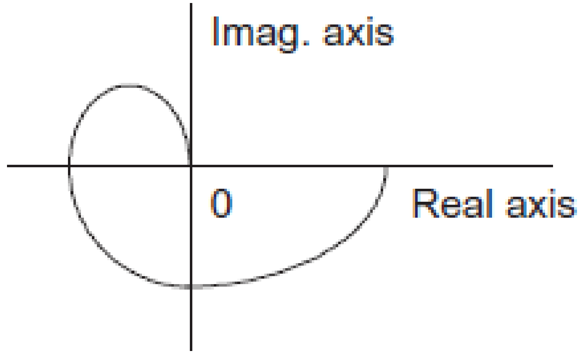


$G(s)H(s) = K/s(s+a)(s+b)$; this is values unstable with large K but can become stable if K is reduced, the point at which the plot crosses the axis being $-K/(a+b)$ and so stability is when $-K/(a+b) > -1$.

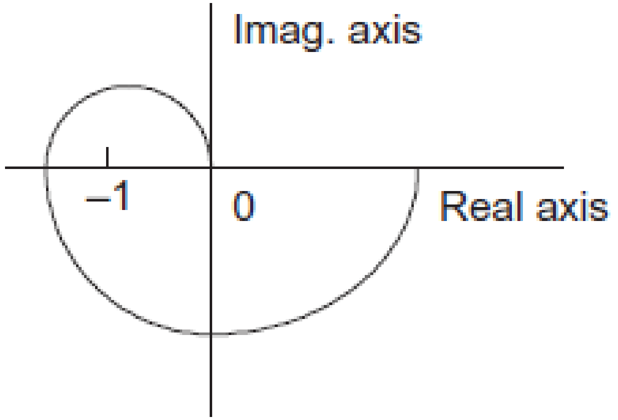
1. The system giving the Nyquist diagram shown
   A. Stable for all frequencies
   B. Stable only at low frequencies
   C. Unstable at all frequencies
   D. Marginally stable



2. The system giving the Nyquist diagram shown has a value of $K/10$ where it cuts the negative real axis and so is:
   A. Stable when K is greater than 10
   B. Stable when K is equal to 10
   C. Stable when K is less than 10
   D. Stable for all values of K



3. The system giving the Nyquist diagram shown i
   A. Stable for all frequencies
   B. Stable only at low frequencies
   C. Unstable at all frequencies
   D. Marginally stable



1. A
2. C
3. C

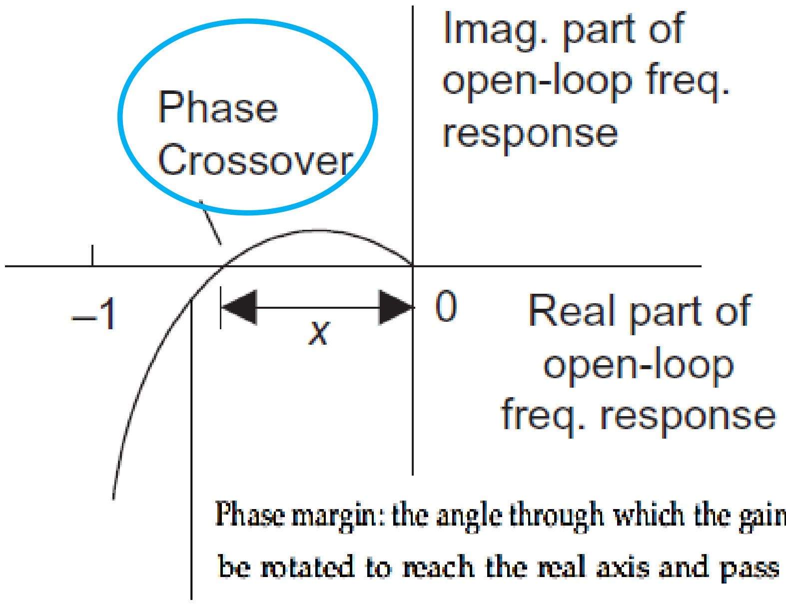
1. *Gain margin*

The phase crossover frequency is the frequency at which the phase angle first reaches $-180°$ and thus is the point where the Nyquist plot crosses the real axis On a Nyquist plot the $(-1, j0)$ point is the point separating stability from instability. The gain margin is the amount by which the actual gain must be multiplied before the onset of instability. Thus if the plot cuts the negative real axis at $-x$, it has to be multiplied by $1/x$ to give the value -1 and so the gain margin, which is expressed in dB, is $20 \lg(1/x)$.

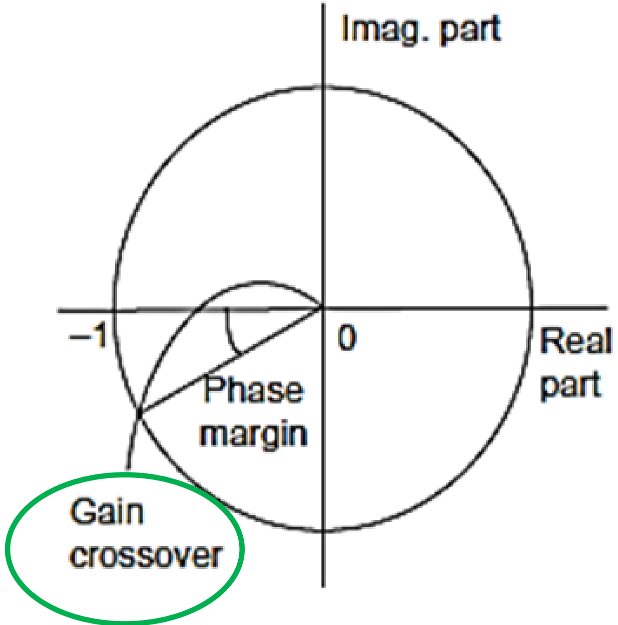
When the open-loop plot goes through the $(-1, j0)$ point the gain margin is 0 dB, the system being on the margin of instability. When the open-loop plot goes to the left of $(-1, j0)$ point the gain margin is negative in dB, the system being unstable. When the open-loop plot goes to the right of $(-1, j0)$ point the gain margin is positive in dB, the system being stable. When the open-loop plot does not intersect the negative real axis the gain margin is infinite in dB.

2. *Phase margin*

The phase margin is defined as the angle in degrees by which the phase angle is smaller than $-180°$ at the gain crossover, the gain crossover being the frequency at which the open-loop gain first reaches 1. Thus, with a Nyquist plot, if we draw a circle of radius 1 centred on the origin, then the point at which it intersects the Nyquist line gives the gain crossover. The phase margin is the angle through which this gain crossover line must be rotated about the origin to reach the real axis and pass through the $(-1, j0)$ point

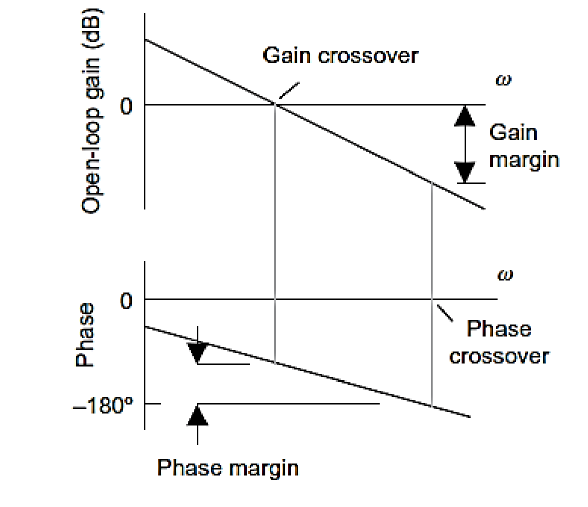


Phase margin: the angle through which the gain crossover line must be rotated to reach the real axis and pass through the (−1, j0) point.

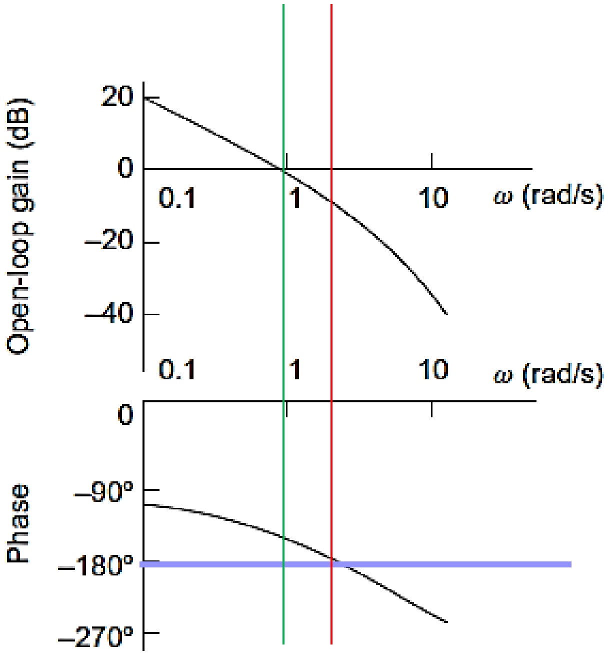


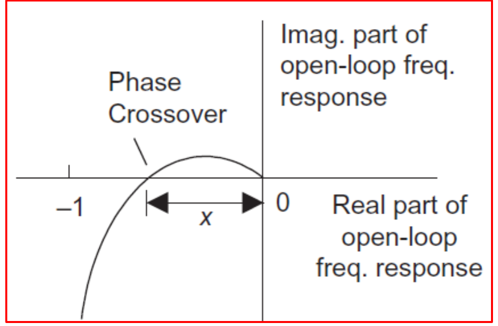
6. For the Bode plot shown determine (a) whether the system is stable, (b) the gain margin and (c) the phase margin.

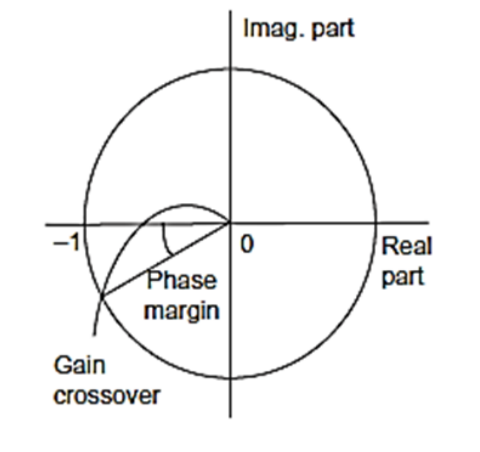
(a) The system is stable because it has an open-loop gain less than 1 when the phase is $-180°$.
(b) The gain margin is about 12 dB.
(c) The phase margin is about 30°.



Stability and Bode plots.







7. Determine the gain margin and the phase margin for a system that gave the following open-loop experimental frequency response data: at frequency 0.005 Hz a gain of 1.00 and phase $-120°$, at 0.010 Hz a gain of 0.45 and phase $-180°$.

The gain margin is the factor by which the gain must be multiplied at the phase crossover to have the value 1. The phase crossover occurs at 0.010 Hz and so the gain margin is $1.00/0.45 = 2.22$. The phase margin is the number of degrees by which the phase angle is smaller than $-180°$ at the gain crossover. The gain crossover is the frequency at which the open-loop gain first reaches the value 1 and so is 0.005 Hz. Thus, the phase margin is $180° - 120° = 60°$.

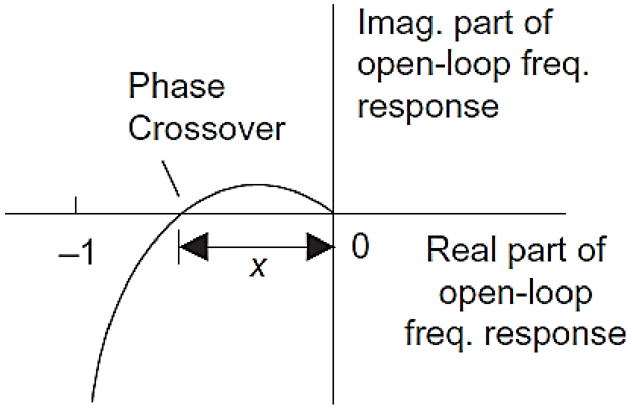
4. Sketch the Nyquist diagram for a system having an open-loop transfer function of $1/[s(s+1)]$.

5. With a Nyquist diagram for the open-loop frequency response for a system, what is the condition for the system to be stable?

6. Determine the gain margin and the phase margin for a system which gave the following open-loop frequency response:

Freq. rad/s	1.4	2.0	2.6	3.2	3.8
Magnitude	1.6	1.0	0.6	0.4	0.2
Phase deg.	−150	−160	−170	−180	−190

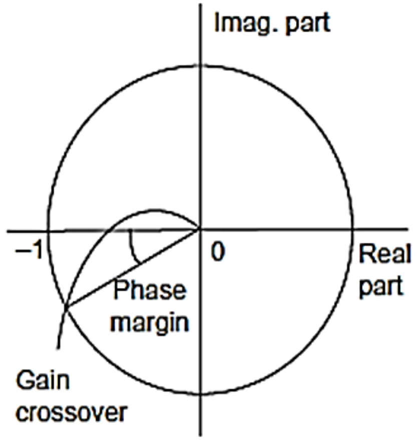
20 log (1 / 0.4) = 20 log 2.5 = 8 db



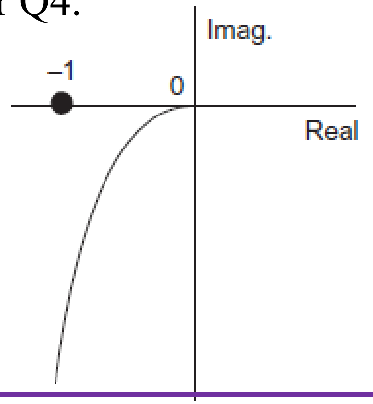
7. Determine the gain margin and the phase margin for a system which gave the following open-loop frequency response:

Freq. rad/s	4	5	6	8	10
Gain	3.2	2.3	1.7	1.0	0.6
Phase in deg.	−140	−150	−157	−170	−180

20 log (1 / 0.6)= 20 log 1.666 = 4.4 db



Answer for Q4.



<u>Answers</u>

5. (−1, j0) point not to be enclosed

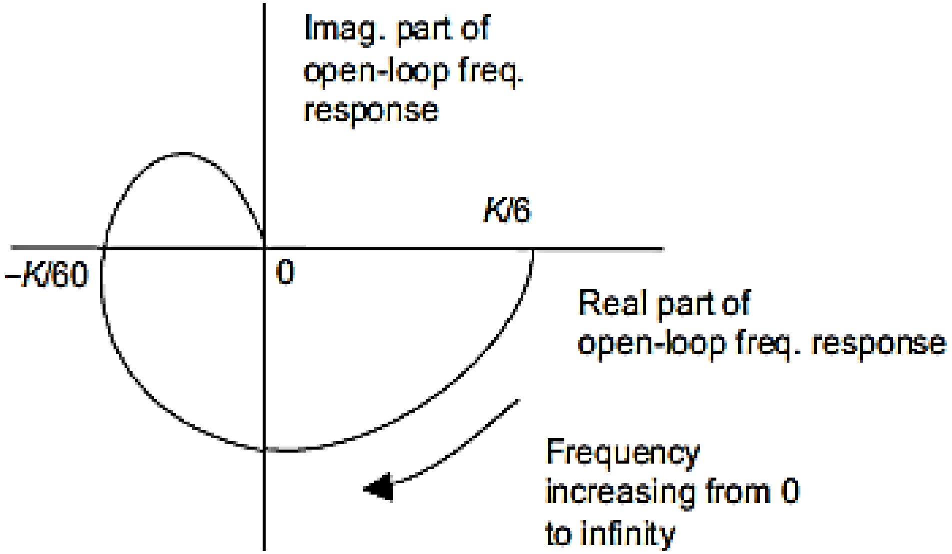
6. 8.0 dB, 20°

7. 4.4 dB, 10°

Plot the Nyquist diagram for a system with the open-loop transfer function $K/[(s+1)(s+2)(s+3)]$ and consider the value of K needed for stability.

The open-loop frequency response is:

$$\frac{K}{(j\omega+1)(j\omega+2)(j\omega+3)}$$



The magnitude and phase are:

$$\text{Magnitude} = \frac{K}{\sqrt{(\omega^2+1)(\omega^2+4)(\omega^2+9)}}$$

$$\text{Phase} = \tan^{-1}\left(\frac{\omega}{1}\right) + \tan^{-1}\left(\frac{\omega}{2}\right) + \tan^{-1}\left(\frac{\omega}{3}\right)$$

When $\omega = 0$ then the magnitude is $K/6$ and the phase is 0°. When ω = infinity then the magnitude is 0 and the phase is 270°. We can use these and other points to plot the polar graph.

Alternatively we can consider the frequency response in terms of real and imaginary parts. We can write the open-loop frequency function as:

$$\frac{6K(1-\omega^2)}{(\omega^2+1)(\omega^2+4)(\omega^2+9)} + j\frac{\omega K(\omega^2-1)}{(\omega^2+1)(\omega^2+4)(\omega^2+9)}$$

When $\omega = 0$ then the imaginary part is zero and the real part is $K/6$. When ω = infinity then the imaginary part is zero and the real part is 0. The imaginary part will be zero when $\omega = \sqrt{11}$. This is a real part, and hence magnitude, of $-K/60$ and is the point at which the plot crosses the real axis. Thus for a stable system we must have $-K/60$ less than -1, i.e. K must be less than 60. shows the complete Nyquist plot (not to scale).

Determine the gain margin and the phase margin for a system with the open-loop transfer function $K/(s+1)(s+2)(s+3)$ with $K=20$. This system was discussed earlier in this chapter (see Figure 12.11 for the Nyquist plot).

The open-loop frequency response is:

$$\frac{K}{(j\omega+1)(j\omega+2)(j\omega+3)}$$

and this can be rearranged to give:

$$\frac{6K(1-\omega^2)}{(\omega^2+1)(\omega^2+4)(\omega^2+9)}+j\frac{\omega K(\omega^2-11)}{(\omega^2+1)(\omega^2+4)(\omega^2+9)}$$

The imaginary part will be zero when $\omega=\sqrt{11}$ and thus the real part is $-K/60$ and is the point at which the plot crosses the real axis. Hence, if we have $K=20$ then the plot intersects the negative real axis at $-20/60=-1/3$. The gain can thus be increased by a factor of 3 in order to reach the -1 point. The gain margin is thus $20 \lg 3 = 9.5$ dB.

The magnitude is:

$$\frac{K}{\sqrt{(\omega^2+1)(\omega^2+4)(\omega^2+9)}}$$

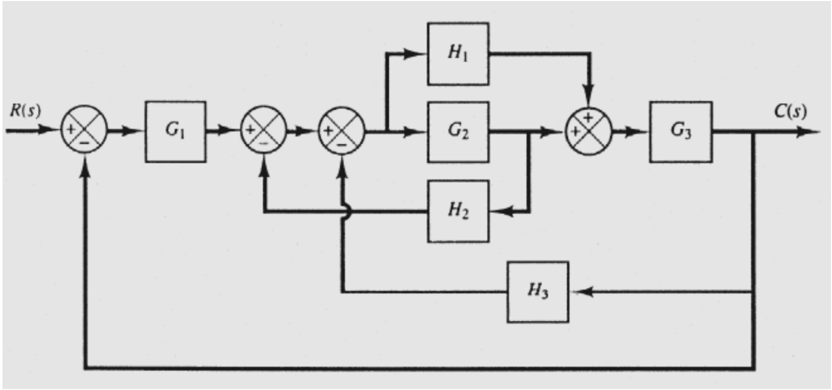
Thus, for $K=20$, the magnitude is 1 when $\omega=1.84$ rad/s. The phase is given by:

$$\text{Phase}=\tan^{-1}\left(\frac{\omega}{1}\right)+\tan^{-1}\left(\frac{\omega}{2}\right)+\tan^{-1}\left(\frac{\omega}{3}\right)$$

and so, at this frequency, the phase is $-135.5°$. Thus the phase margin is $44.5°$.

2 : Block Diagrams

Block Diagrams



Control Signal

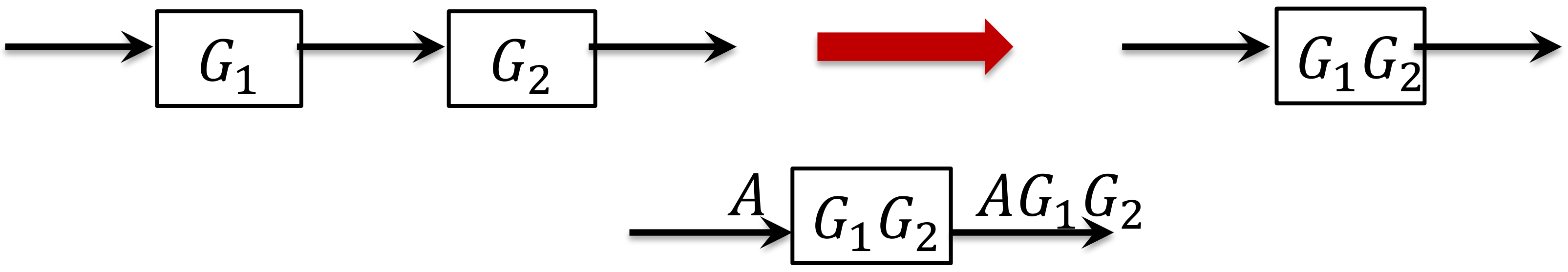
Transfer Function

Objectives

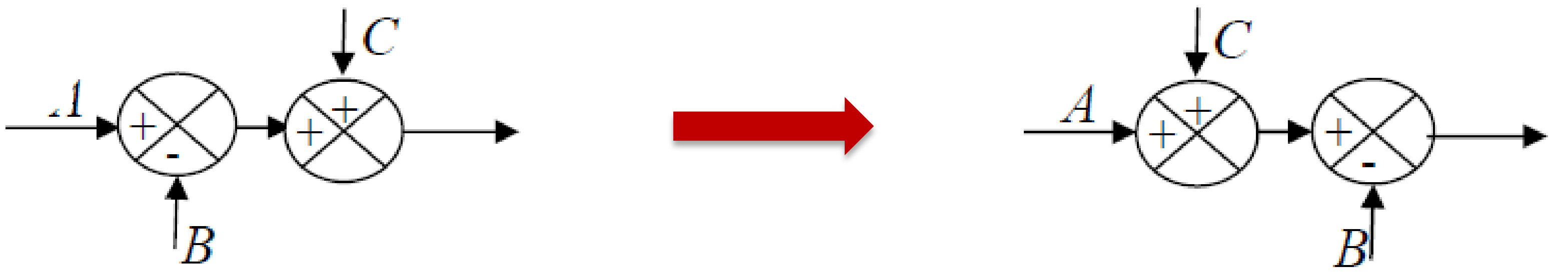
- Understand the relationship between control signals and transfer functions
- Simplify the block diagrams
- Establish the relationship between the input and output control signals

General Rules

1. Multiply

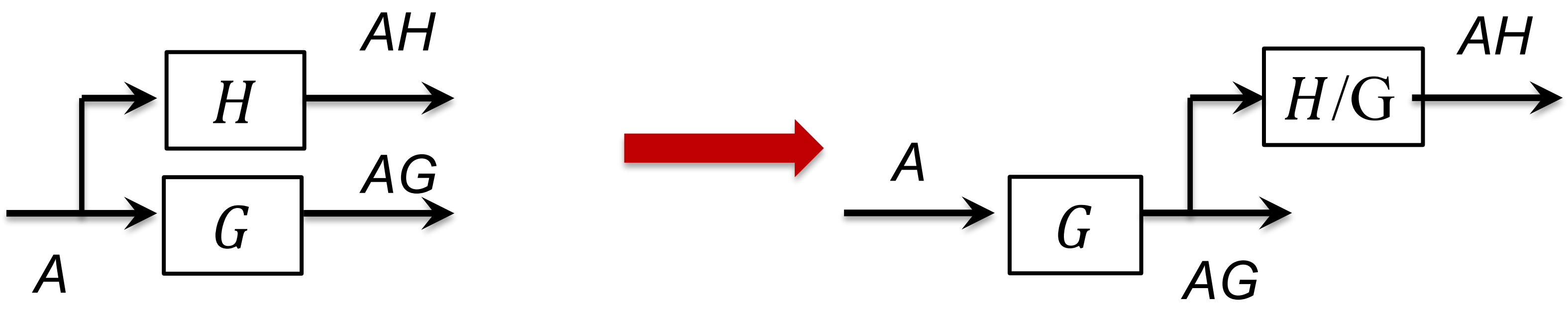


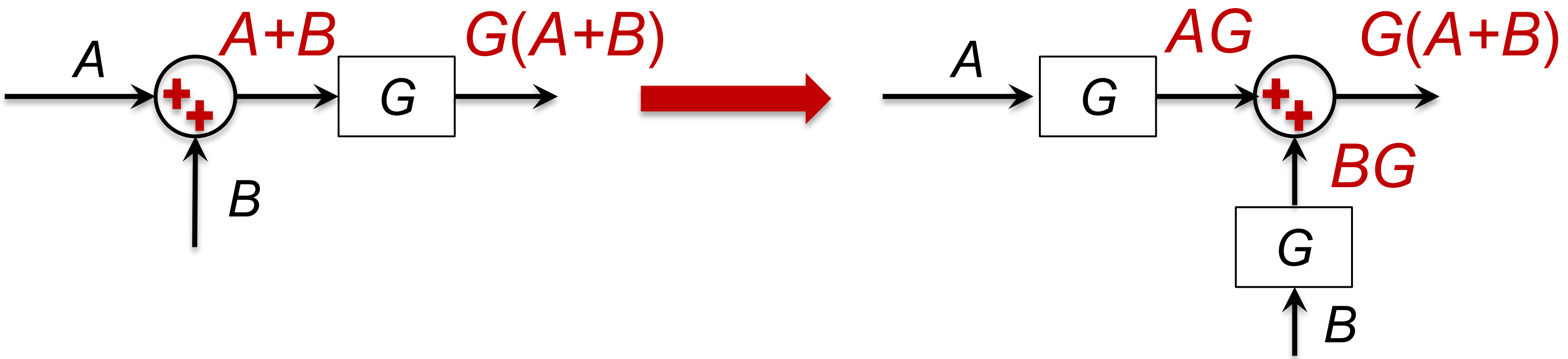
2. Summation



General Rules

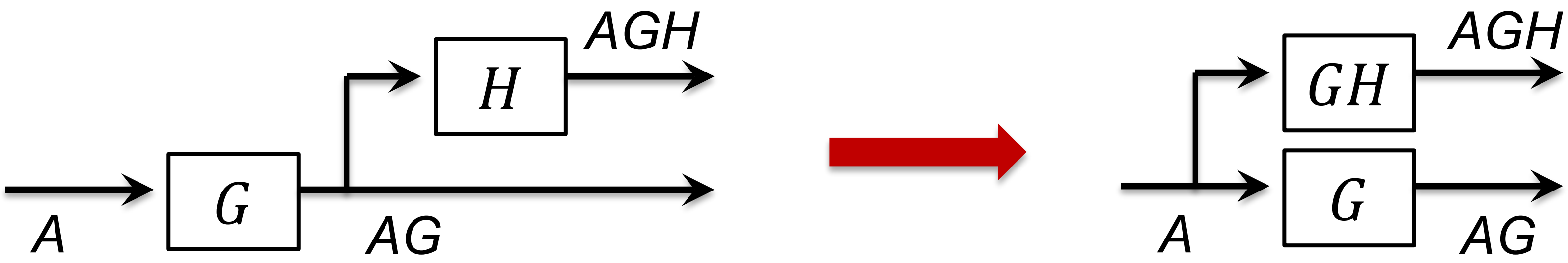
3. Pre-Multiply (G) -> Post-Multiply (G)

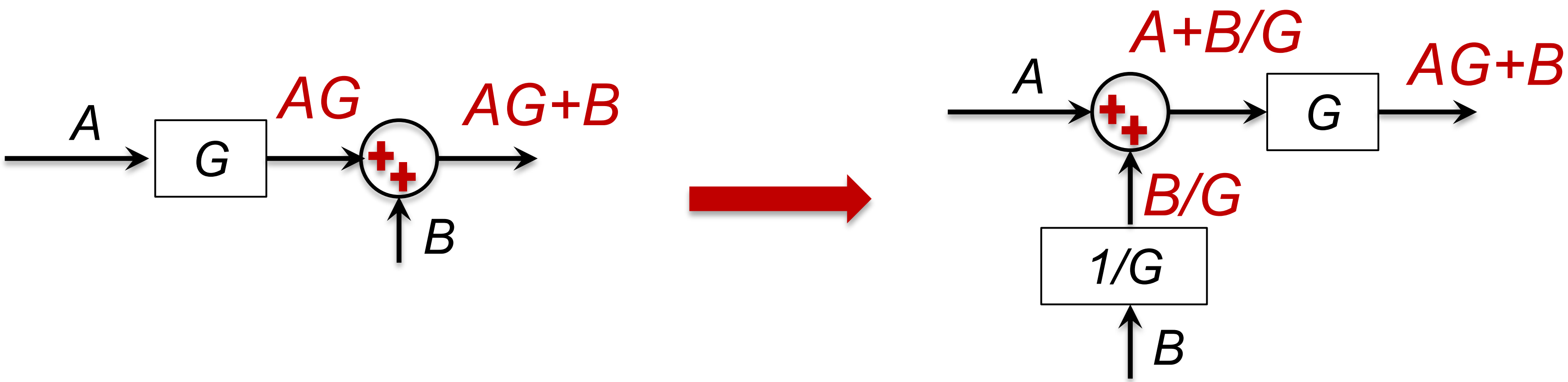




General Rules

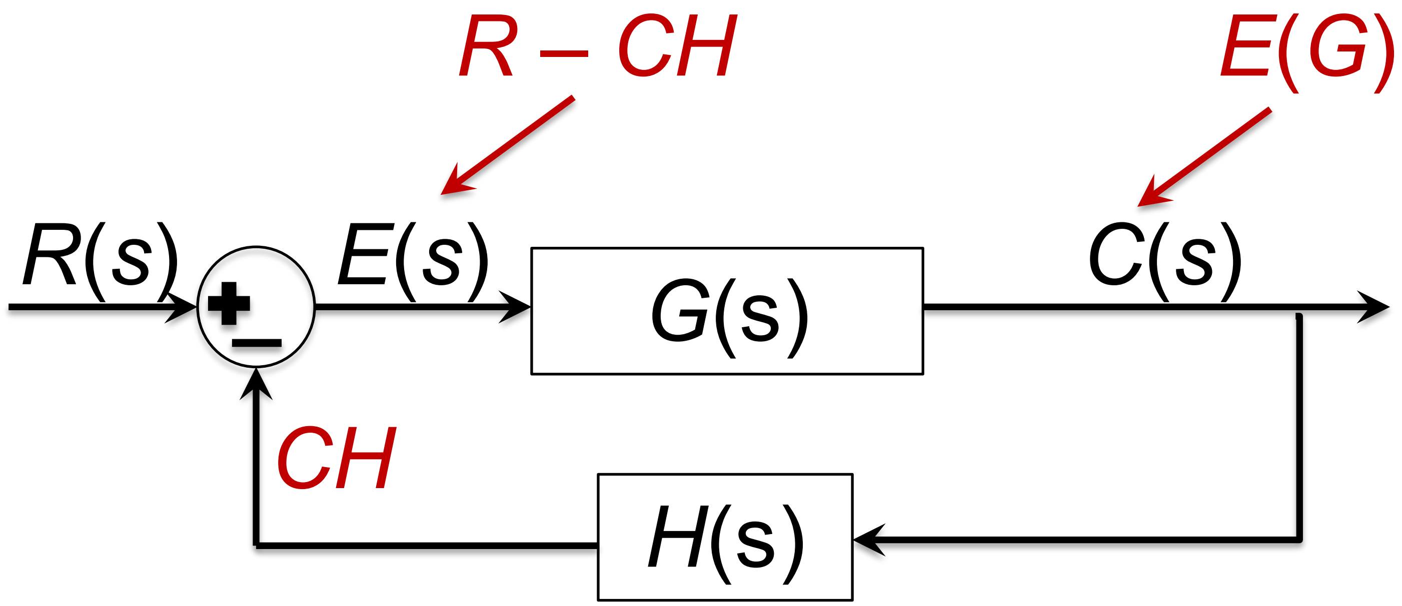
4. Post-Multiply (G) -> Pre-Multiply (G)





General Rules

5. Closed-loop



$$\boxed{\frac{C}{R} = \frac{G}{1 + GH}}$$

$EG = C$

$(R - CH)G = C,$

$RG = C(1+HG)$

G = direct T.F = Forward T.F

H = feedback T.F

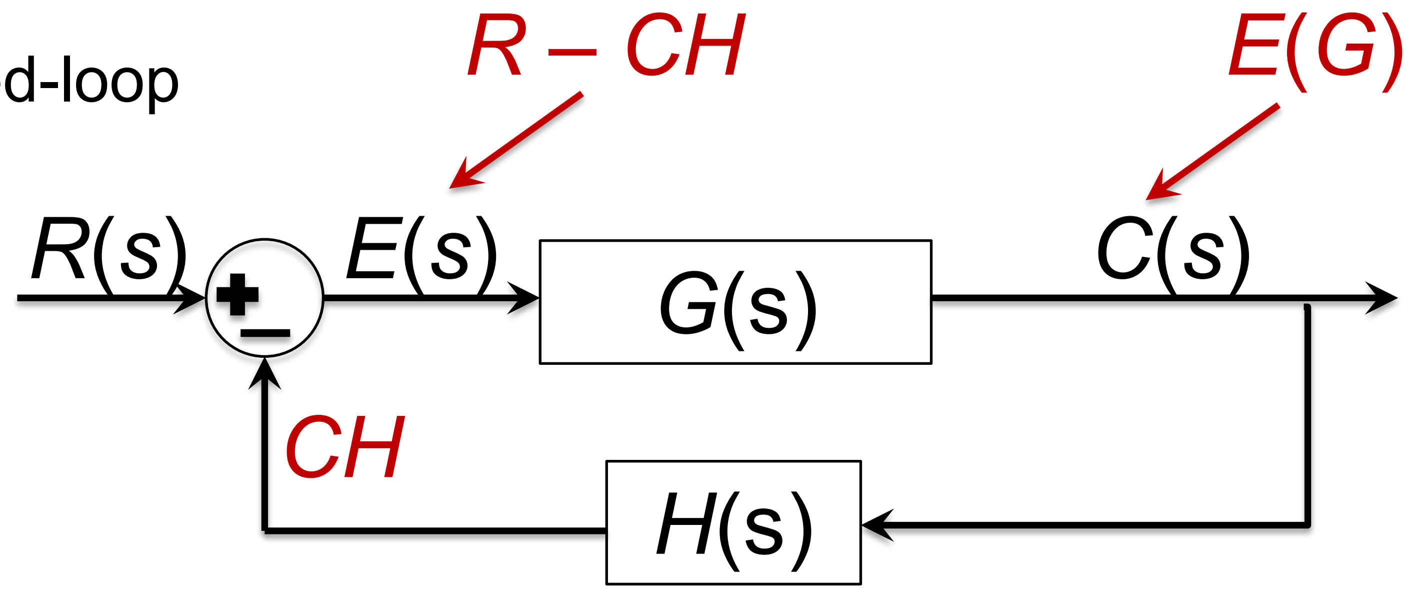
GH = loop T.F = O/L T.F

C/R = C/L T.T

E/R = 1/1+GH (Actuating Signal Ratio)

General Rules

5. Closed-loop



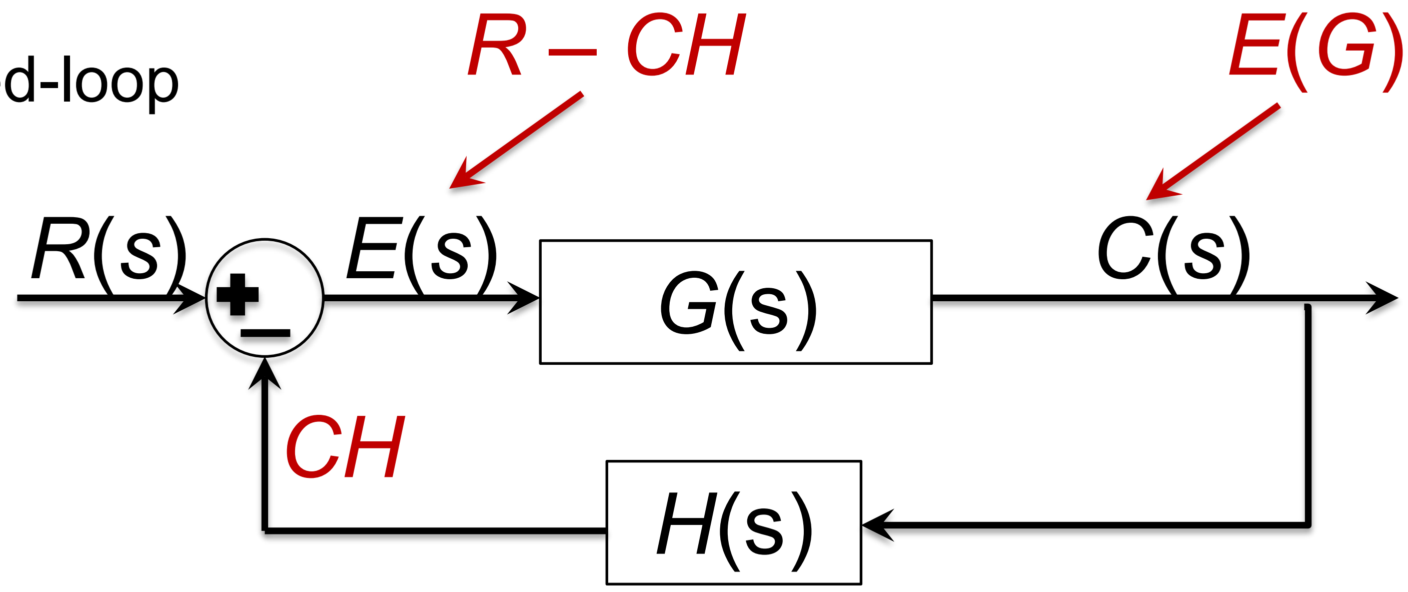
$$EG = C$$

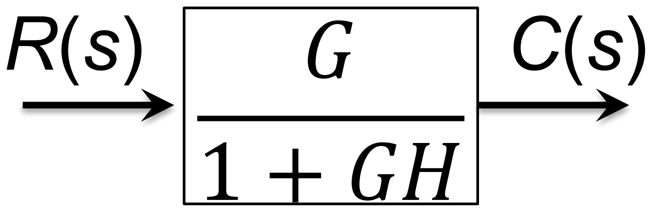
$$(R - CH)G = C, \quad RG = C(1+HG)$$

$$\boxed{\frac{C}{R} = \frac{G}{1+GH}}$$

General Rules

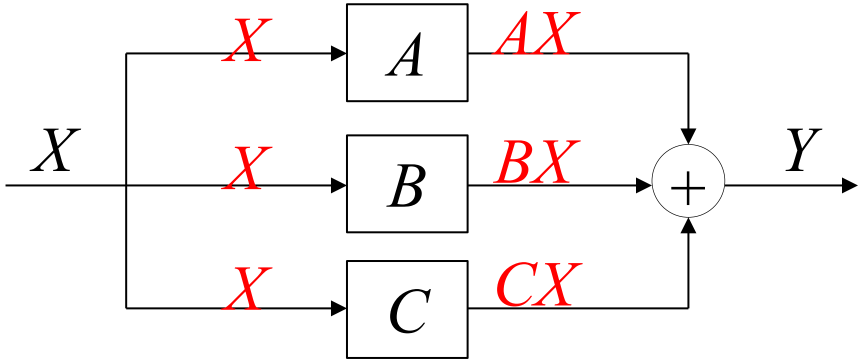
5. Closed-loop





Example I

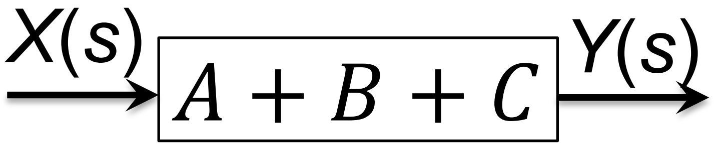
Aim: Establish the relationship between $Y(s)$ and $X(s)$



$$Y = AX + BX + CX$$

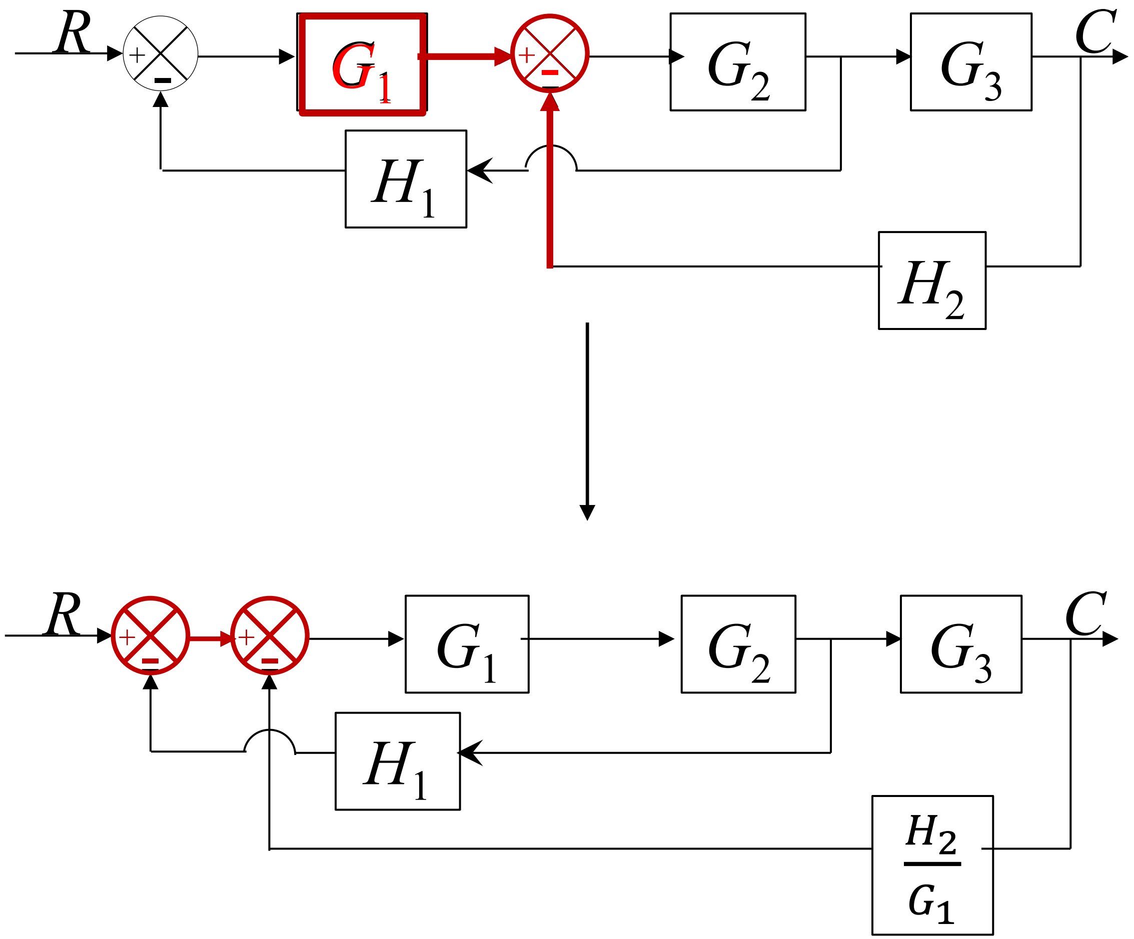
$$Y = (A + B + C)X$$

$$\frac{Y}{X} = A + B + C$$



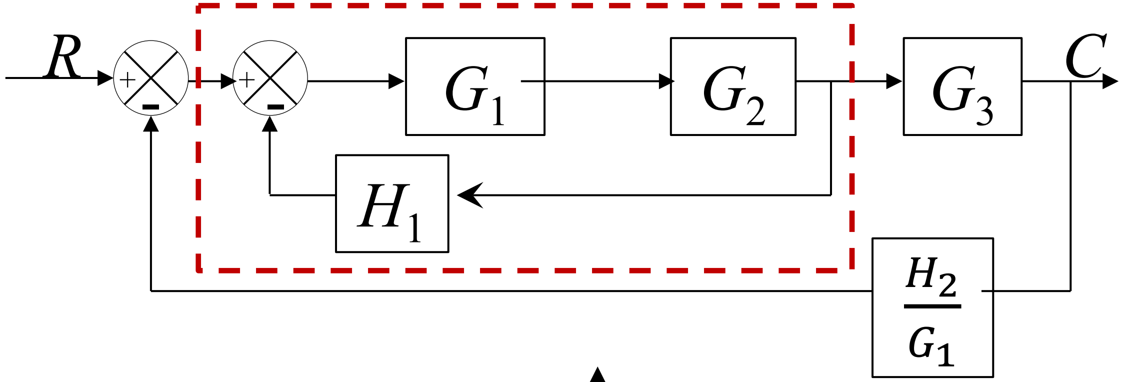
Example II -a

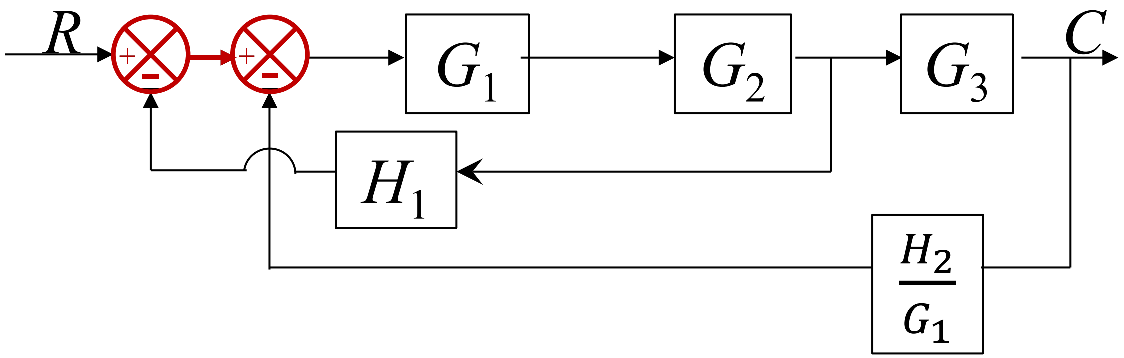
Simplify the system and find C/R



Example II -b

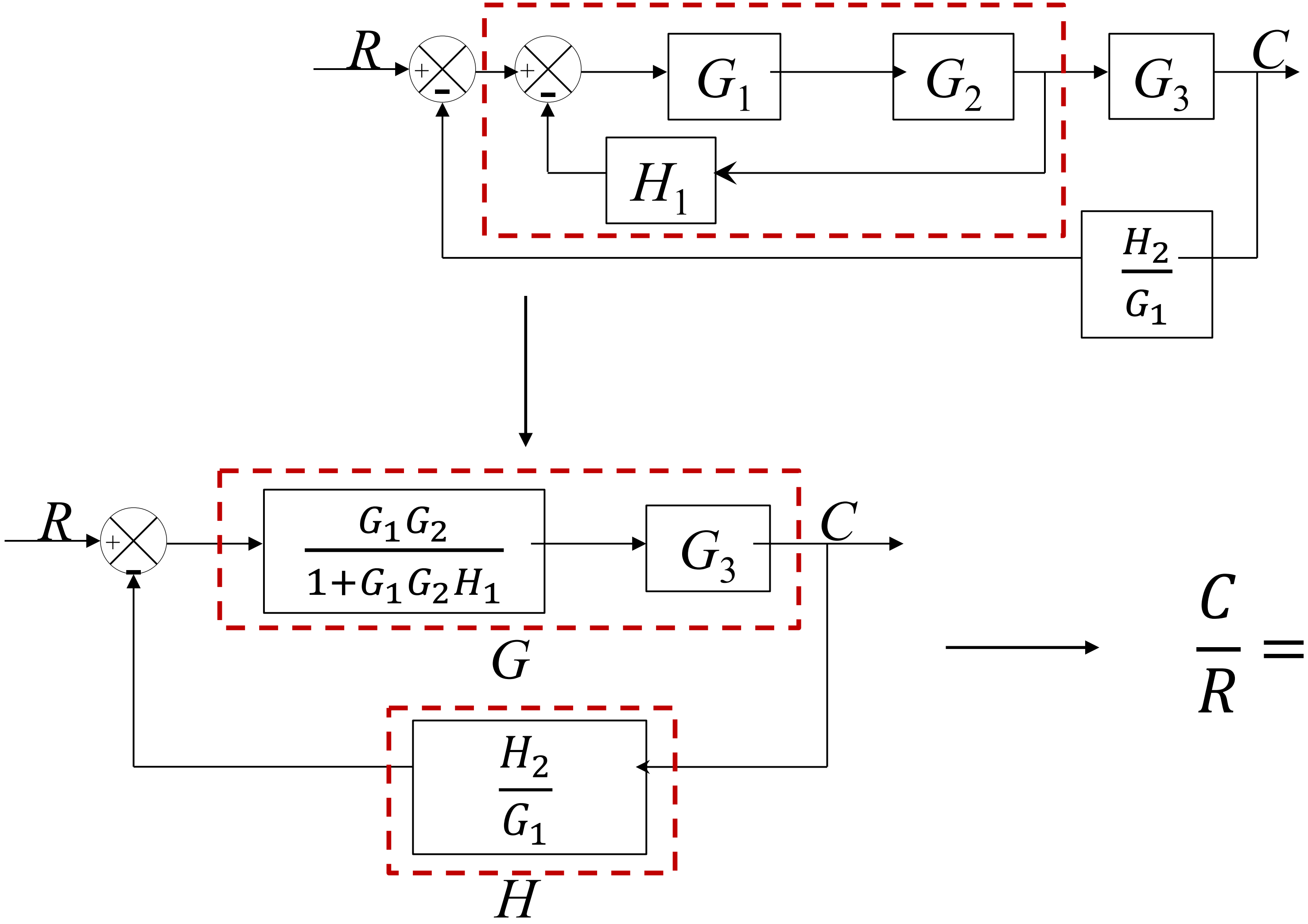
Simplify the system and find C/R





Example II - c

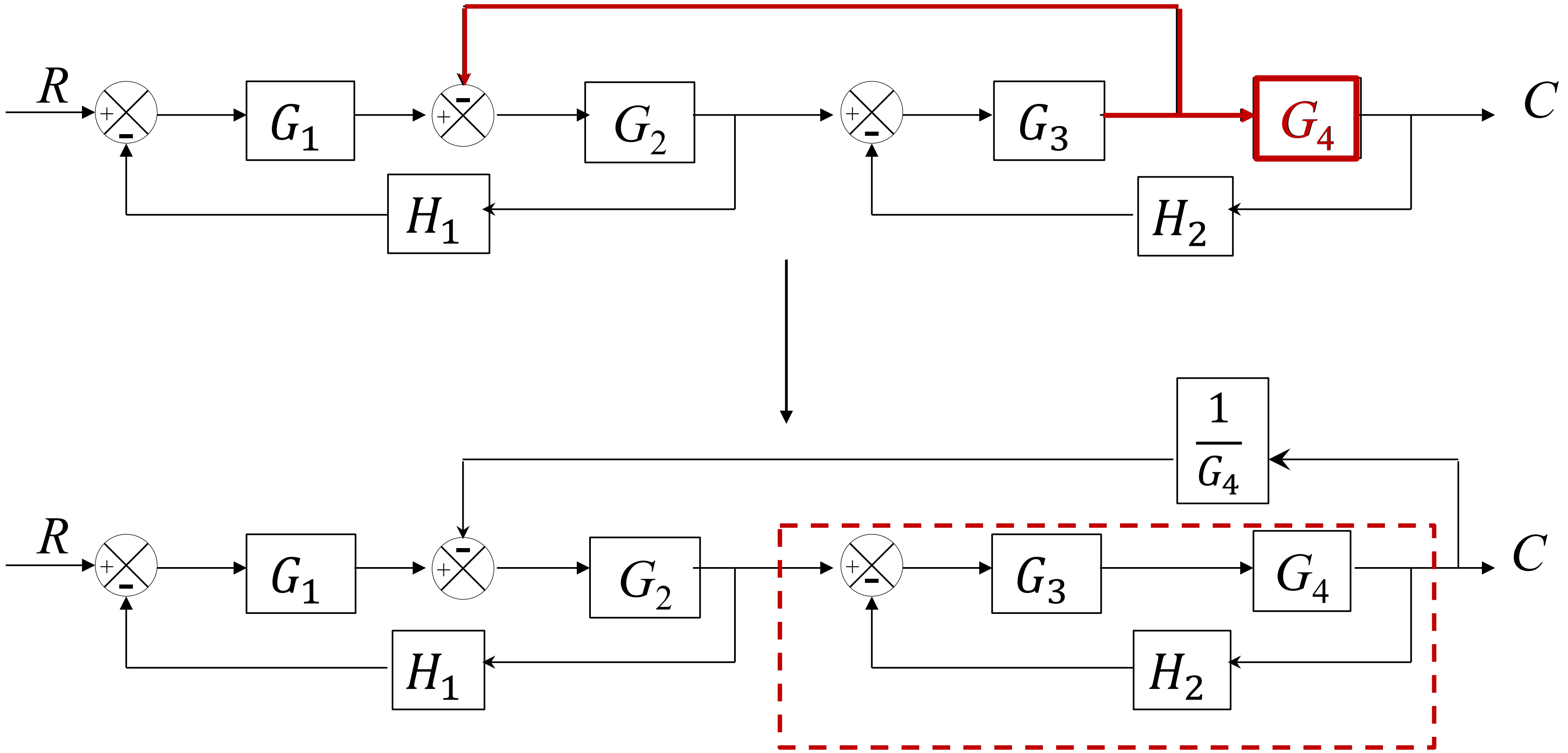
Simplify the system and find C/R



$$\frac{C}{R} = \frac{G}{1 + GH}$$

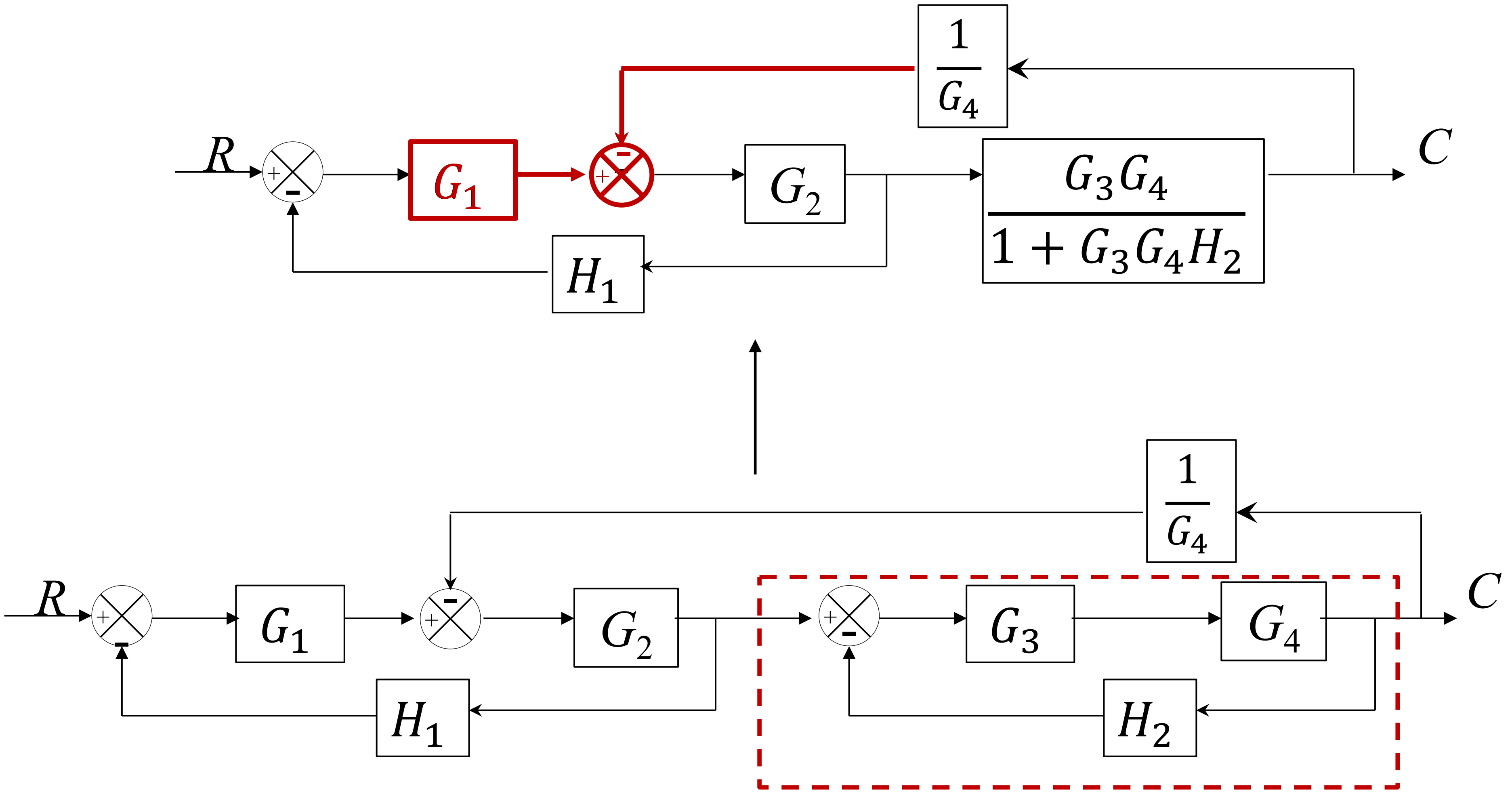
Example III - a

Simplify the system and find C/R



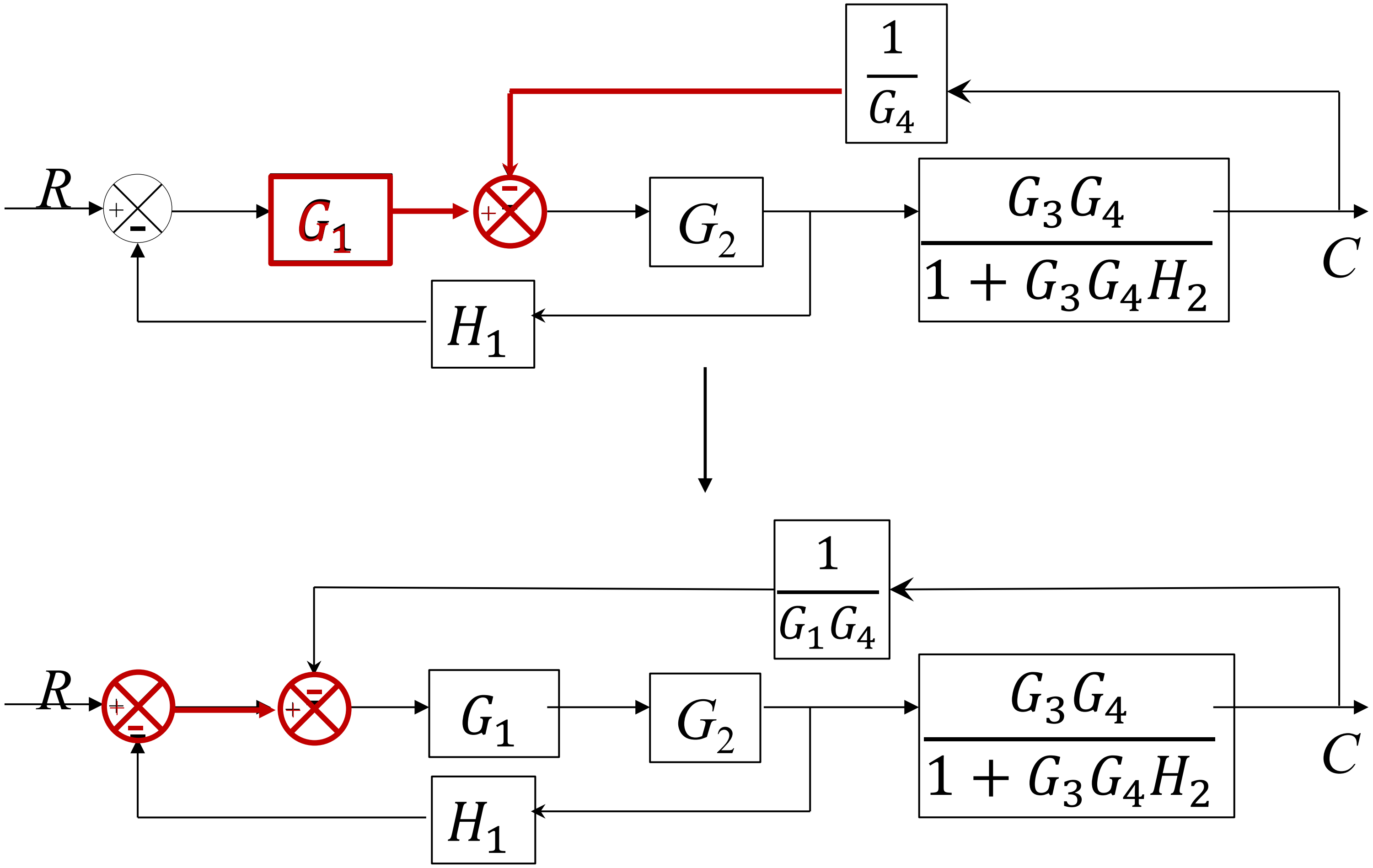
Example III - b

Simplify the system and find C/R



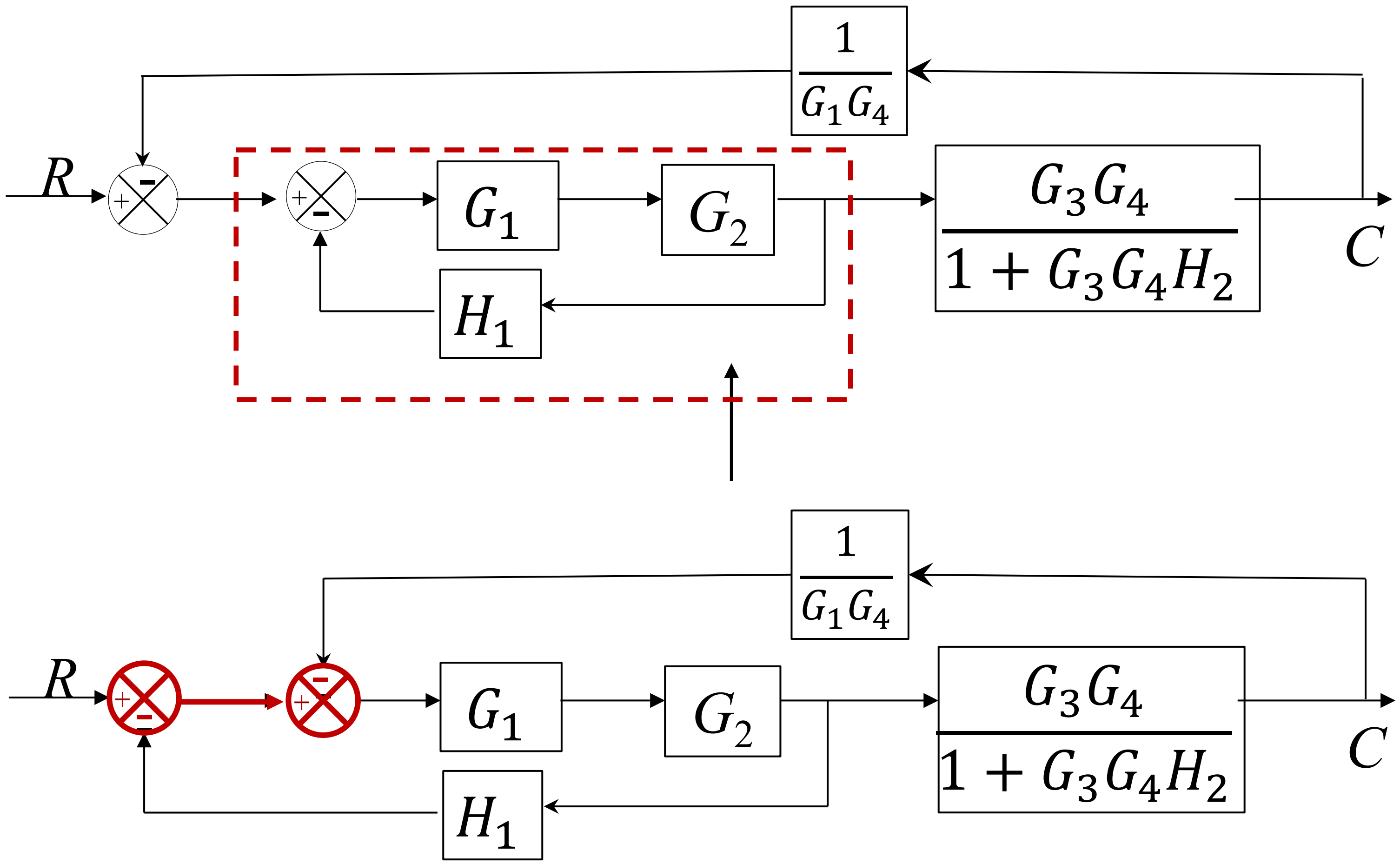
Example III - c

Simplify the system and find C/R



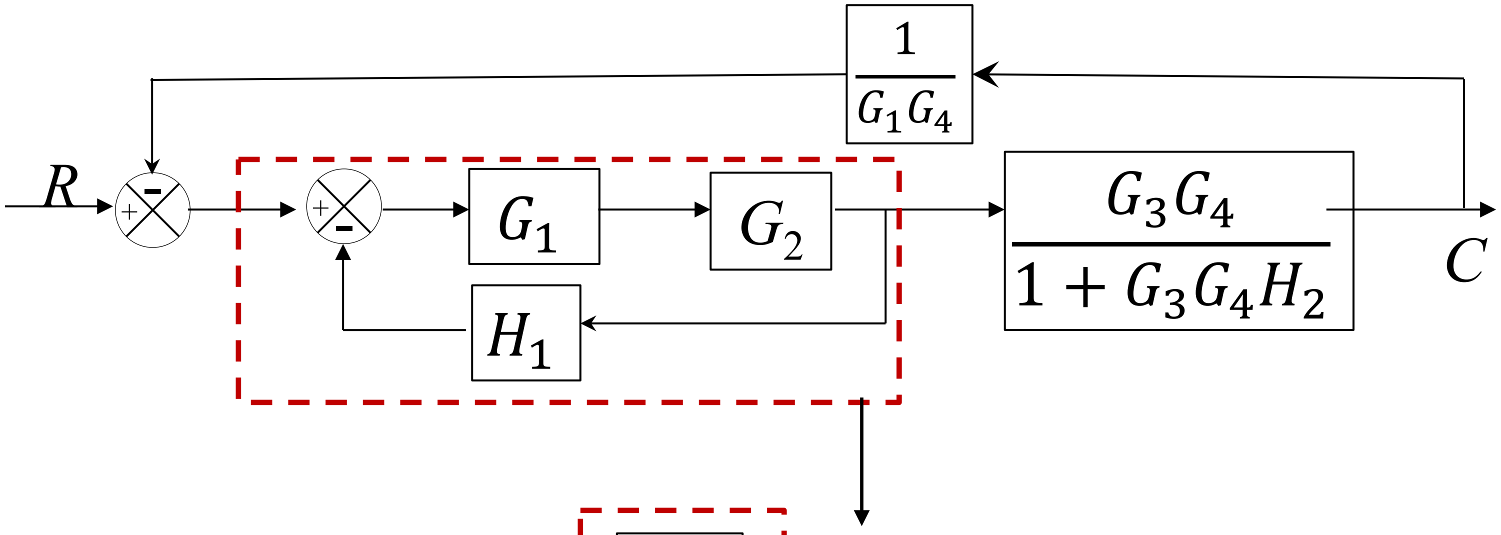
Example III - d

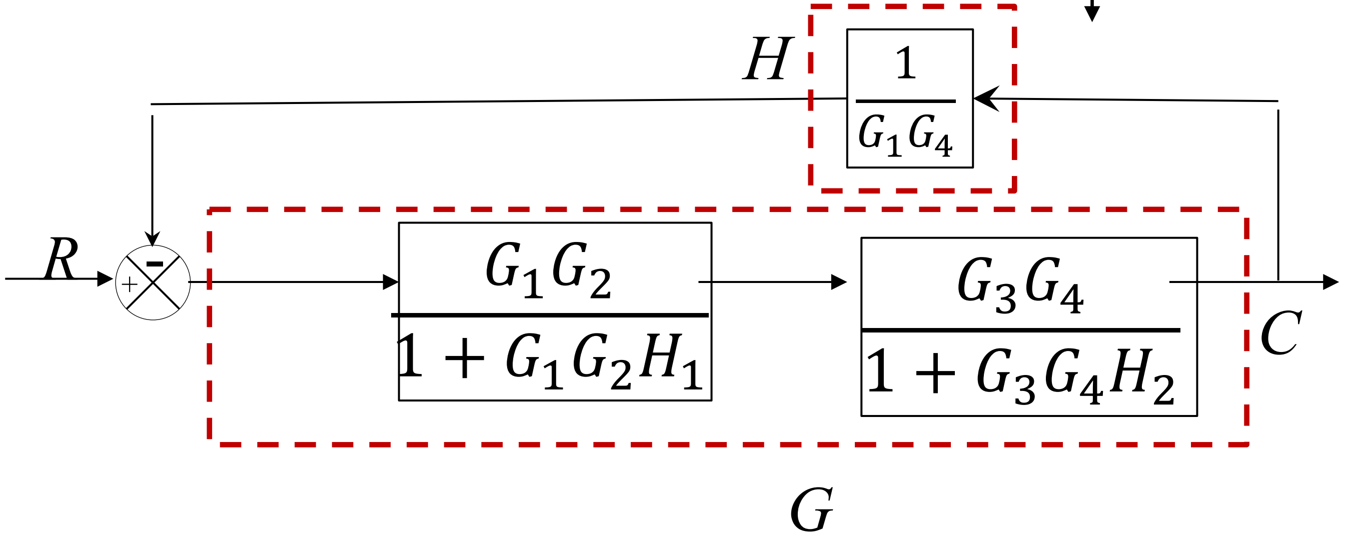
Simplify the system and find C/R



Example III - e

Simplify the system and find C/R

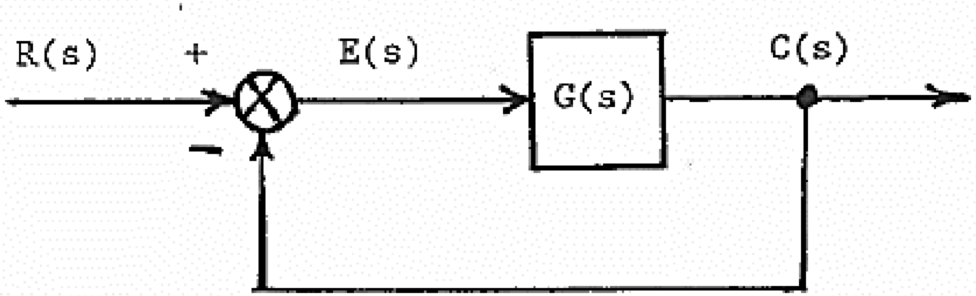




$$\frac{C}{R} = \frac{G}{1+GH}$$

Closed Loop Transfer Function

2.10.1 Unity Feedback



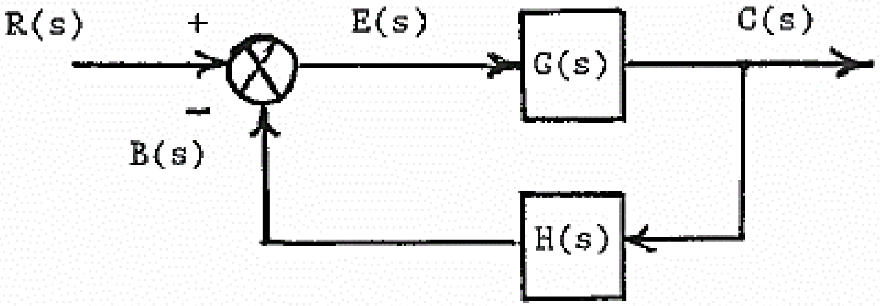
$$C(s) = G(s).\ E(s) \quad \text{-----------(1)}$$

$$E(s) = R(s) - C(s) \quad \text{-----------(2)}$$

$$\therefore\ C(s) = G(s)\ [R(s) - C(s)]$$

Rearranging : C/L T.F : $\frac{C(s)}{R(s)} = \frac{C(s)}{1 + C(s)}$

Non Unity Feedback



$$E(s) = R(s) - B(s)$$

$$B(s) = C(s)\ H(s)$$

$$C(s) = G(s)\ E(s)$$

$$\therefore\ C/L\ T.F = \frac{C(s)}{1 + G(s)\ H(s)}$$

G = direct T.F = Forward T.F

H = feedback T.F

GH = loop T.F = O/L T.F

C/R = C/L T.T

E/R = 1/1+GH (Actuating Signal Ratio)

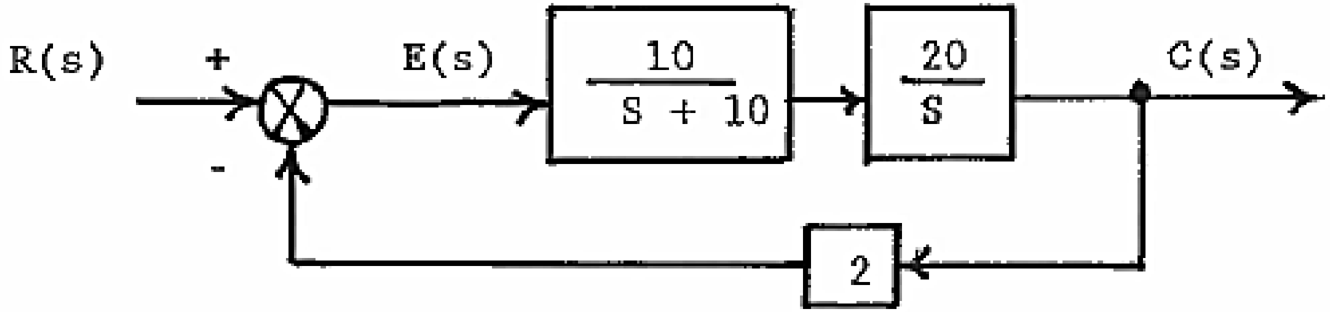
9. A negative feedback system has a forward path gain of 12 and a feedback path gain of 0.1. What is the overall gain of the system?

Using the equation derived above for the system gain:

$$\text{System gain} = \frac{G}{1+GH} = \frac{12}{1+0.1\times 12} = 5.45$$

The overall gain is thus 5.45.

- Direct Transmission Gain : G(s)



$$G(s) = \frac{10}{(S+10)} \cdot \frac{20}{S} = \frac{200}{S(S+10)}$$

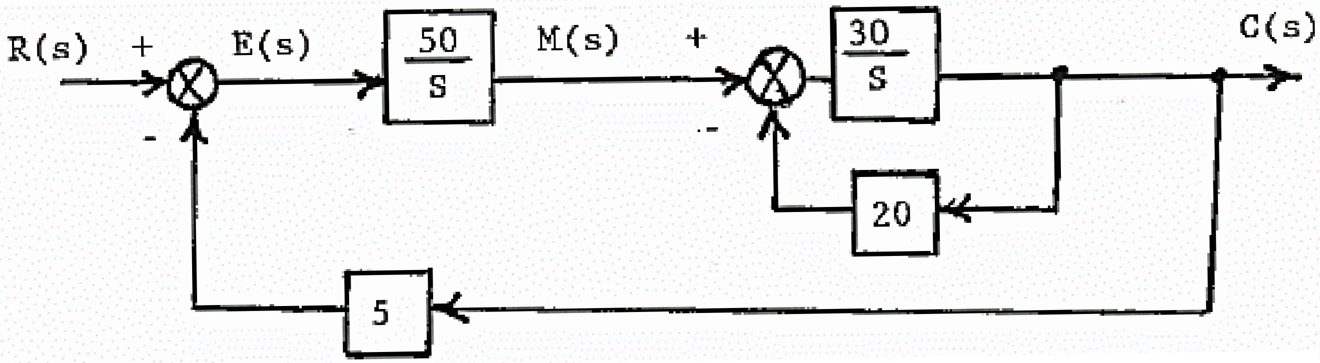
- O/L T.F: $GH(s) = \frac{200}{S(S+10)} \cdot 2 = \frac{400}{S(S+10)}$

- C/L T.F: $\frac{C}{R}(s) = \frac{G(s)}{1 + \text{O/L T.F}} = \frac{200}{S^2 + 10s + 400}$

- Characteristic Equations is : $S^2 + 10S + 400 = 0$

Reduced System:

$$R(s) \longrightarrow \boxed{\frac{200}{S^2 + 10S + 400}} \longrightarrow C(s)$$



- $G(s) = 1500/S(S+600)$

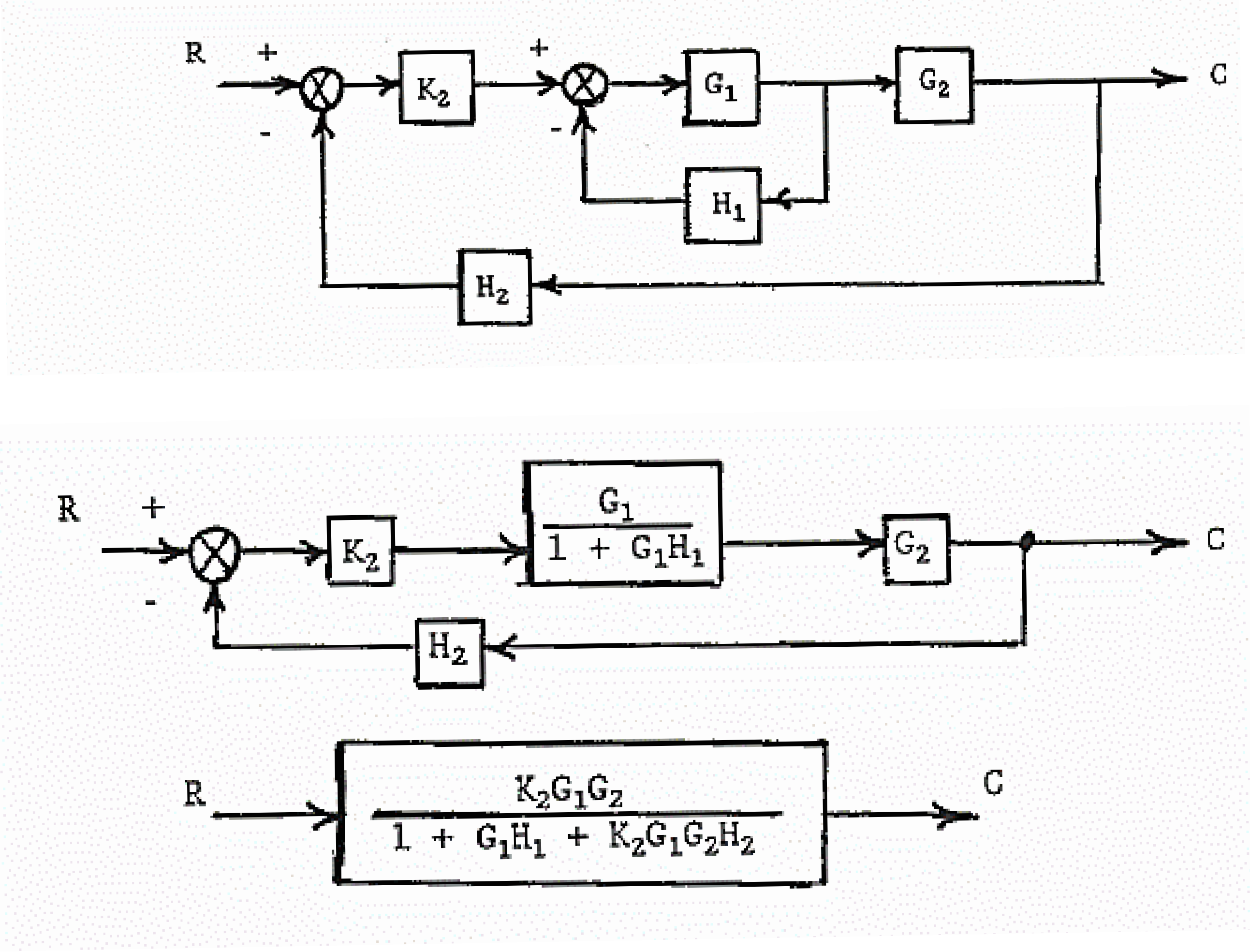
- O/L T.F: $GH(s) = 7500/S(S+600)$

- C/L T.F = $C/R\ (s) = 1500/(S^2 + 600S + 7500)$

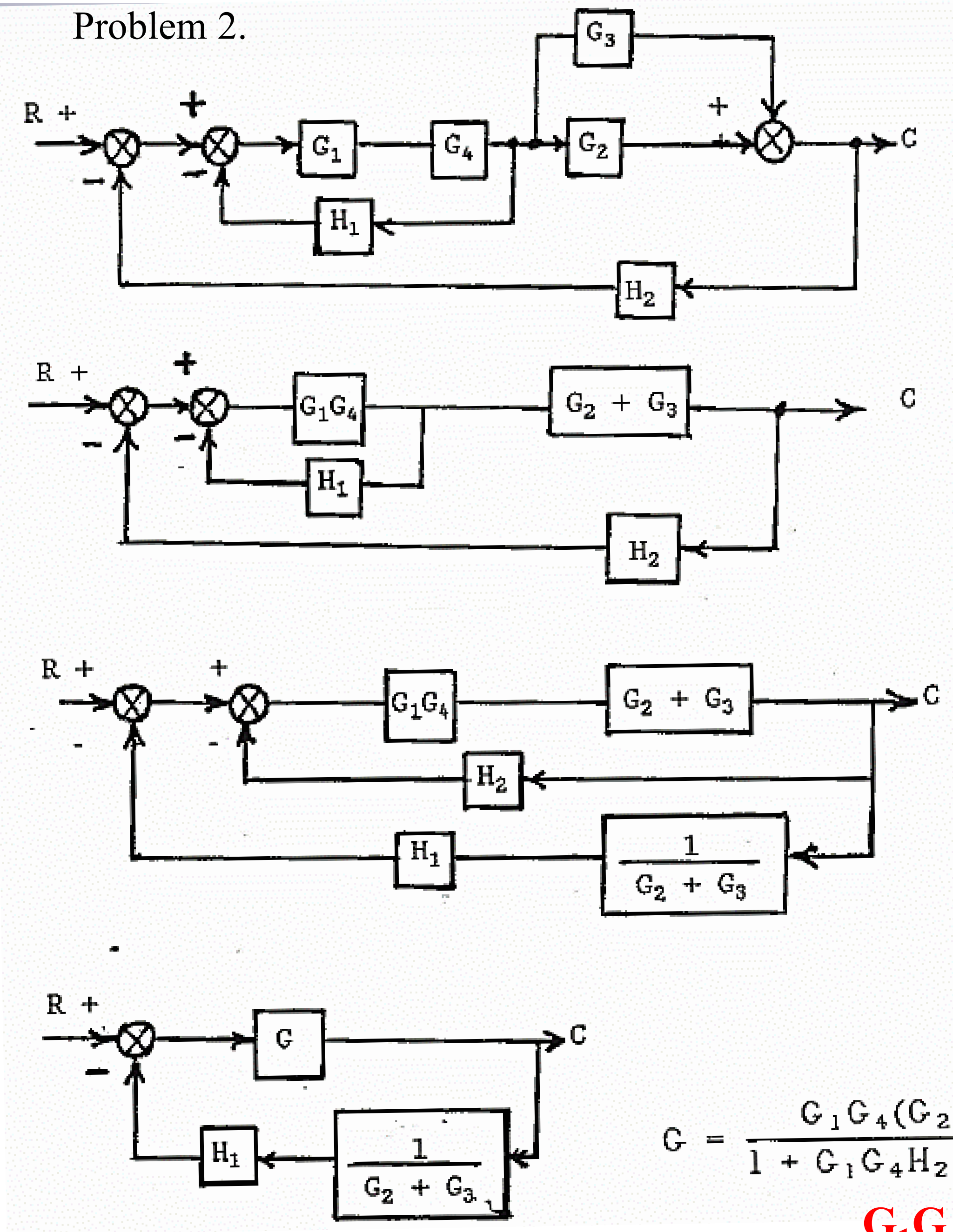
- Characteristic Equation : $S^2 + 600S + 7500 = 0$

Problem1.

Simplify The Block Diagram



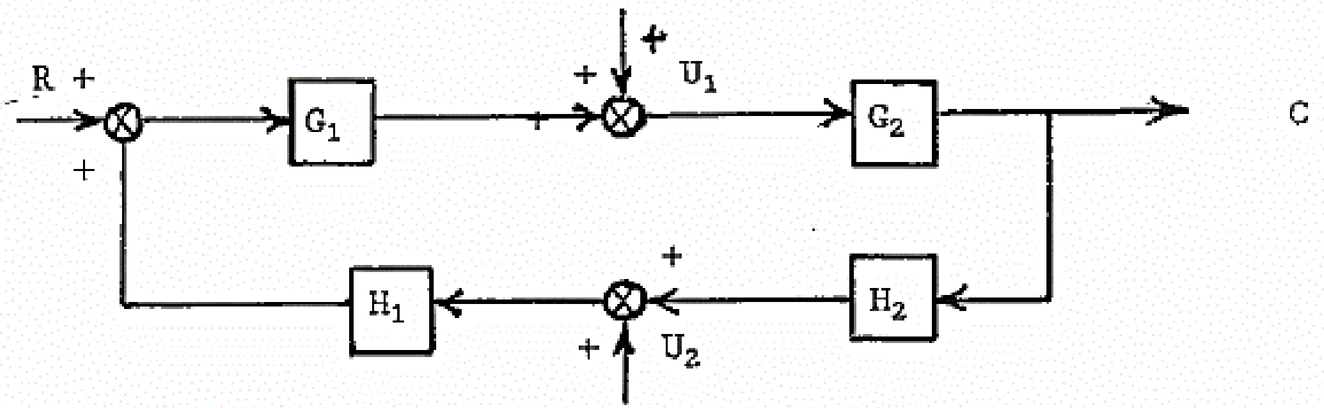
Problem 2.



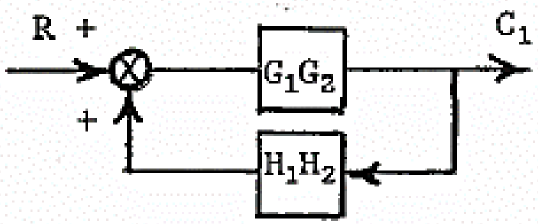
$$G = \frac{G_1 G_4 (G_2 + G_3)}{1 + G_1 G_4 H_2 (G_2 + G_3)}$$

$G_1 G_4 H_1$

A2.5 Find the output in following block diagram having 3 inputs, R, U_1 & U_2

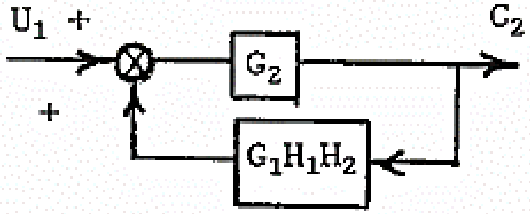


R only:



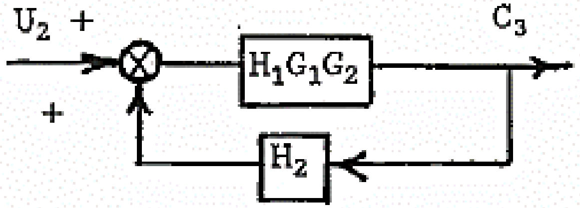
$$: \; C_1 = R \cdot \frac{G_1 G_2}{1 - G_1 G_2 H_1 H_2}$$

U_1 only:



$$: \; C_2 = U_1 \cdot \frac{G_2}{1 - G_1 G_2 H_1 H_2}$$

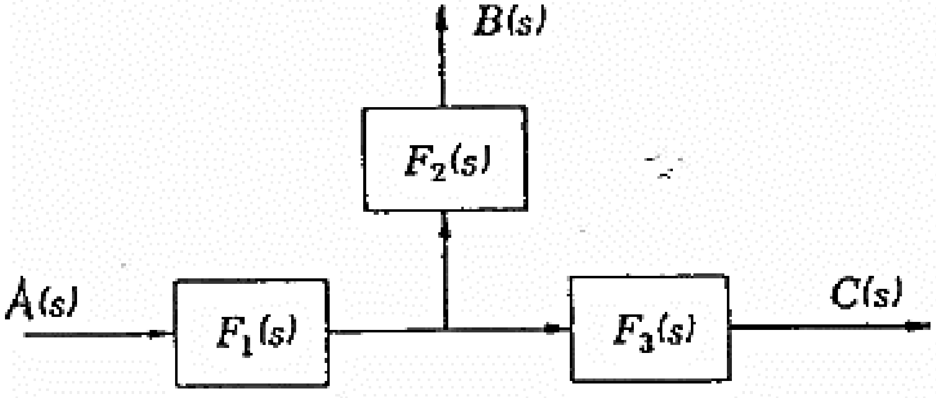
U_2 only:



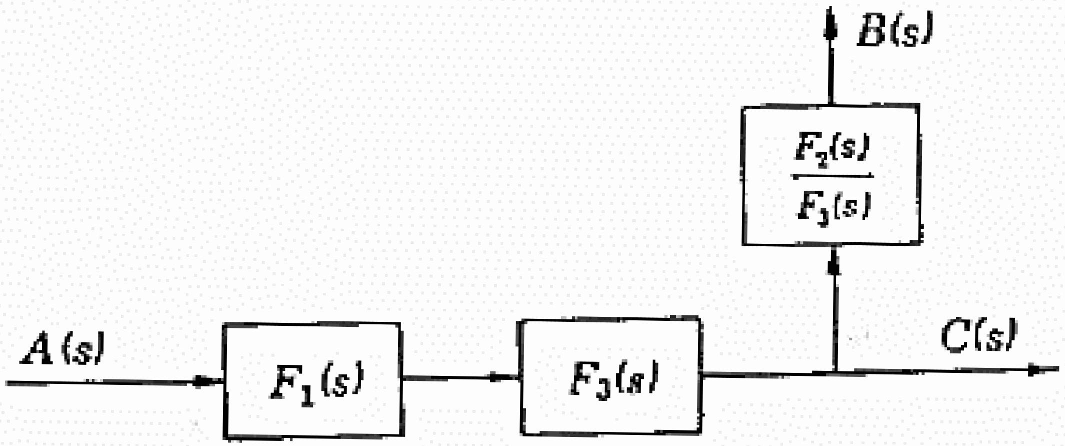
$$: \; C_3 = U_2 \cdot \frac{H_1 G_1 G_2}{1 - G_1 G_2 H_1 H_2}$$

$$\therefore \; C = \frac{R\, G_1 G_2 + U_1 G_2 + U_2 H_1 G_1 G_2}{1 - G_1 G_2 H_1 H_2}$$

A2.6 Moving a take-off point around a block



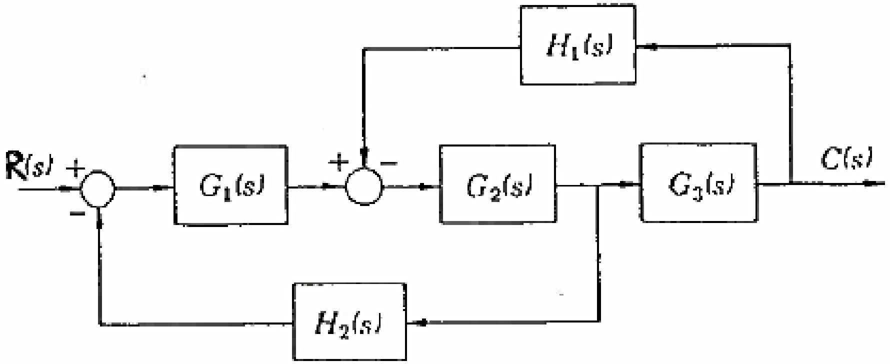
(a)



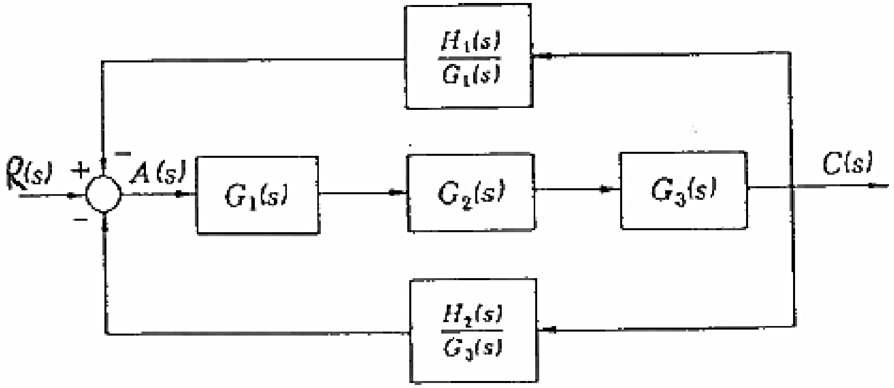
AHE/smc(Misc.1:AHE(2))

(b)

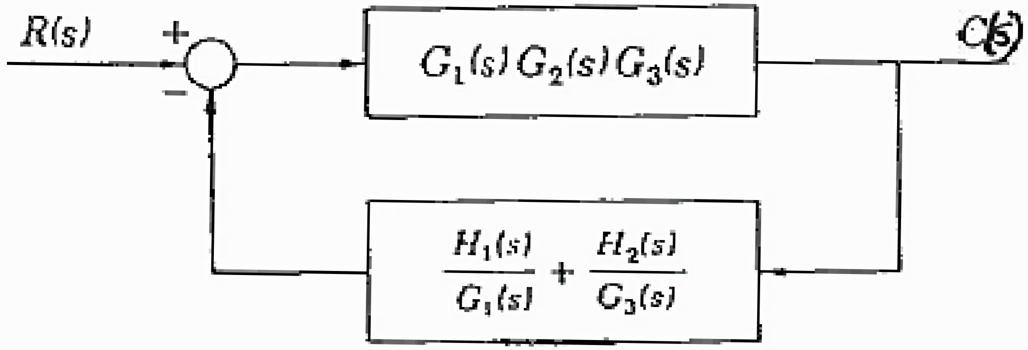
A2.7 Block Manipulation

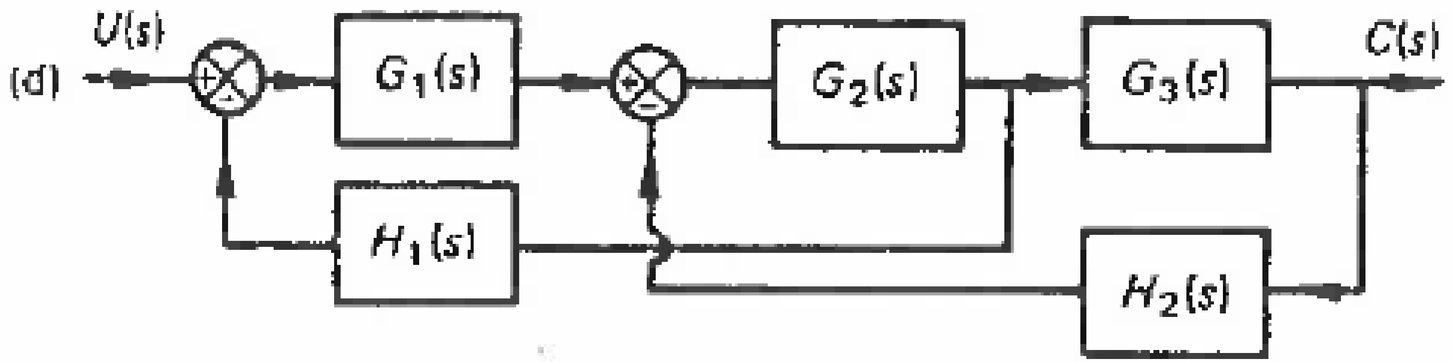


(a)

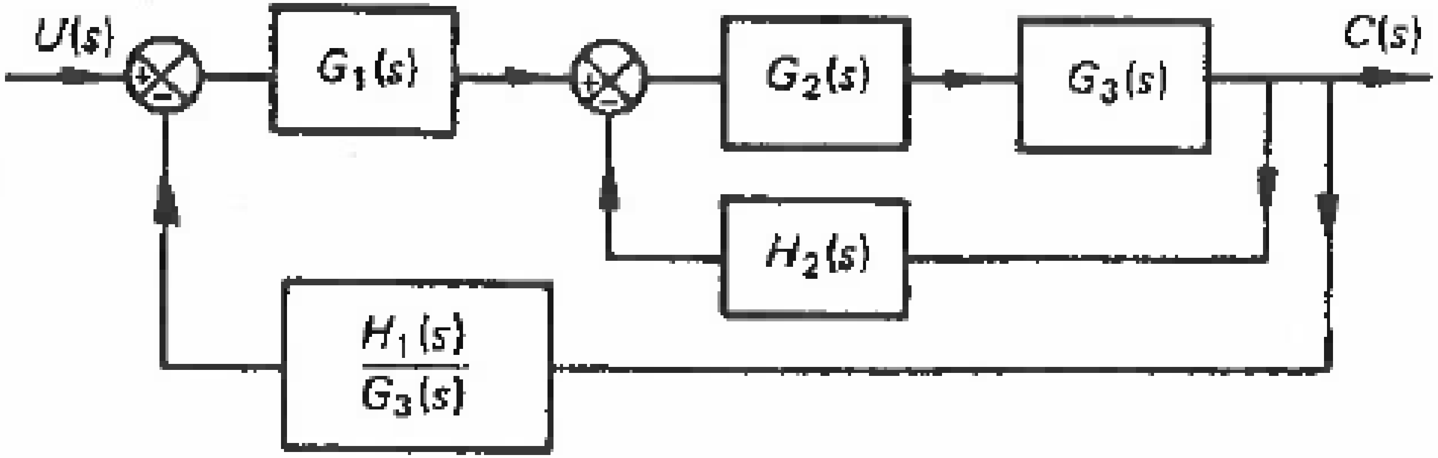


(b)

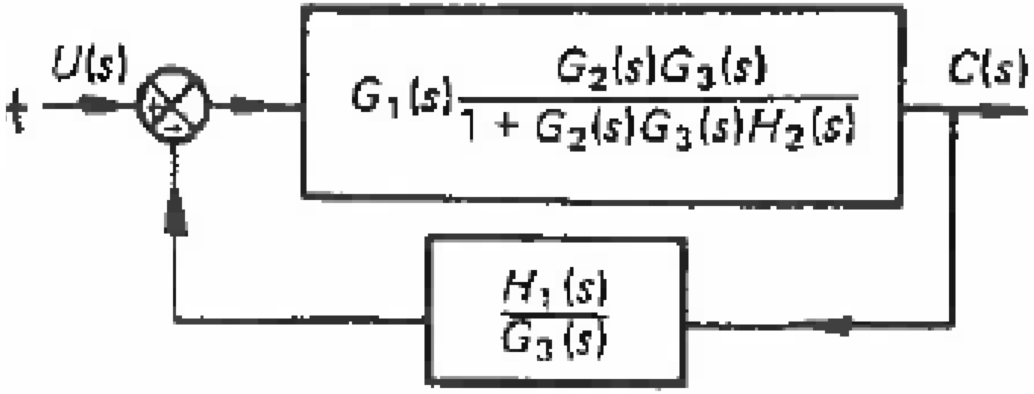




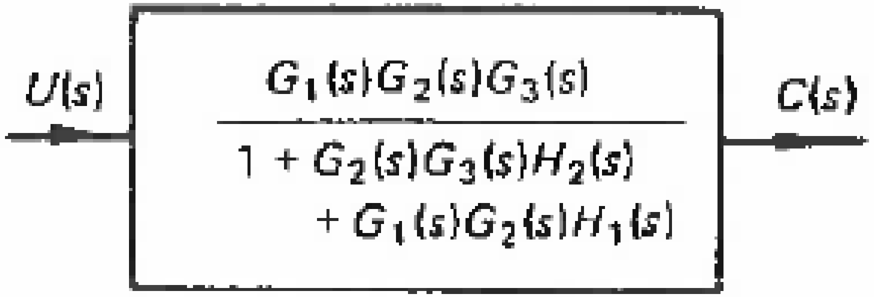
can be rearranged thus to avoid the interlinking loops

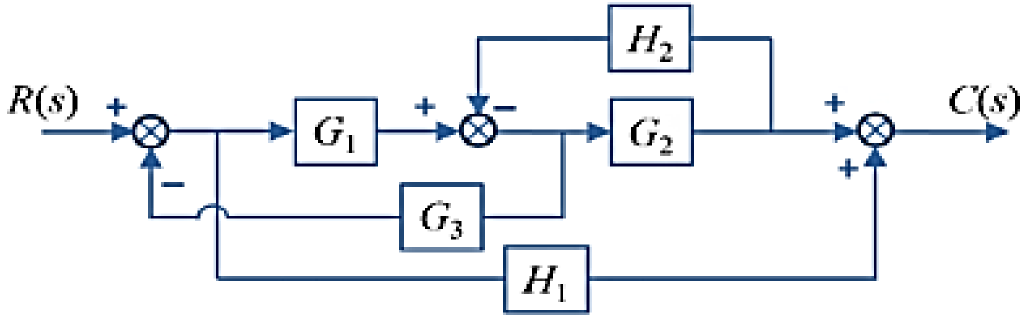


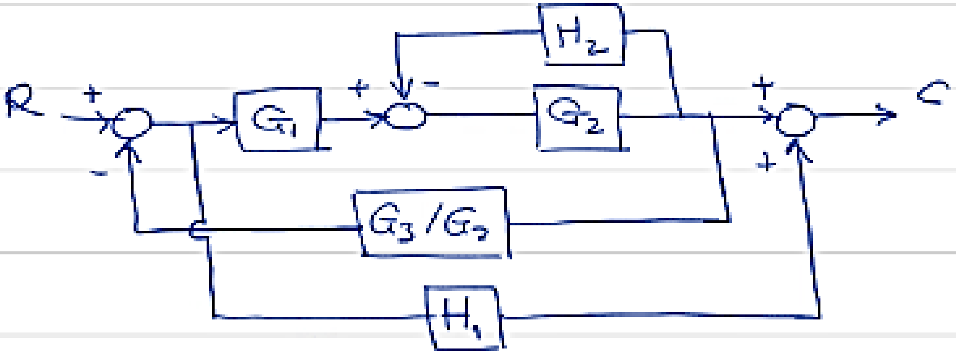
Which is equivalent to

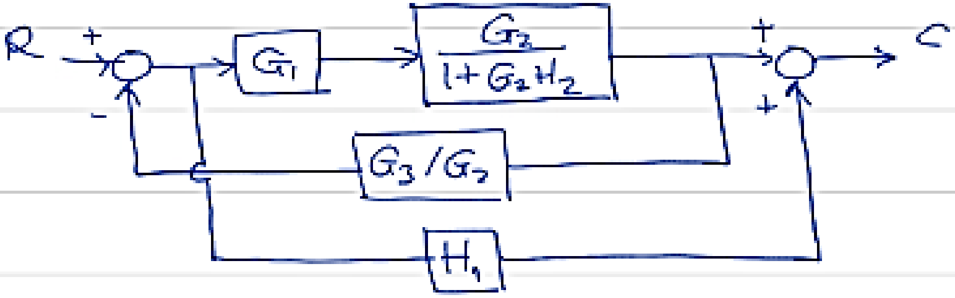


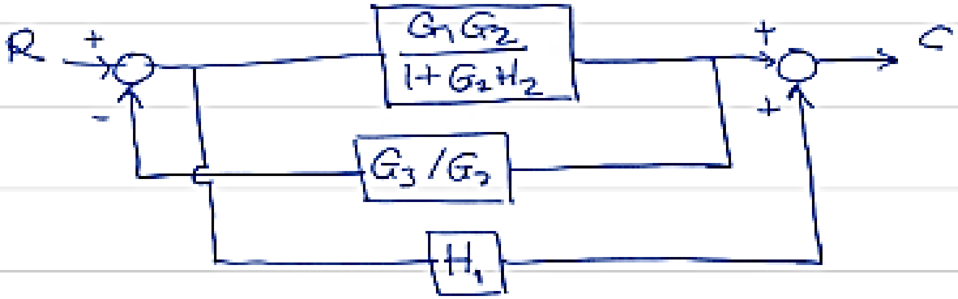
or

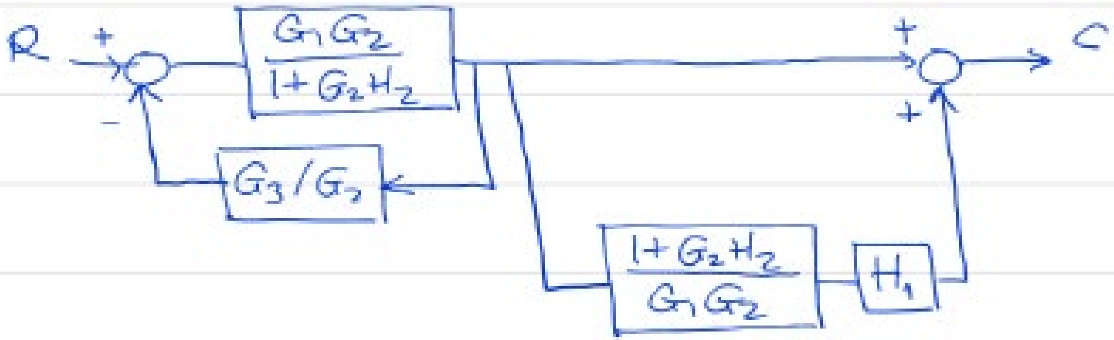


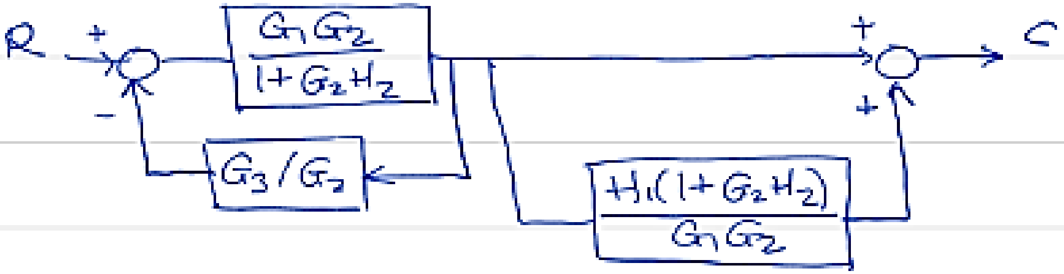

R(s)
C(s)
G1
G2
G3
H1
H2

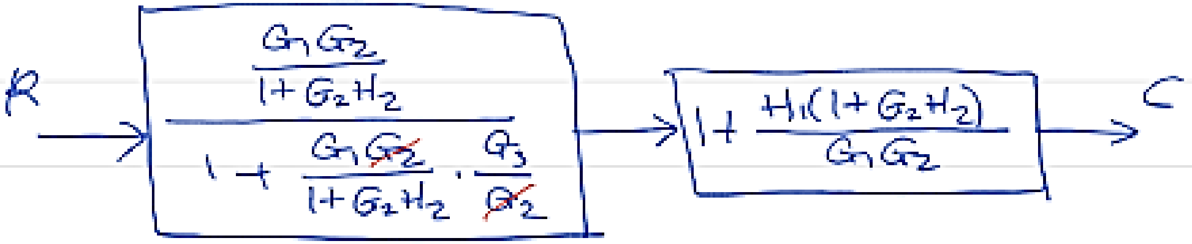

R
C
G1
G2
H2
G3 / G2
H1

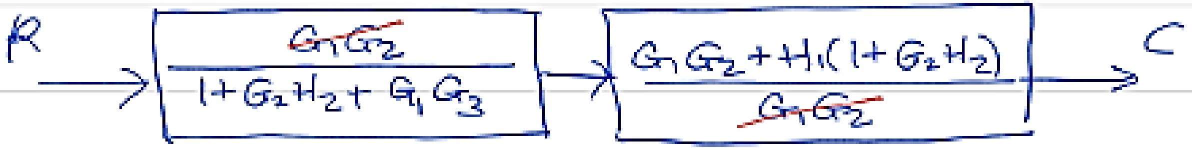

R
C
G1
G2 / (1+G2H2)
G3 / G2
H1

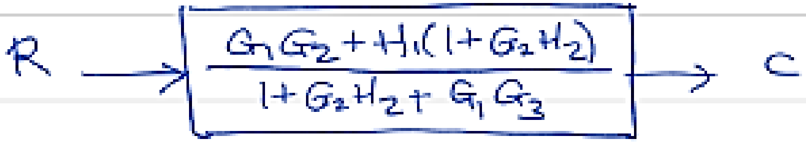

R
C
G1G2 / (1+G2H2)
G3 / G2
H1

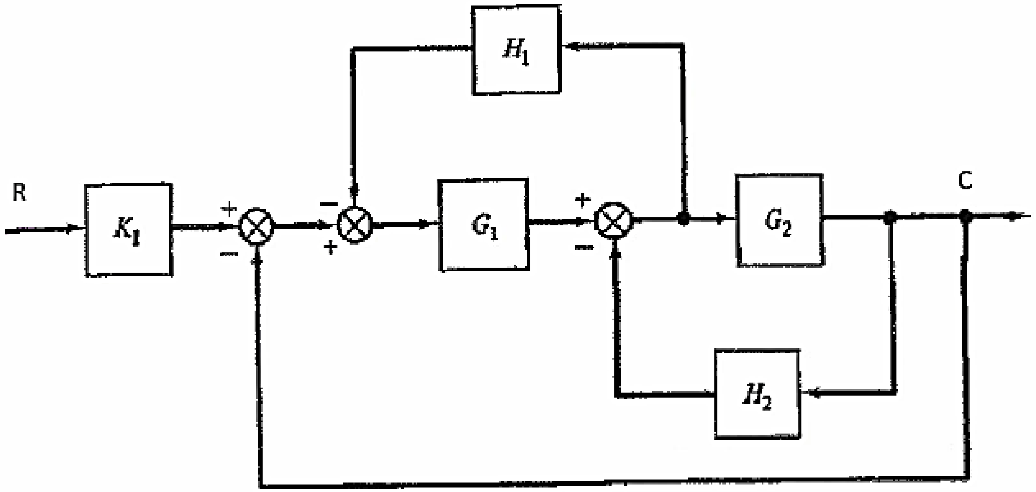

R
C
G1G2 / (1+G2H2)
G3 / G2
(1+G2H2) / G1G2
H1


R
C
G1G2 / (1+G2H2)
G3 / G2
H1(1+G2H2) / G1G2

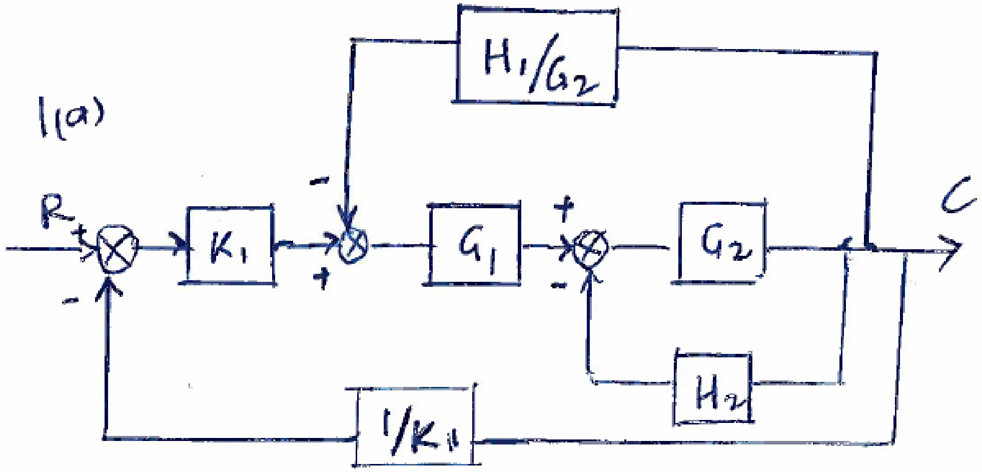

R
C
[G1G2 / (1+G2H2)] / [1 + G1G2/(1+G2H2) · G3/G2]
1 + H1(1+G2H2) / G1G2

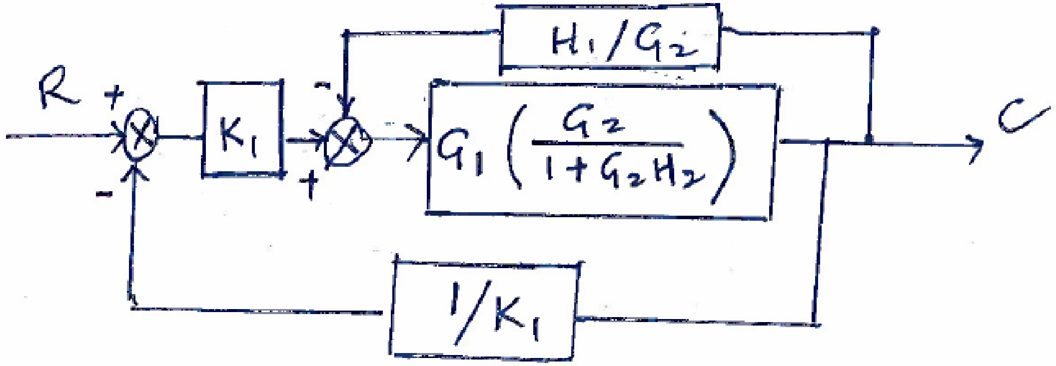

R
C
G1G2 / (1+G2H2+G1G3)
(G1G2+H1(1+G2H2)) / G1G2

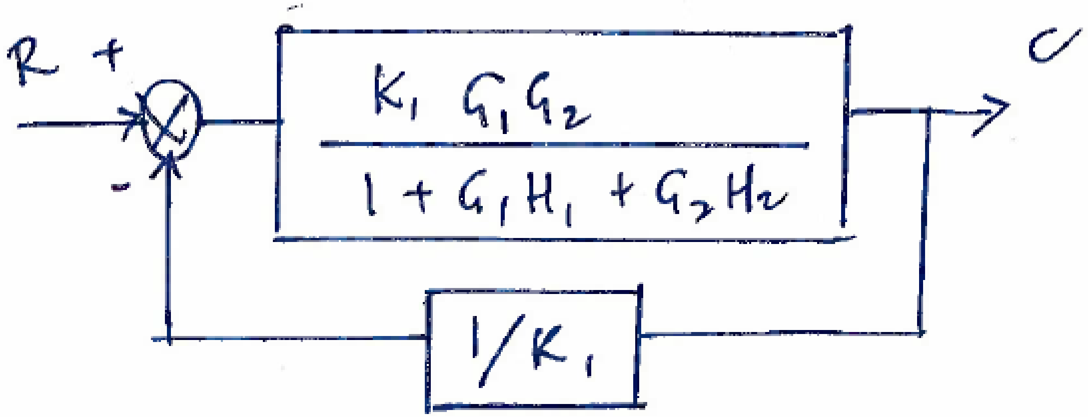

R
C
(G1G2+H1(1+G2H2)) / (1+G2H2+G1G3)



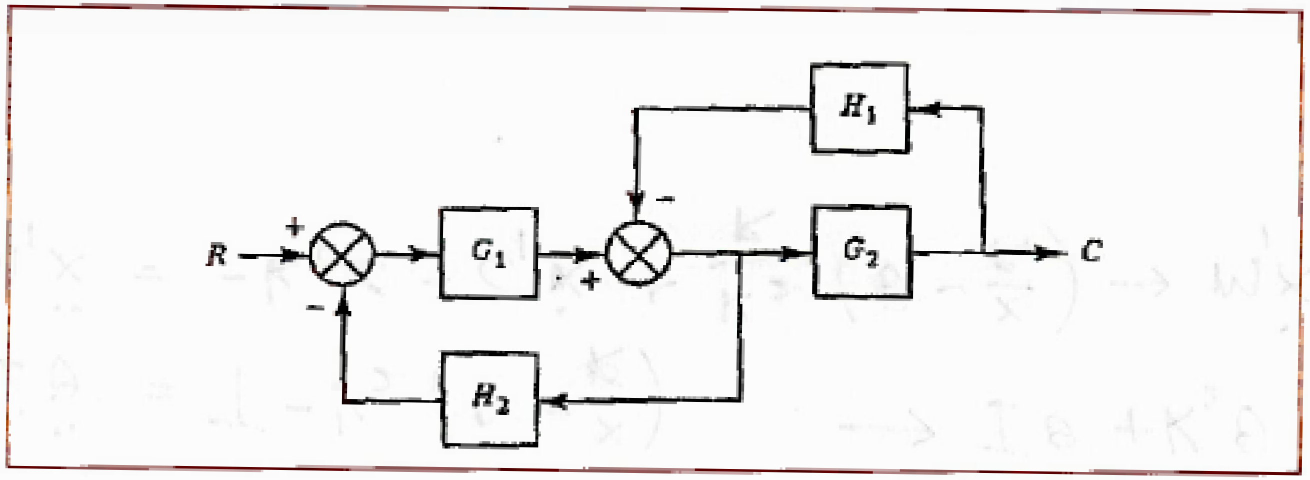
1(a)

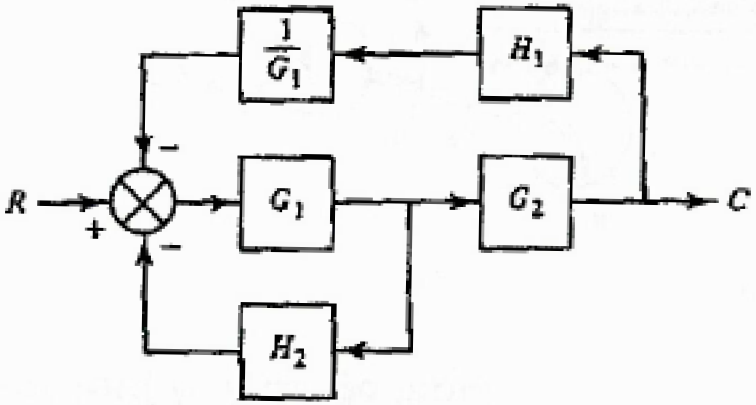




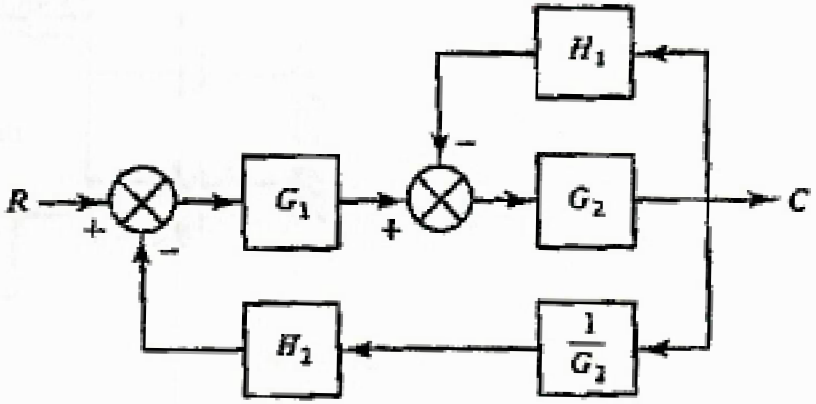


$$\frac{C}{R} = \frac{K_1 G_1 G_2}{1 + G_1 H_1 + G_2 H_2 + G_1 G_2}$$





(a)



(b)

closed-loop transfer function

$$\frac{C}{R} = \frac{G_1G_2}{1 + G_1H_2 + G_2H_1}$$

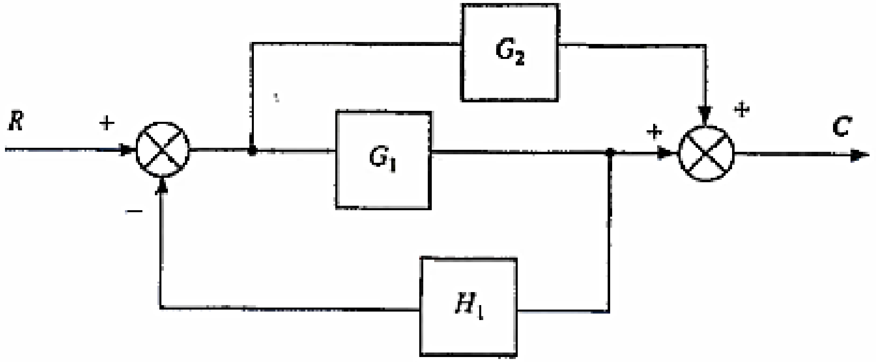
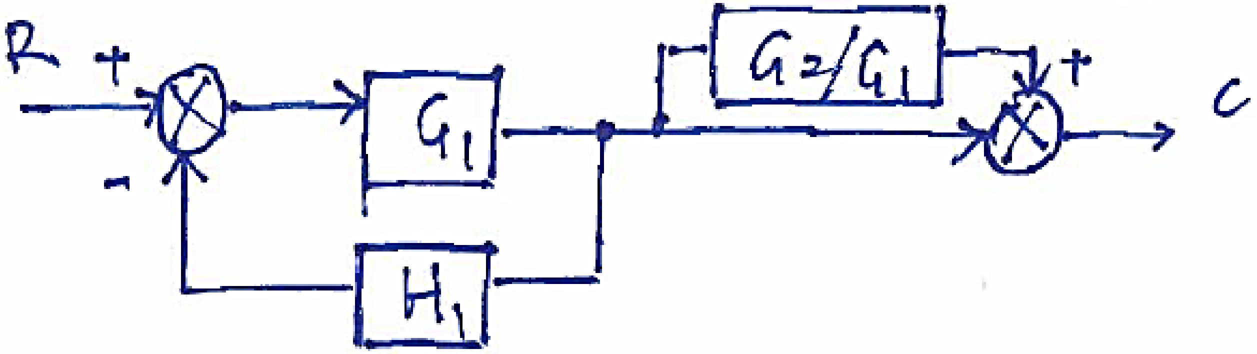
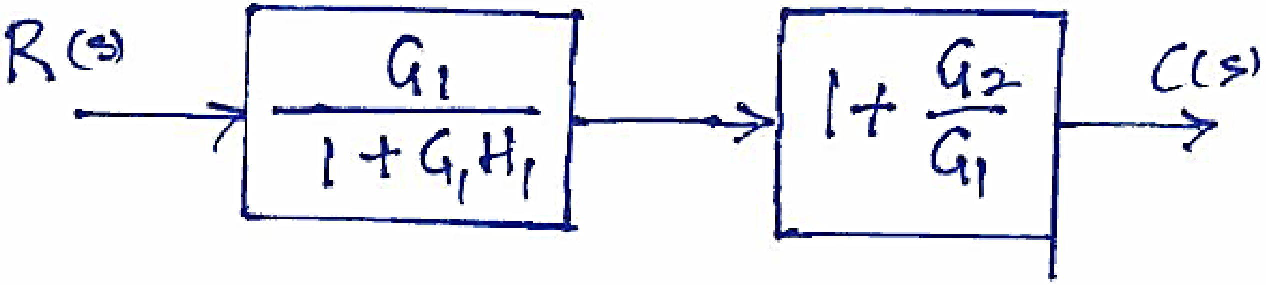
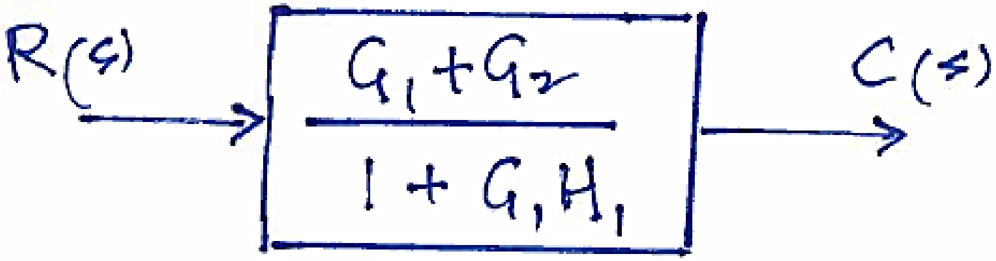
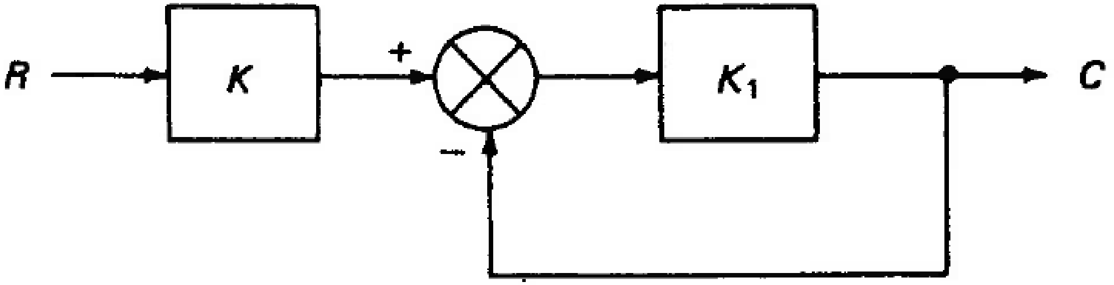


Fig 1

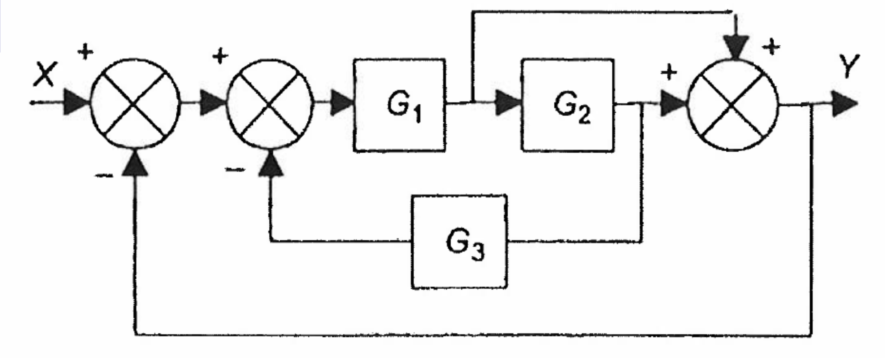


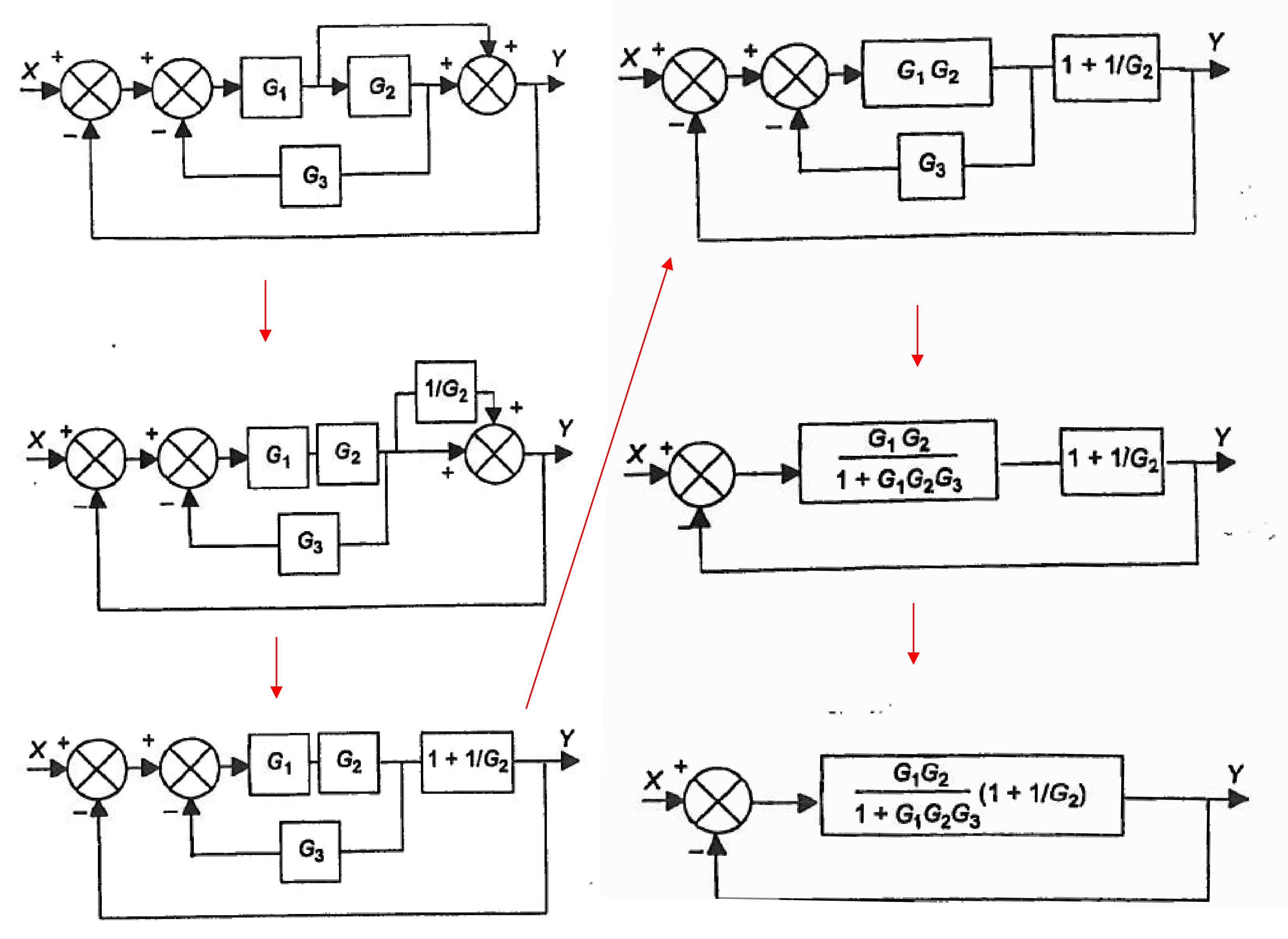




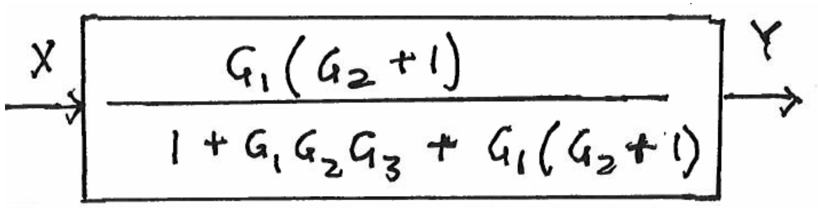


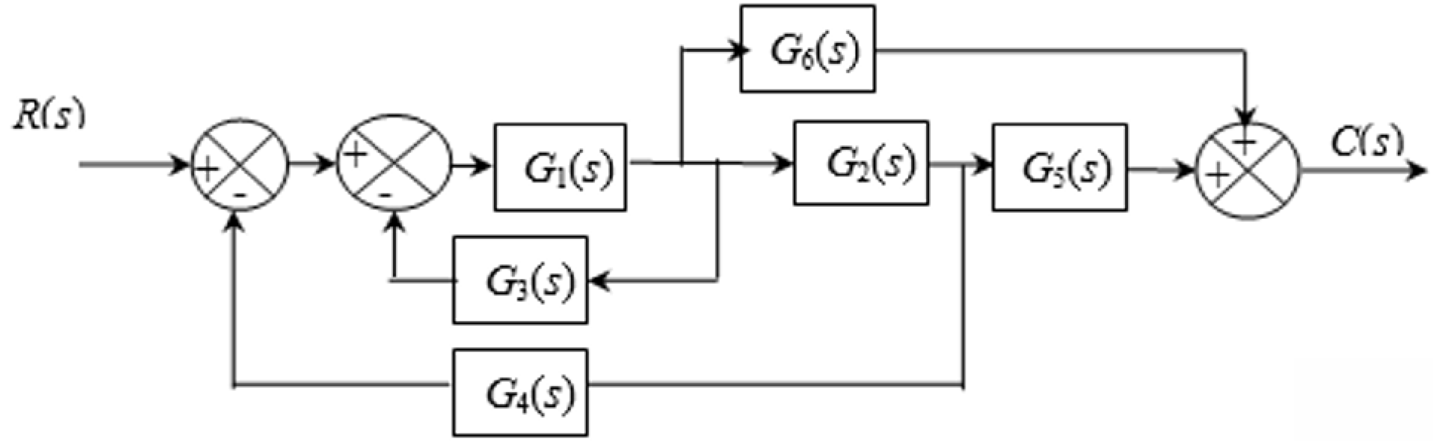
$$\frac{C}{R} = K\left[\frac{K1}{1+K1}\right]$$

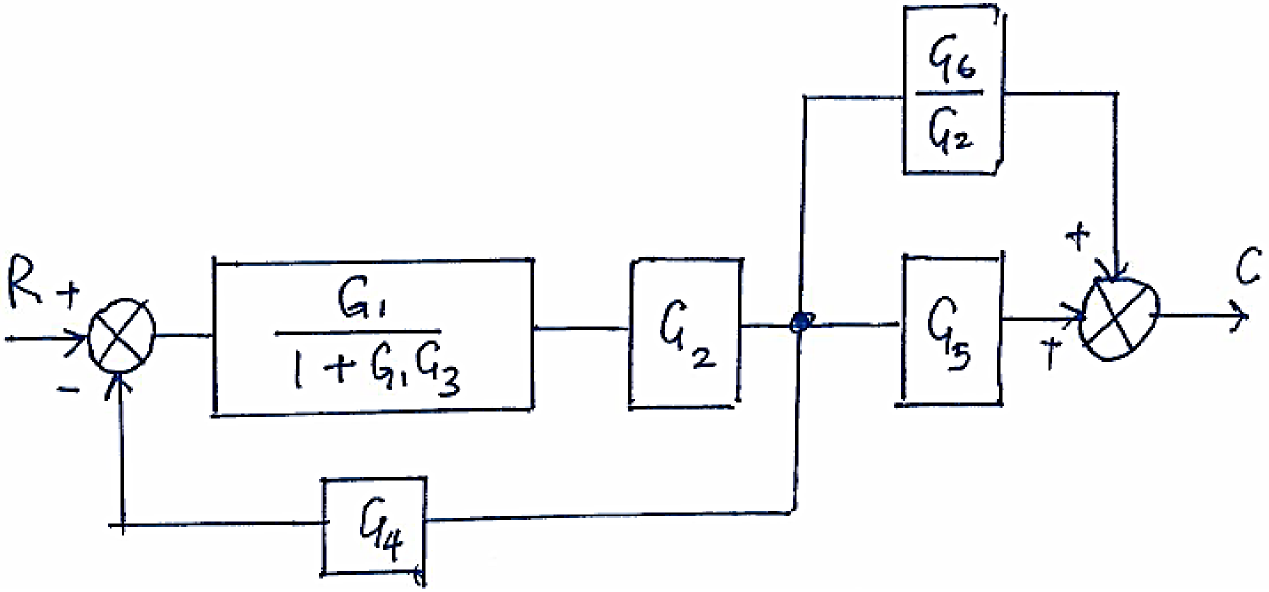




Transfer function of output to input : $\frac{Y}{X}$



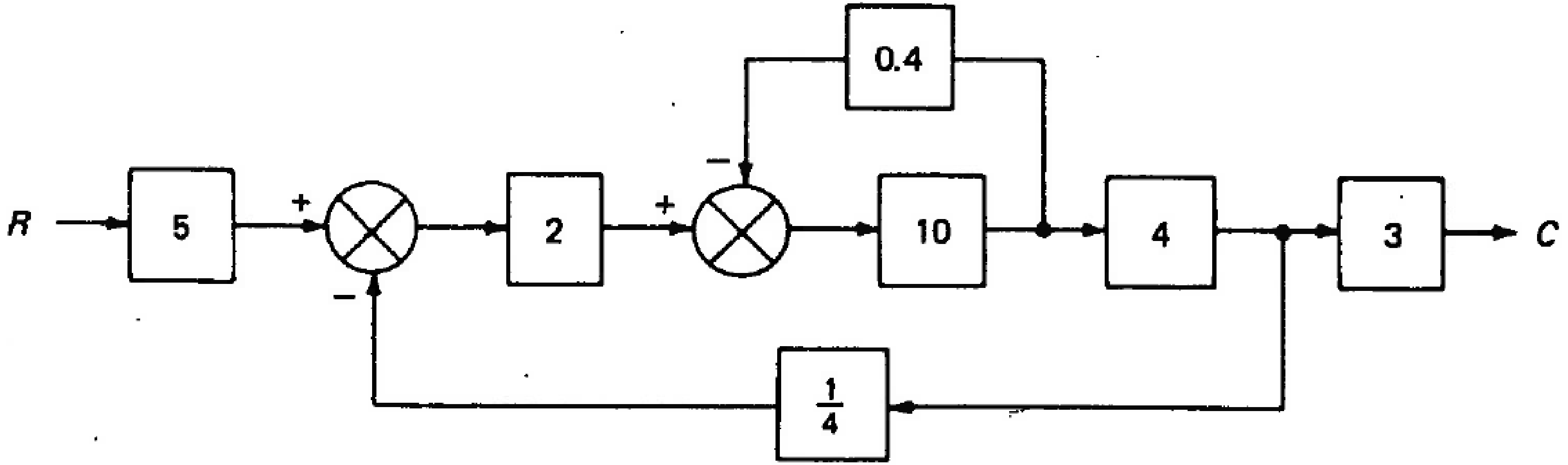


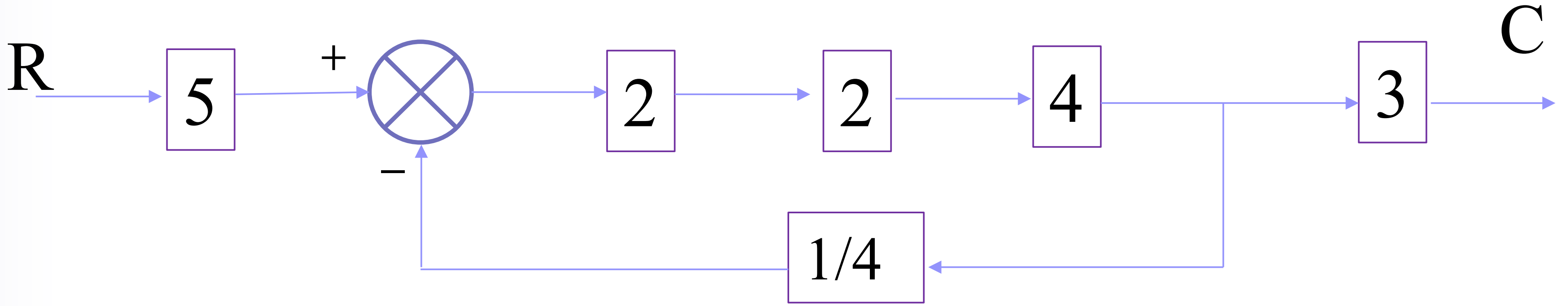


$$\frac{C}{R} = \left[\frac{\dfrac{G_1G_2}{1+G_1G_3}}{1+\dfrac{G_1G_2G_4}{1+G_1G_3}}\right]\left[G_5+\frac{G_6}{G_2}\right]$$

$$= \left[\frac{G_1G_2}{1+G_1G_3+G_1G_2G_4}\right]\left[\frac{G_2G_5+G_6}{G_2}\right]$$

$$\frac{C}{R} = \frac{G_1G_2G_5+G_1G_6}{1+G_1G_3+G_1G_2G_4}$$





$$\frac{C}{R} = 5 \times 16/5 \times 3 = 48$$

Summary

- Analysis of block diagrams
  - Manipulate control signals and transfer functions
  - Establish basic rules
  - Simplify the block diagram
- Concepts are demonstrated via examples